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THE THEORY

OF

NAUTICAL ASTRONOMY

AND

NAVIGATION.

PART II.

CONTAINING THE INVESTIGATIONS AND PROOFS OF THE PRINCIPAL RULES AND CORRECTIONS.

With Practical Examples.

Designed for Beginners and advanced Students.

By H. W. JEANS, F.R.A.S.

LATE MATHEMATICAL MASTER AND EXAMINER AT THE ROYAL NAVAL COLLEGE, PORTSMOUTH;
AUTHOR OF A WORK ON
"PLANE AND SPHERICAL TRIGONOMETRY;" "HANDBOOK OF THE STARS;" "PROBLEMS IN ASTRONOMY, NAVIGATION, ETC. WITH SOLUTIONS:"
FORMERLY MATHEMATICAL MASTER IN THE ROYAL MILITARY ACADEMY, WOOLWICH; AND AN EXAMINER OF OFFICERS IN THE MERCHANT-SERVICE IN NAUTICAL ASTRONOMY, ETC.

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PREFACE.

The First Part of this Work — consisting of Practical Rules in Navigation and Nautical Astronomy, with a series of examples under each rule—was originally drawn up for the use of beginners, and as introductory to some of the larger works on the subject; but recent additions render it complete in itself, and it will now be found to contain ample directions for the guidance of the practical navigator. The rules are adapted to any of the standard Nautical Tables now in use, such as Norie's, Raper's, or Riddle's; but as the collection of Tables published by the late Dr. Inman* is almost universally adopted in the Royal Navy, these tables have in most cases been used in working out the examples. The present volume, Part II., may be considered as the scientific part of the subject, consisting of the analytical investigations and proofs of the principal rules and corrections in Navigation and Nautical Astronomy. The Author trusts that the

* This venerable and most useful public servant died on the 8th of February 1859, at his residence at Southsea near Portsmouth, at the advanced age of eighty-three years. The Reverend James Inman, D.D., was upwards of thirty years Professor of Mathematics at the Naval College. He was the oldest of Cambridge senior wranglers, and long possessed a just celebrity in naval circles for his application of science to navigation and ship-building. He laboured very many years unobtrusively but zealously in his country's service. He sailed round the world, having been appointed to the expedition under Flinders as astronomer; was wrecked with him; and returning to Europe in the fleet of East-India ships under the command of Commodore Dance, was present in that celebrated action in which a fleet of merchantmen repulsed the French Admiral Linois. While Professor of Mathematics at the Royal Naval College he reduced to system the previously ill-arranged methods of navigation, and published several valuable works on the subject; but he was best known by his having been the first person in this country who built ships on scientific principles. Dr. Inman's translation of Chapman, with his valuable annotations, is the text-book on which all subsequent writers have proceeded. His pupils, a long list of distinguished naval officers, will remember him as a type of the high-minded scholar,—of the loyal, the truthful, and independent man.

plan followed by him in this Part, namely, to exhibit a geometrical figure or diagram of each problem, then to give the analytical investigation from which the rule is derived, followed by a numerical example worked out to show the application of the formula, together with nearly 300 examples for practice dispersed throughout the volume, will render the book useful to beginners (for whom, in fact, it is chiefly intended), as well as deserving the attention of more advanced students, and increasing the confidence of the practical navigator. The Author's wish was to exclude from the present work every problem requiring a mathematical knowledge beyond algebra and trigonometry; he has, however, been induced to depart from this in one or two instances, at the request of several Naval Instructors, who thought that it would render the work more useful to introduce the problems for finding the longitude by an occultation, and for determining the spheroidal figure of the earth from actual measurements on its surface. These two problems belong to the more advanced part of Nautical Astronomy, which the Author contemplated at one time to publish under the title of "Nautical Astronomy, Part III." Part II. however, it is hoped, will enable the naval student to comprehend without difficulty the *principal* rules in Nautical Astronomy; and this he will more readily do, if he has previously made himself acquainted with the Author's volume of Astronomical Problems, which work may be looked upon as introductory to the more important subject of Nautical Astronomy, explained and illustrated at large in the present volume.

Langstone House, Havant,
July 10, 1868.

CONTENTS.

The student is recommended to confine his attention at first to the problems and articles not marked with an asterisk (*).

PROBLEMS ON TIME.

PROBLEMS FOR DETERMINING THE LATITUDE.

PROBLEMS FOR DETERMINING THE LONGITUDE.

PROBLEMS FOR DETERMINING THE VARIATION OF THE COMPASS.

CORRECTIONS IN NAUTICAL ASTRONOMY.

PROOF OF RULES IN PLAIN SAILING.

SHORE OBSERVATIONS FOR TIME AND LONGITUDE.

NAUTICAL ASTRONOMY.

CHAPTER I.

ASTRONOMICAL AND NAUTICAL TERMS AND DEFINITIONS.

1. NAUTICAL Astronomy teaches the method of finding the *place* of a ship, that is, its latitude and longitude, by means of astronomical observations.

2. The following are the principal terms in Nautical Astronomy: the definitions of these terms should be thoroughly understood and carefully committed to memory.

True place of a heavenly body.
Apparent place of a heavenly body.
Axis of the earth.
Terrestrial equator.
Poles of the earth.
Axis of the heavens.
Celestial equator.
Poles of the heavens.
The ecliptic.
Obliquity of the ecliptic.
True latitude of spectator.
Reduced or central latitude of spectator.
Meridians of the earth.
True zenith.

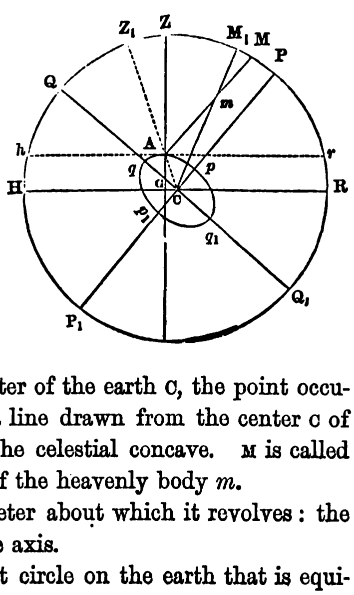
Reduced zenith.
Visible horizon.
Rational horizon.
Poles of the horizon.
Vertical circles, or circles of altitude.
Celestial meridian.
North and south points.
Prime vertical.
East and west points.
Circles of declination.
Circles of latitude.
Right ascension of a heavenly body.
Declination of a heavenly body.
Longitude of a heavenly body.
Latitude of a heavenly body.
Altitude of a heavenly body.
Azimuth or true bearing of a heavenly body.
Amplitude of a heavenly body.
Hour-angle of a heavenly body.
Solar year.
Sidereal year.
Mean solar year.
Sidereal day.
Apparent solar day.
Mean sun.
Mean solar day.
Sidereal time.
Apparent solar time.
Mean solar time.
Equation of time.
Sidereal clock.
Mean solar clock or chronometer.

3. By the combination of theory with astronomical observations, the motions of the sun, moon, and planets have been determined with great accuracy, so that their places may be computed beforehand. The places and relative positions of the heavenly bodies, that is, their right ascensions, declinations, &c., at certain given times, are printed every year in England in the *Nautical Almanac*. In France a similar work is published, called the *Connaissance des Tems*.

Definitions of the preceding Terms in Nautical Astronomy.

4. To a spectator on the earth the sun, moon, and stars seem to be placed on the interior surface of a hollow sphere of great but indefinite magnitude. The interior surface of this sphere is called the *celestial concave*, the center of which may be supposed to be the same as that of the earth.

5. The heavenly bodies are not in reality thus situated with respect to the spectator; for they are interspersed in infinite space at very different distances from him: the whole is an optical deception, by which an observer, wherever he is placed, is induced to imagine himself to be the center of the universe. For let us suppose the elliptical figure $p\ q\ p_1\ q_1$ to represent the earth, $P\ Q\ P_1\ Q_1$ the celestial concave, and m a heavenly body. Then a spectator at A not being able to estimate the distance of m, would imagine it to be in the celestial concave at M.



This figure will enable us to explain the terms *true* and *apparent place* of a heavenly body. The body m viewed from the surface of the earth would appear to a spectator A to be at M in the celestial concave: but if it could be seen from the center of the earth C, the point occupied by m would be M_1, the extremity of a line drawn from the center C of the earth through the heavenly body to the celestial concave. M is called the *apparent* place, and M_1 the *true* place of the heavenly body m.

6. The *axis of the earth* is that diameter about which it revolves: the *poles* of the earth are the extremities of the axis.

7. The *terrestrial equator* is that great circle on the earth that is equidistant from each pole.

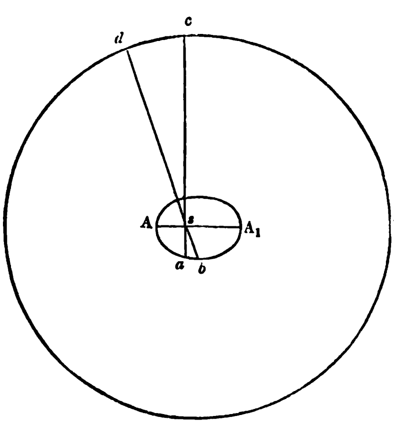
8. A spectator on the earth, not being sensible of the motion by which in fact he describes daily a circle from west to east with the spot on which he stands, views in appearance the heavens moving past him in the opposite direction, or from east to west. The sphere of the fixed stars, or, as it is more usually called, the *celestial concave*, thus appears to revolve from east to west round an imaginary line which is the axis of the earth produced

to the celestial concave: this line is therefore called the *axis of the heavens*.

9. *The poles of the heavens* are the extremities of the axis of the heavens.

10. *The celestial equator* is that great circle in the celestial concave which is perpendicular to the axis of the heavens; or it may be defined to be the terrestrial equator expanded or extended to the celestial concave. The poles of the celestial equator and the poles of the heavens are therefore identical.

11. While the earth thus performs its daily revolution, it is carried with great velocity from west to east round the sun, and describes an elliptic orbit once every year. This *annual* motion of the earth round the sun causes the latter body, to a spectator on the earth, insensible of his own change of place, to appear to describe a great circle in the celestial concave from west to east. This may be explained by a figure. Let A b A_1 be the earth's orbit, s the sun, and $c\,d\,e\,l$ the celestial concave; then, to a spectator at a the sun is seen at a point c in the celestial concave: but when the earth has arrived at b the spectator (not being sensible of his motion from a to b) imagines the sun to be at d, and thus it would seem to have described the arc c d in the time the earth actually moved from a to b. It appears from this, that when the earth has arrived again at a, the sun will again be at c, having described one complete circle in the celestial concave among the fixed stars. The great circle thus described by the sun is called the *ecliptic*.



12. The axis of the earth as it is thus carried round the sun, continues always parallel to itself, and it may be assumed without any sensible error, on account of the smallness of the earth's orbit (small, when compared with the distance of the heavenly bodies), to be always directed to the same points in the celestial concave, namely, the *poles* of the heavens.

13. From observation, the celestial equator is found to be inclined to the ecliptic at an angle of about 23° 28′. This inclination of the equator to the ecliptic is called the *obliquity of the ecliptic*. The axis of the earth, therefore, which is perpendicular to the equator, is inclined to the ecliptic, or, as it is in the same plane, to the earth's orbit, at an angle of 66° 32′.

14. In consequence of the whirling motion of the earth about its axis, the parts near the equator, which have the greatest velocity, acquire thereby a greater distance from the center than the parts near the poles. By actual measurement of a degree of latitude in different parts of the earth, it has

been computed that the equatorial diameter is longer than the axis or polar diameter by 26 miles: the former being about 7924 miles; the latter about 7898 miles, and that the form of the earth is that of an *oblate spheroid.* It is usual, however, in drawing the figure of the earth to exaggerate very much its ellipticity; this is done for the sake of drawing the lines about the figure with greater clearness; for if it were constructed according to its true dimensions, the line $p\,p_1$ (fig. art 5) (being only about the $\frac{1}{300}$th part of itself less than $q\,q_1$), would appear to the eye of the same length as $q\,q_1$, and we should see that the figure that more nearly resembles the earth would be a sphere.

15. If A G, a perpendicular to the earth's surface, be drawn passing through A, the angle A G q formed by the line A G with the plane of the equator is the *latitude,* or true latitude of the point A.

16. If A C be a line drawn from A to C, the center of the earth, then the angle A C q is called the reduced or central latitude of the point A. The difference between the true and reduced latitude is not great: it is, however, of importance in some of the problems in Nautical Astronomy. This correction has accordingly been calculated, and forms one of the Nautical Tables.

17. Sections of the earth passing through the poles, as p A p_1, are called *meridians* of the earth. If the earth is considered as a sphere (which it is very nearly), the meridians will be circles: on this supposition, moreover, the perpendicular A G would coincide with A C, and the latitude of a place on the surface of the earth may, on this supposition, be defined to be the arc of the meridian passing through the place, intercepted between the place and the equator. If G A be produced to meet the celestial concave at Z, the point Z is the zenith of the spectator at A. If C A be produced to the celestial concave at Z′, then Z′ is called the *reduced* zenith of the spectator at A. The point opposite to Z in the celestial concave is called the *Nadir.* In the figure the terrestrial equator $q\ q_1$ is extended to the celestial concave, and therefore Q c Q_1 is the plane of the *celestial* equator.

By means of this figure we may define the zenith, reduced zenith, latitude, and reduced latitude, as follows:

18. The *zenith* is that point in the celestial concave which is the extremity of the line drawn perpendicular to the place of the spectator, as Z.

19. The *reduced* zenith is that point in the celestial concave which is the extremity of a straight line drawn from the center of the earth, through the place of the spectator, as Z′.

20. The *latitude* of a place A on the surface of the earth, is the inclination of the perpendicular A G to the plane of the equator: thus the angle A G Q is the latitude of A. The arc Z Q in the celestial concave measures the angle A G Q; hence Z Q, *or the distance of the zenith from the celestial equator, is equal to the latitude of the spectator.*

21. The *reduced latitude* of the place A is the inclination of Z′ C or A C

to the plane of the equator: or it is the angle A C Q or arc Z′ Q, which measures the angle. Since the curvature of the earth diminishes from the equator to the poles, the reduced latitude Z′ Q must be always less than the true latitude Z Q, and therefore the difference Z Z′ must be subtracted from the true latitude to get the reduced latitude.

The formula for computing the difference between the true and reduced latitude of any place will be investigated hereafter.

22. The *visible horizon* is that circle in the celestial concave which touches the earth where the spectator stands, as *h* A *r*; and a circle parallel to the *visible* horizon, and passing through the center of the earth, is called the rational horizon: thus H C R is the rational horizon. These two circles, however, form one and the same great circle in the celestial concave: thus R and *r* in the figure must be supposed to coincide. This may be readily conceived, when we consider that the distance of any two points on the surface of the earth will make no sensible angle at the celestial concave; therefore either of these two circles is to be understood by the word horizon. The *poles* of the horizon of any place are manifestly the zenith and nadir.

23. Great circles passing through the zenith are called *circles of altitude* or *vertical* circles. That circle of altitude which passes through the poles of the heavens is called the *celestial meridian.* The points of the horizon through which the celestial meridian passes are called the *north* and *south* points. A circle of altitude at right angles to the meridian is called the *prime vertical.* This last circle cuts the horizon in two points called the *east* and *west* points. The east and west points are manifestly the poles of the celestial meridian.

24. Since the horizon and celestial equator are both perpendicular to the celestial meridian, the points where the horizon and celestial equator intersect each other, must be 90° distant from every part of the meridian (Jeans' *Trig.*, P. II., art. 65); that is, the celestial equator must cut the horizon in the east and west points.

25. The ecliptic (art. 11) is divided into twelve parts, called signs, which receive their names from constellations lying near them. These divisions or signs are supposed to begin at that intersection of the celestial equator and ecliptic which is called the *first point* of Aries.

26. Great circles passing through the poles of the heavens are called *circles of declination;* and great circles passing through the poles of the ecliptic are called *circles of latitude.*

27. *Parallels of declination* and of *latitude* are small circles parallel respectively to the celestial equator and ecliptic.

28. The *declination* of a heavenly body is the arc of a circle of declination passing through its place in the celestial concave, intercepted between that place and the celestial equator.

29. The *right ascension* of a heavenly body is the arc of the equator, intercepted between the first point of Aries and the circle of declination

passing through the place of the heavenly body in the celestial concave, measuring from the first point of Aries, eastward, from 0° to 360°.

30. The *latitude* of a heavenly body is the arc of a circle of latitude passing through its place in the celestial concave, intercepted between that place and the ecliptic.

31. The *longitude* of a heavenly body is the arc of the ecliptic intercepted between the first point of Aries and the circle of latitude passing through the place of the heavenly body in the celestial concave, measuring from the first point of Aries, eastward, from 0° to 360°.

32. The *altitude* of a heavenly body is the arc of a circle of altitude passing through the place of the body intercepted between the place and the horizon.

33. The *azimuth*, or bearing of a heavenly body, is the arc of the horizon intercepted between the north or south points and the circle of altitude passing through the place of the body; or it is the corresponding angles at the zenith between the celestial meridian and the circle of altitude passing through the body.

34. The *amplitude* of a heavenly body is the distance from the east point at which it rises, or the distance from the west point at which it sets, the arcs or distances being measured on the horizon.

35. The *hour angle* of a heavenly body, is the angle at the pole between the celestial meridian and the circle of declination passing through the place of the body.

Practical Exercises on the preceding Definitions.

In order more clearly to understand the definitions in Nautical Astronomy, the student should construct a figure or diagram *for each*, and explain the same in the manner pointed out in the answers to the following questions.

1. Construct a figure, and show what is meant by the *true* and *apparent* places of a heavenly body.

2. Construct a figure, and show what is meant by the *axis* of the earth, the *poles* of the earth, and the *terrestrial equator.*

3. Construct a figure, and show what is meant by the *axis* of the heavens, the *poles* of the heavens, and the *celestial equator.*

4. Construct a figure, and show what is meant by the *ecliptic.*

5. Construct a figure, and show what is meant by the *true* and *reduced latitude* of a place on the earth, and also by the *true* and *reduced zenith.*

6. Construct a figure, and show what is meant by *circles of altitude*, the *prime vertical*, the *celestial meridian*, and the north, south, east, and west points of the horizon.

7. Construct a figure, and show what is meant by *circles of declination*, *circles of latitude*, and the *obliquity of the ecliptic.*

8. Construct a figure, and show how to represent on the plane of the

horizon the pole of the heavens and the celestial equator for a given latitude.

9. Construct a figure, and show what is meant by the *right ascension, declination, longitude*, and *latitude*, of a heavenly body.

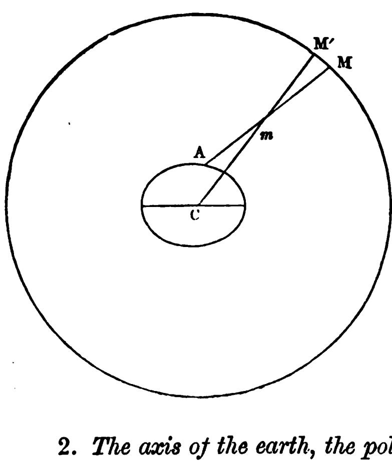
10. Construct a figure, and show what is meant by the *altitude* of a heavenly body, its *polar distance, zenith distance, hour-angle*, and *azimuth* or true bearing.

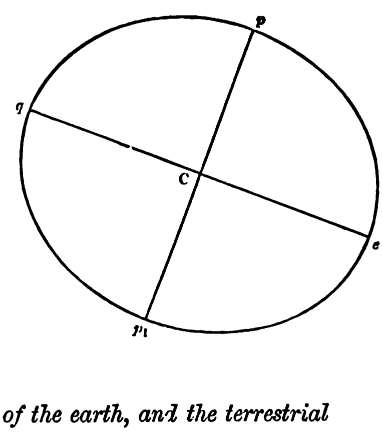
11. Construct a figure, and show what is meant by the *amplitude* of a heavenly body.

ANSWERS TO THE FOREGOING QUESTIONS.

1. *The true and apparent place of a heavenly body.*

Let m be a heavenly body, and A a spectator on the surface of the earth. Through m draw the straight lines AM and CM′ from the surface and center of the earth to the celestial concave at M and M′. Then M is called the *apparent* place, and M′ the *true* place of the heavenly body m.





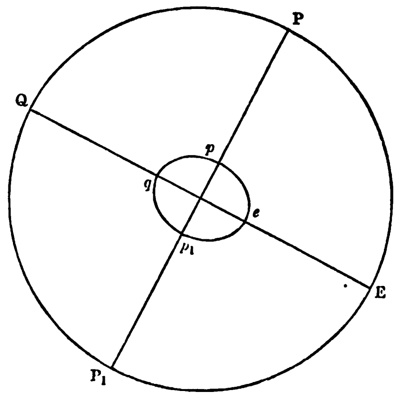
2. *The axis of the earth, the poles of the earth, and the terrestrial equator.*

Assuming the earth to be an oblate spheroid (chapter iv.), let pqp_1e represent a section of the earth passing through the center C; then if pp_1 be that diameter about which the earth revolves, the line pp_1 is called the *axis* of the earth, the extremities p and p_1 are the *poles*, and the line eq, drawn at right angles to pp_1, will be in the plane of the equator, and may be taken to represent the *terrestrial equator* itself.

3. *The axis of the heavens, the poles of the heavens, and the celestial equator.*

Let PQP_1E represent the celestial concave, pqp_1e the earth, pp_1 the axis

of the earth, and eq the terrestrial equator. Expand the terrestrial equator eq to the celestial concave in Q and E, and produce pp_1 to P and P_1; then PP_1 is the *axis* of the heavens, the extremities P and P_1 are the *poles* of the heavens, and EQ is the *celestial equator.*

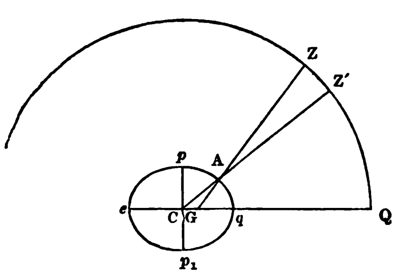


4. *The ecliptic.*

Let AbA_1 (fig. p. 4) represent the earth's orbit, and s the sun, a and b the earth in two points of its orbit. Then, when the earth is at a, the sun's place in the celestial concave is c; when the earth is at b, the sun's place is d: while the earth, therefore, describes the arc ab of its orbit, the sun appears to describe an arc of a great circle cd; and when the earth has completed its revolution about the sun, the latter will appear to have described in the celestial concave a great circle cel. This circle is called the *ecliptic.*

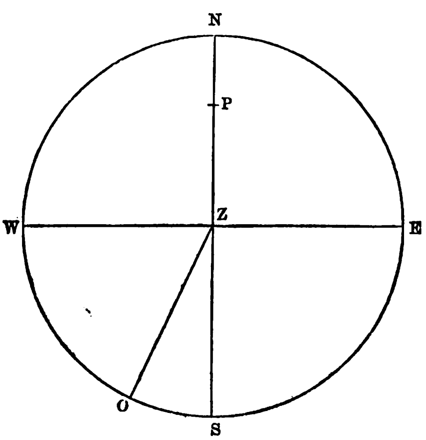
5. *The true and reduced latitude of a place on the earth; the true and reduced zenith.*

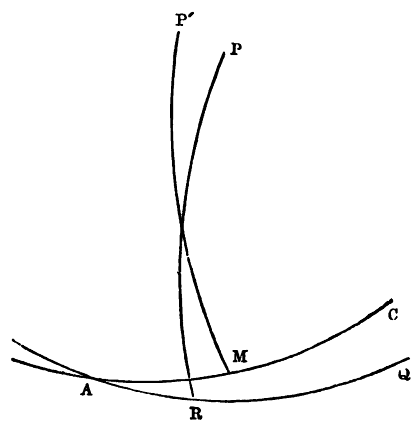
Let pep_1q be a section of the earth passing through the center, which, since the earth is an oblate spheroid, will be an ellipse (chapter iv.); A the given place; through A draw AG perpendicular to the tangent passing through A; join AC, and produce GA and CA to the celestial concave at Z and Z′; let eQ be the plane of the celestial equator; then the arc ZQ or angle ZGQ is the *true latitude* of A, and Z′Q or the angle Z′CQ is the *reduced latitude* of A. The point Z is the *true zenith,* and Z′ the *reduced zenith,* of the spectator at A.



6. *Circles of altitude, the prime vertical, the celestial meridian, and the north, south, east, and west points of the horizon.*

These definitions are more easily explained by projecting the figure on the plane of the horizon. Thus, let NWSE represent the horizon of the spectator; then the point Z, considered as the pole of the horizon, will be the zenith, and great circles WE, NS, ZO, will be *circles of altitude* or vertical circles. Let the circle of altitude ZN pass through the pole of the heavens P: then NZS is the *celestial meridian* of the spectator, whose zenith is Z; and the circle of altitude WE, drawn at right angles to the celestial meridian, is the *prime vertical.* If P be the north pole, then the point of the horizon N intersected by the celestial meridian is the *north* point, and the opposite point of the horizon S is the *south* point; the points of the horizon W, E, intersected by the prime vertical, are the *west* and *east* points.





7. *Circles of declination, circles of latitude, and the obliquity of the ecliptic.*

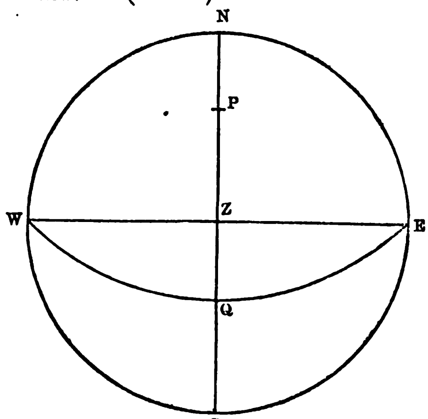
Since the equator and ecliptic are inclined to each other at an angle of about 23° 28′ (p. 4); let AQ represent the celestial equator, and AC the ecliptic. Let P be the pole of the heavens, and P′ the pole of the ecliptic, and through P and P′ draw PR perpendicular to AQ and P′M perpendicular to AC; then PR is a *circle of declination* and P′M a *circle of latitude,* and the angle CAQ is the *obliquity* of the ecliptic.

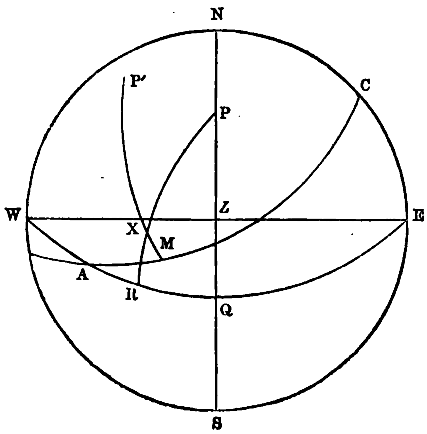
8. *Represent on the plane of the horizon the pole of the heavens and the celestial equator for a given latitude.**

Let NWSE represent the horizon, NS the celestial meridian, and WE the prime vertical; then, since the distance of the zenith from the celestial

* This projection will be very often used in future problems in Nautical Astronomy.

equator is equal to the latitude of the spectator (p. 5), take ZQ = the given latitude, and through the points W, Q, E draw a great circle WQE: this will represent the celestial equator, since the equator must pass through the east and west points (p. 6). From Q take QP=90°; then P will represent the pole of the heavens, since the pole of the celestial equator is the pole of the heavens (art. 10).



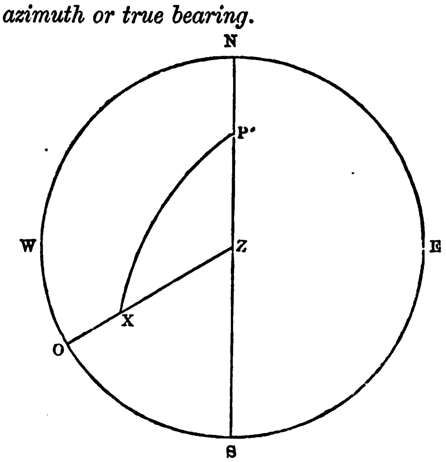


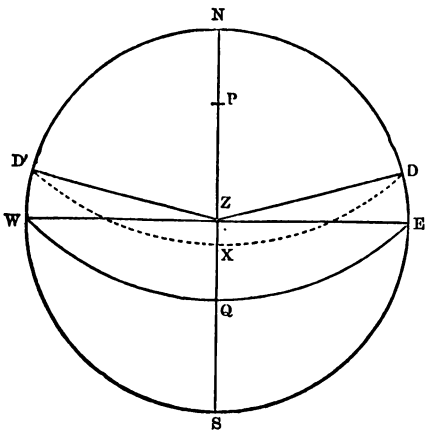
9. *The right ascension, declination, longitude and latitude, of a heavenly body.*

Let NWSE represent the horizon, P the pole, Z the zenith, and X the place of a heavenly body. Draw the equator WQE and the ecliptic AC, and let P′ be the pole of the ecliptic AC and A the first point of Aries. Through X draw a circle of declination PR and a circle of latitude P′M. Then AR is the *right ascension*, XR is the *declination*, AM is the *longitude*, and XM is the *latitude* of the heavenly body X.

10. *The altitude of a heavenly body, its polar distance, zenith distance, hour-angle, and azimuth or true bearing.*

Let NWSE represent the horizon, NS the celestial meridian, P the pole, and Z the zenith. Let X be the place of a heavenly body; through X draw the circle of altitude ZO and circle of declination PX. Then XO is the *altitude* of the heavenly body, ZX its *zenith distance*, and PX its *polar distance*. The angle ZPX is the *hour-angle*, and the angle PZX or SZX, or the arc of the horizon NO or SO, is its *azimuth* or true bearing.





11. *The amplitude of a heavenly body.*

Let NWSE represent the horizon, and the small circle DXD′ the parallel of declination described by the heavenly body X from its rising at D to setting at D′; then the angle DZE or D′ZW is the amplitude of X, or the arc DE or D′W, which measures these angles.

DEFINITIONS AND PROBLEMS ON TIME.

The solar year, and sidereal year.

36. A *solar year* is the interval between the sun's leaving the first point of Aries and returning to it again.

37. A *sidereal year* is the interval between the sun's leaving a fixed point, as a star, and returning to that point again.

The equinoctial points have an annual motion of $50''\cdot1$, by which they are carried back to meet the sun in its apparent motion among the fixed stars from west to east.

On this account a solar year is shorter than a sidereal year by the time the sun takes to describe $50''\cdot1$.

38. The length of the solar years is found to differ a little from each other, on account of certain irregularities in the sun's apparent motion, and that of the first point of Aries. The *mean length* of several solar years is therefore the one made use of in the common division of time, and called the *mean solar year.*

To find the length of the mean solar year.

39. By comparing observations made at distant periods, it was found that the sun had described 36000° 45′ 45″ of longitude in 36525 days. Now in one solar year the sun separates from the first point of Aries 360° (taking into consideration its own apparent motion from west to east, and the actual motion of the first point of Aries in the opposite direction). Let therefore $x =$ the length of a mean solar year;

$$\text{then, } 36000^\circ\ 45'\ 45'' : 360^\circ :: 36525^d : x$$

$$\text{whence, } x = 365^d\ 5^h\ 48^m\ 51^s\cdot6 = 365^d\cdot242264.*$$

* According to Bessel, the formula for determining the length of the mean solar or tropical year is

$$365^d\cdot2422013 - \cdot00000006686 \times t$$

where t is the number of years since 1800.

To find the length of the sidereal year.

40. Since the first point of Aries moves with a slow annual motion of about $50''\cdot 1$ from east to west to meet the sun, the arc of the ecliptic described by the sun from the first point of Aries to the first point of Aries again must be $360^\circ - 50''\cdot 1 = 359^\circ\ 59'\ 9''\cdot 9$, and this is the arc described by the sun in a mean solar year; but in a sidereal year the sun describes 360°; hence a sidereal is greater than a solar year by the time the sun takes to move over an arc of $50''\cdot 1$. Hence the proportion,

$$\text{sidereal year} : \text{mean solar year} :: 360^\circ : 360^\circ - 50''\cdot 1$$
$$\text{or, sidereal year} : 365^d\cdot 242264 :: 360^\circ : 359^\circ\ 59'\ 9''\cdot 9.$$
$$\text{Therefore, sidereal year} = 365^d\ 6^h\ 9^m\ 11^s\cdot 5.*$$

The sidereal day, the apparent solar day, and the mean solar day.

41. The *sidereal day* is the interval between two successive transits of the first point of Aries over the same meridian. It begins when the first point of Aries is on the meridian.

The *apparent solar day* is the interval between two successive transits of the sun's center over the same meridian. It begins when that point is on the meridian.

42. The length of an apparent solar day is variable chiefly from two causes:

1st. From the variable motion of the sun in the ecliptic.

2d. From the motion of the sun being in a circle inclined to the equator.

43. To explain briefly these causes of variation, let us suppose the two circles A Q, A I, to represent the celestial equator and ecliptic, and S S′ the arc of the ecliptic described by the sun in one day. The angle at P, between the two circles of declination, is measured, not by the arc S S′ described by the sun, but by the arc R R′ of the equator. Now,

1st. The velocity or motion of the sun in the ecliptic is variable, on account of the earth moving in an elliptic orbit; it sometimes describes an arc of 57′ in a day; at other times the arc described is about 61′; this is one cause of the inequality in the length of the solar days.

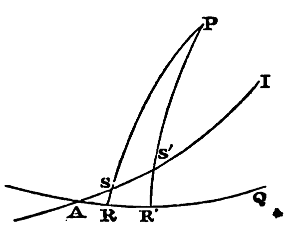


Fig 1.

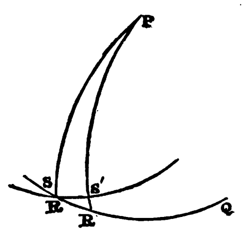


Fig 2.

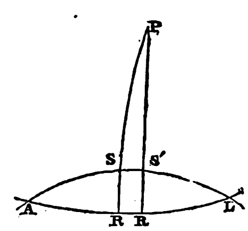


Fig 3.

2d. But even supposing the arcs of the ecliptic described by the sun to

* The length of a sidereal year according to Bessel is
$365^d\cdot 256374322 = 365^d\ 6^h\ 9^m\ 10^s\cdot 7423$ mean time.

be equal, yet the angles at P between the meridians, as R P R′ (in the three figures) will not be so, since these angles are measured by the arc R R′ of the equator, to which S S′ will be differently inclined according to the place of the sun in the ecliptic. At the equinoxes, or when the sun is at R (fig. 2), the arcs S S′ and R R′ will be inclined to each other at an angle of about 23° 27′. At the solstices they are parallel (see fig. 3). This is the second cause of the inequality.

44. To obtain, therefore, a proper measure of time, we must proceed as follows. An imaginary, or as it is called a *mean sun*, is supposed to *move uniformly in the celestial equator with the mean velocity of the true sun.* A *mean solar day* may therefore be defined to be the interval between two successive transits of the mean sun over the same meridian. It begins when the mean sun is on the meridian.

To find the daily motion of the mean sun in the celestial equator.

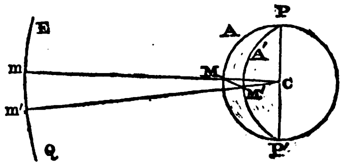
45. The mean solar year, or the time the sun takes to return again to the first point of Aries, has been found to be equal to $365^d{\cdot}2422$. Let us suppose the mean sun to describe the *equator* in this time, then we shall find its daily motion in the equator as follows: Let x = daily motion,

$$365^d{\cdot}2422 : 1^d :: 360^\circ : x = 0^\circ{\cdot}9856472 = 59'\ 8''{\cdot}33$$

or, the mean sun's daily motion in the celestial equator from west to east is 59′ 8″·33.

To find the arc described by a meridian of the earth, in a mean solar day.

46. Let P A M P′ represent the meridian of a spectator A, drawn in some plane; as, for instance, that of the paper; M M′ the terrestrial, and E Q the celestial equator, which must therefore be supposed at right angles to the paper.



Suppose the mean sun to be at m on the meridian of A, and therefore in the plane of the paper; and let $m\ m'$ be the arc of the equator described by the mean sun in one day, namely, 59′ 8″·33. Join C m', cutting the terrestrial equator at M M′. Now, let the earth be supposed to revolve about P P′, from west to east, until the meridian again passes through the mean sun, which has arrived at m', having moved through the arc $m\ m'$ in the same time. Then the whole number of degrees described by the meridian of the spectator will evidently be one complete revolution, or 360° (by which it is again brought into the plane of the paper), together with the arc M M′ = $m\ m'$, or 59′ 8″·33. Hence in a mean solar day a meridian describes 360° 59′ 8″·33; and therefore a mean solar day is longer than a sidereal day in the ratio of 360° 59′ 8″·33 to 360°, or 24 mean solar hours = $24^h\ 3^m\ 36^s{\cdot}555$ in sidereal time. (See Prob. IV.)

Sidereal time, apparent solar time, and mean solar time.

47. *Sidereal* time is the angle at the pole of the heavens between the celestial meridian and a circle of declination passing through the first point of Aries, measuring from the meridian westward.

48. *Mean solar time* is the angle at the pole between the celestial meridian and a circle of declination passing through the mean sun, measuring from the meridian westward.

49. *Apparent solar time* is the angle at the pole between the celestial meridian and a circle of declination passing through the place of the sun's center, measuring from the meridian westward.

The equation of time is the difference in time between the places of the true and mean sun.

Sidereal clock, and mean solar clock.

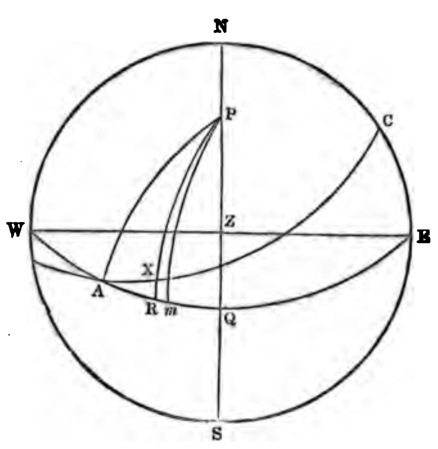
50. A *sidereal clock* is a clock adjusted so as to go 24 hours during one complete revolution of the earth; that is, during the interval of two successive transits of a fixed star: or supposing the first point of Aries to be invariable between two successive transits of the first point of Aries.

51. A *mean solar clock* is a clock adjusted to go 24 hours during one complete revolution of the mean sun; or while a sidereal clock is going $24^h\ 3^m\ 56^s{\cdot}555$.

QUESTIONS ON TIME.

52. *Construct a figure, and show what is meant by sidereal time, apparent solar time, mean solar time, and the equation of time.*

Let N W S E represent the horizon, P the pole, A E the equator, A the first point of Aries, and A C the ecliptic. Let X be the place of the sun in the ecliptic, and m the mean sun; through X and m draw the circles of declination P R and Pm. Then sidereal time is the angle Q P A, or arc Q A; apparent solar time is the angle Q P R, or arc Q R; and mean solar time is the angle Q P m, or arc Qm,—these angles or arcs being always measured from the meridian N Z S westward. Also the angle mP R, or arc m R, is the equation of time.



It may not be amiss sometimes for the young student to answer questions on the definitions more in detail; that is, *by referring to the definitions themselves* whilst constructing the figures: thus the last question may be answered more fully as follows.

THE LAST QUESTION REPEATED.

53. *Construct a figure, and show what is meant by sidereal time, apparent solar time, mean solar time, and the equation of time.*

Let NWSE (last fig.) represent the horizon, P the pole, AE the equator, A the first point of Aries, and AO the ecliptic. Let X be the place of the sun in the ecliptic, and *m* the mean sun: through X and *m* draw the circles of declination PR and P*m*. Then sidereal time is the angle QPA, or arc QA; since, by definition, sidereal time is the angle at the pole between the celestial meridian and a circle of declination passing through the first point of Aries, measuring from the meridian westward. Apparent solar time is the angle QPR, or arc QR; since, by definition, apparent solar time is the angle at the pole between the celestial meridian and a circle of declination passing through the place of the sun, measuring from the meridian westward. Mean solar time is the angle QP*m*, or arc Q*m*; since, by definition, mean solar time is the angle at the pole between the celestial meridian and a circle of declination passing through the mean sun, measuring from the meridian westward. The equation of time is the angle *m*PR, or arc *m*R; since, by definition, the equation of time is the difference in time between the places of the true and mean sun, or the angle at the pole between the circles of declination passing through the place of the sun and the mean sun.

In a similar manner may other questions on the definitions in pages 1 and 2 be extended or amplified when deemed necessary.

Examples on the Julian and Gregorian years.

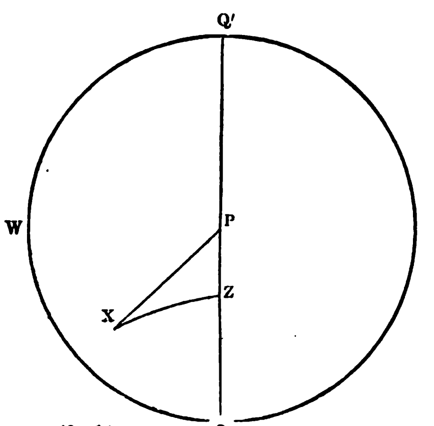
54. If the length of the mean solar year is 365·242264 days (page 12), and we assume its length to be 365·25 days (the length of the common or *Julian* year); find the error on this account in 400 years. *Ans.* 3·0944 days.

55. To diminish the error committed by making the year to be 365 days 6 hours, a correction is made in the Gregorian Calendar, or New Style, to the Julian year, as follows. Every centenary year of which the two first figures are not divisible by 4 (such as 1800, 1900, 2100, &c.), is to be considered as a common year of 365 days; thereby deducting 3 days in every 400 years. Now supposing this correction to be made, it is required to find the number of years that must elapse before the error arising from making the year to consist of 365 days 6 hours (so corrected) shall amount to one day. *Ans.* 4237 years.

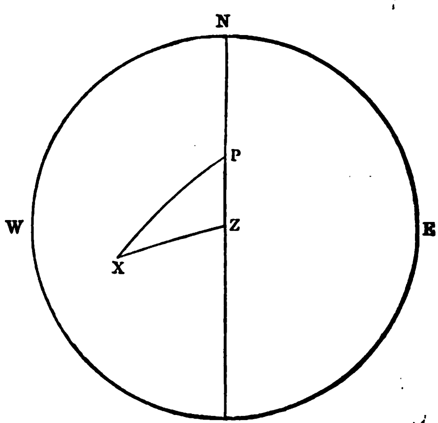
CONSTRUCTION OF ASTRONOMICAL DIAGRAMS.

The diagrams to the examples in the following pages need not be projected by the student with very great accuracy; the quantities given, namely, the arcs and angles, may be drawn after a little practice sufficiently near the truth by estimating with the eye the length or magnitude of each.

56. The figures or diagrams in Nautical Astronomy are usually projected on one of the three following planes: *the plane of the horizon, the plane of the celestial equator, or the plane of the celestial meridian.* The first is the one most frequently adopted, as it shows the positions of the heavenly body during the whole of its diurnal motion. When several parts of the celestial equator are required to be seen, then the diagram should be projected on the plane of the celestial equator. In some cases it is indifferent which projection is used: thus, if it be required to represent by means of a figure the polar distance, zenith distance, and hour-angle of a heavenly body, we may project the figure on either the plane of the equator or the plane of the horizon. If we project on the plane of the celestial equator QQ'E (first figure), then the center P will be the pole of the heavens; and if Z be the zenith of



(fig. 1.)



(fig. 2.)

the spectator, and X the place of the heavenly body, then PX is the polar distance, ZX is the zenith distance, and the angle ZPX is the hour-angle of the heavenly body. If we project the figure on the plane of the horizon NWSE (fig. 2), the center Z will be the zenith; and if P be the pole of the heavens, and X the place of the heavenly body, PX is the polar distance, ZX is the zenith distance, and P is the hour-angle of the heavenly body. An example of a figure projected on the plane of the celestial meridian is given in the volume of *Solutions of Astronomical Problems*, p. 212 (published by the author as a key to the problems in *Trigonometry*, Part I.).

CHAPTER II.

PROBLEMS ON TIME.

PROBLEM I.*

Given mean time and the equation of time, to find apparent time: or, Given apparent time and the equation of time, to find mean time.

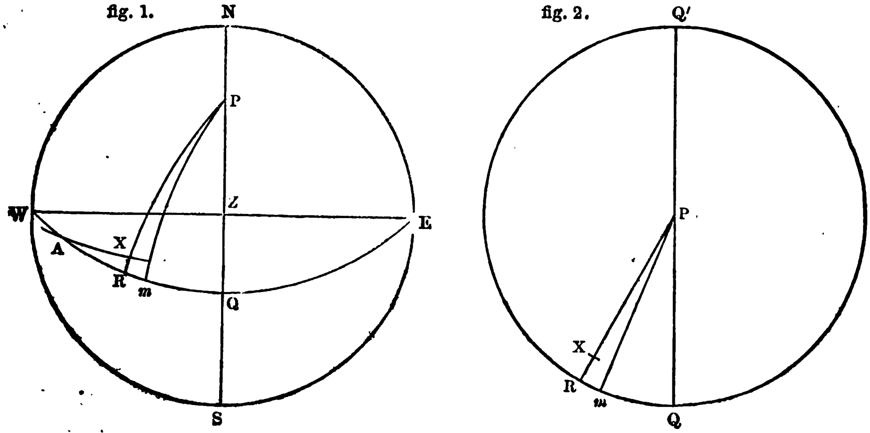
Fig. 1 is a construction on the plane of the horizon, fig. 2 on the plane of the celestial equator. The sun's declination is supposed to be about 10° N. The declination is given to enable the student to insert the sun's place in the figure.

Construction.

Let P represent the north pole, WQE in fig. 1, or QRQ′ in fig. 2, the celestial equator, PQ the celestial meridian, X the place of the sun 10° N. of the equator, and m the mean sun in the equator. Through X and m draw

fig. 1.

fig. 2.



the circles of declination PR and Pm: then the angle QPR, or arc QR, is apparent time; the angle mPR, or arc mR, is the equation of time; and the angle QPm, or arc Qm, is mean time.

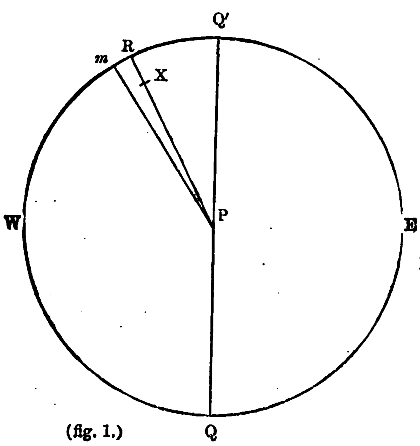
* This and some of the subsequent problems are given chiefly to afford the student a few very easy exercises in the construction of astronomical problems.

By the fig., $QR = Qm + mR$,
or apparent time = mean time + equation of time,

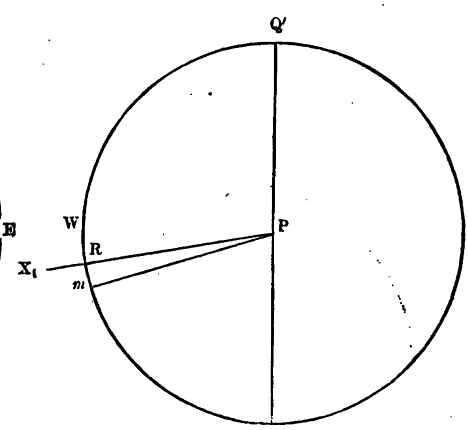
From which formula, apparent time or mean time may be found when the other two quantities are given.

As the mean sun moves uniformly in the equator with the mean angular velocity of the true sun (p. 14), it is manifest that it will be sometimes in advance and sometimes behind the place of the true sun; the above formula must therefore be modified to suit the relative positions of m and X. When the equation of time is subtractive from mean time to get apparent time, m must be on the other side of R; when it is additive, the above figures will represent the correct positions of the bodies. The equation of time, with its proper sign, is given in the *Nautical Almanac* for every day at noon.

In the following easy examples (inserted chiefly for the sake of affording the student useful exercises in *constructing diagrams*), the positions of the true and mean suns are to be determined as nearly as possible by the eye and without the assistance of mathematical instruments.



(fig. 1.)



(fig. 2.)

57. Given mean time$=10^h\,14^m\,15^s$ P.M., and the equation of time$=2^m\,30^s$ *additive* to mean time; construct a figure, and find by calculation apparent time (fig. 1).

The figure is to be constructed on the plane of the equator, and the declination is supposed to be about 10° N.

Let QWQ′ represent the celestial equator, P the north pole, and QPQ′ the celestial meridian. Take $Qm=10^h\,14^m\,15^s$=mean time; then m is the mean sun; and since the equation of time is additive to mean time, let mR represent the equation of time$=2^m\,30^s$. Draw Pm, PR, circles of declination,

through m and R; then the true sun will be in the circle PR. Take RX$=10°$; then X is the place of the sun.

By the fig., QR$=$Q$m+m$R,

or apparent time$=$mean time$+$equation of time;

and mean time$=$	10^h	14^m	15^s
equation of time$=$		2	30
∴ apparent time$=$	10	16	45

58. Given apparent time$=5^h\ 10^m$ P.M., and the equation of time$=15^m\ 10^s$ subtractive from apparent time, the sun's declination being about $20°$ S; construct a figure, and find by calculation mean time.

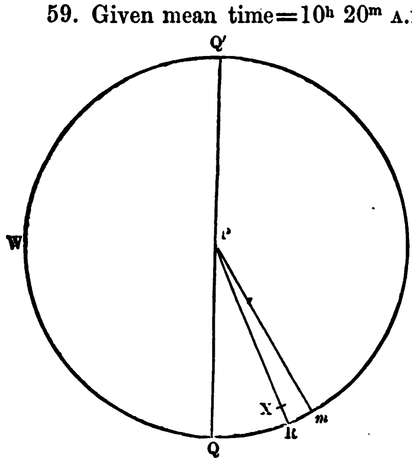
Let QWQ′ represent the celestial equator, P the north pole, and QPQ′ the celestial meridian. Take QR$=5^h\ 10^m=$apparent time, and R$m=15^m\ 10^s$ the equation of time. Draw the circles of declination Pm and PX through m and R, and take RX$=20°$ to the south of the equator; then X is the place of the sun.

By the fig., Q$m=$QR$-$Rm,

or mean time$=$apparent time$-$equation of time;

apparent time$=$	5^h	10^m	0^s
equation of time$=$		15	10
∴ mean time$=$	4	54	50

59. Given mean time$=10^h\ 20^m$ A.M.$=22^h\ 20^m$ P.M., and the equation of time$=3^m\ 13^s$ additive to mean time; required apparent time, decl. $10°$ N. Construct the figure on the plane of the equator. Let QWQ′ represent the celestial equator, P the pole and QPQ′ the celestial meridian. Take QQ′$m=22^h\ 20^m$ to represent mean time; and since the equation of time is additive to mean time, take mR$=3^m\ 13^s$. Through m and R draw circles of declination Pm, PR, and take RX$=10°$; then X is the place of the sun.



By the fig., QQ′R$=$QQ′$m+m$R,

or apparent time$=$mean time$+$equation of time;

mean time$=$	22^h	20^m	0^s
equation of time$=$		3	13
∴ apparent time$=$	22	23	13
or$=$	10	23	13 A.M.

EXAMPLES FOR PRACTICE.

60. Given apparent time $= 4^h\ 30^m$ P.M., and equation of time $= 3^m\ 0^s$ additive to apparent time. Construct a figure, and find by calculation mean time (declination 20° N.). *Ans.* Mean time $= 4^h\ 33^m\ 0^s$.

61. Given mean time $= 2^h\ 30^m$ A.M., and equation of time $= 2^m\ 0^s$ additive to mean time. Construct a figure, and find by calculation apparent time (declination 20° N.). *Ans.* Apparent time $= 2^h\ 32^m$ A.M.

62. Given mean time $= 10^h\ 10^m$ A.M., and equation of time $= 10^m\ 0^s$ subtractive from mean time. Construct a figure, and find by calculation apparent time (declination 20° S.). *Ans.* Apparent time $= 10^h\ 0^m$ A.M.

PROBLEM II.

Given mean time, to find sidereal time.

Let QAQ′ represent the celestial equator, A the first point of Aries, QPQ′ the celestial meridian, and P the pole. Then, if the mean sun is west of meridian, let m (fig. 1) be the mean sun.

If the mean sun is east of the meridian, let m' (fig. 2) be the mean sun.

In the figs. (1) and (2), QA = sidereal time, since it measures the time elapsed from the first point of Aries being on the meridian (p. 15).

In fig. 1, $Qm =$ mean time.

In fig. 2, $m'Q =$ arc that must be described before the mean sun m' arrives at the meridian.

$\therefore\ 24^h - m'Q =$ mean time, $\therefore\ m'Q = 24^h -$ mean time.

Also, Am, or $Am' =$ right ascension of mean sun.

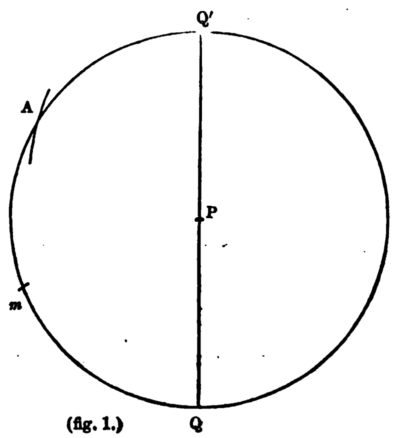
Now (fig. 1) $QA = Am + mQ$,

or sidereal time $=$ RA mean sun $+$ mean time.

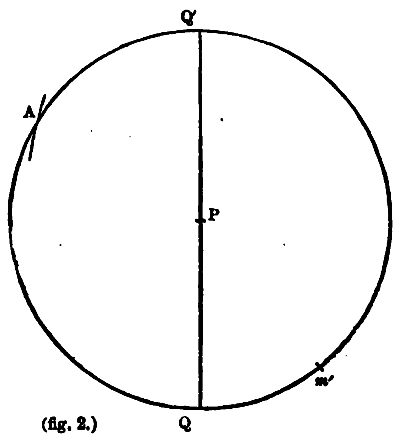
In fig. 2, $QA = Am' - m'Q$,

or sidereal time $=$ RA mean sun $- (24^h -$ mean time$)$

$=$ RA mean sun $+$ mean time $- 24^h$.



(fig. 1.) Q



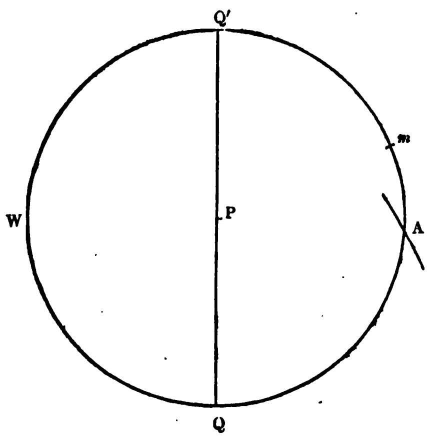
(fig. 2.) Q

The latter expression for sidereal time is the same as the former by adding 24 hours; and, assuming that we can always add or subtract 24 hours, the first equation may be considered true in both cases; and as we shall get the same result for other positions of m and A with respect to the meridian, therefore generally this rule is true; viz.

Sidereal time = RA mean sun + mean time.

If apparent time is given to find sidereal time, reduce apparent time to mean time by Problem I.

The right ascension of the mean sun is given in the *Nautical Almanac* for every day at Greenwich mean noon, being the column marked "sidereal time" in each month.



63. Given mean time = $15^h\ 55^m\ 45^s$, and the right ascension of the mean sun = $2^h\ 22^m\ 58^s$. Construct a figure, and find by calculation sidereal time.

Let QWQ′ represent the celestial equator, P the pole, and QPQ′ the celestial meridian. From Q measure QQ′m = $15^h\ 55^m\ 45^s$; then m is the mean sun. From m take mA = 2^h 22^m 58; then A is the first point of Aries, and the arc QQ′A is sidereal time.

TO FIND SIDEREAL TIME BY FORMULA.

Sidereal time = RA mean sun + mean time.

RA mean sun . .	2^h	22^m	58^s
mean time . . .	15	55	45
∴ sidereal time =	18	18	43

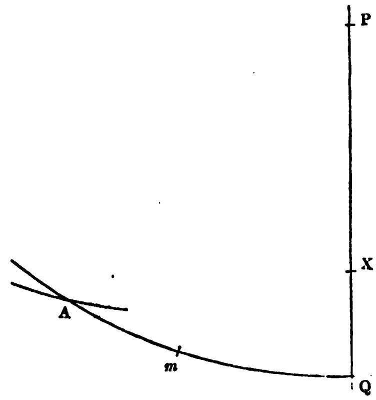
EXAMPLES FOR PRACTICE.

64. Given mean time = $6^h\ 10^m$ P.M., and the right ascension of the mean sun = $2^h\ 22^m\ 41^s$. Construct a figure, and find by calculation sidereal time. *Ans.* Sidereal time = $8^h\ 32^m\ 41^s$.

65. Given mean time = $5^h\ 42^m\ 10^s$ A.M., and the right ascension of the mean sun = $18^h\ 47^m\ 14^s$. Construct a figure, and find by calculation sidereal time. *Ans.* Sidereal time = $12^h\ 29^m\ 24^s$.

PROBLEM III.

Given mean time, or apparent time at a given place; to find what heavenly body will pass the meridian the next after that time.



Let AQ be the celestial equator, PQ the celestial meridian, and X a heavenly body passing the meridian. Therefore the right ascension of the meridian will be the right ascension of the star. Then, knowing the time at the place, we can determine the position of the mean sun with respect to the meridian, and also that of the first point of Aries, since we also know the right Ascension of the mean sun, from the *Nautical Almanac*.

Let, therefore, m be the mean sun, A the first point of Aries.

Then $\text{Q}m$ = mean time at the place, $\text{A}m$ = right ascension of mean sun,

and AQ = right ascension of the heavenly body X
= right ascension of the meridian;

and by the figure, $\text{AQ} = \text{A}m + m\text{Q}$,

or star's RA = RA mean sun + mean time = sidereal time (Prob. II.)

$\therefore$ sidereal time = right ascension of meridian.

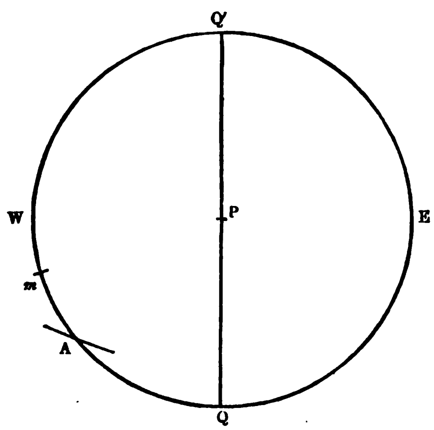
It appears from this, that in order to find the time when a heavenly body passes the meridian, we have only to find sidereal time by the last problem, and this will be the right ascension of the meridian, and therefore of any heavenly body on the meridian. Then the star in the astronomical catalogue in the *Nautical Almanac* whose right ascension is the next greater, will evidently be the next principal star to pass the meridian, and is therefore the one required.

If it is required to know what principal stars will pass a given meridian between any two given dates, we must proceed as above to find sidereal time at each date. Then all the stars in the catalogue whose right ascensions lie between the sidereal times thus determined will be the stars required.

66. Given mean time $= 4^h\ 42^m$ P.M., and the right ascension of the mean sun $= 22^h\ 16^m\ 46^s$. Construct a figure and find by calculation what bright star will pass the meridian the next after that time.

Let QWQ′ represent the celestial equator, P the pole, and QPQ′ the celestial meridian. From Q measure $\text{Q}m = 4^h\ 42^m$; then m is the mean sun. From m measure an arc $m\text{Q}'\text{A} = 22^h\ 16^m\ 46^s$; then A is the first point of Aries, and the arc AQ is the right ascension of the meridian or sidereal

time, and therefore the right ascension of any star on the meridian.



By Calculation.

RA of meridian = sidereal time

= RA mean sun + mean time (Prob. II.).

RA mean sun . . .	22^h 16^m 46^s
mean time . . .	4 42 0
∴ RA of meridian . . .	2 58 46 (rejecting 24 hours).

By catalogue it appears that the star whose right ascension is the next greater than this is α Persei, ∴ the star that passes the next after 4^h 42^m P.M. is α Persei.

67. What bright stars will pass the meridian of the ship between the hours of 9^h and 12^h P.M., the right ascension of the mean sun at 9 o'clock being 12^h 49^m 41^s, and at 12 o'clock, 12^h 50^m 11^s?

By Calculation.

RA of meridian = RA mean sun + mean time.

RA mean sun at 9^h . .	12^h 49^m 41^s	at 12^h . .	12^h 50^m 11^s
mean time . .	9 0 0	mean time . .	12 0 0
RA of meridian =	21 49 41	RA of mer. =	0 50 11

By inspecting the catalogue, it appears that the stars whose right ascensions lie between those of α Aquarii and β Ceti will pass the meridian of the ship between 9 and 12 o'clock.

EXAMPLES FOR PRACTICE.

68. Given mean time $= 8^h\ 0^m$ P.M., and the right ascension of the mean sun $= 23^h\ 34^m$. Construct a figure, and find by calculation the right ascension of the meridian and the bright star that will pass the meridian the next after that time. *Ans.* RA meridian $= 7^h\ 34^m$: Pollux.

69. Given mean time $= 2^h\ 0^m$ A.M., and the right ascension of the mean sun $= 6^h\ 42^m$. Construct a figure, and find by calculation the right ascension of the meridian, and the bright star that will pass the meridian the next after that time. *Ans.* RA meridian $= 20^h\ 42^m$: α Cephei.

PROBLEM IV.

Given the length of a mean solar day $= 24$ hours; to express its length in sidereal hours, and the converse.

In a mean solar day, a meridian of the earth revolves through $360°\ 59'\ 8{\cdot}33''$ (art. 45): in a sidereal day it revolves through $360°$ (art. 40). A mean solar day is therefore longer than a sidereal day (or 24 sidereal hours) by the portion of sidereal time consumed in describing $59'\ 8{\cdot}33''$. This quantity of time may be found by the following proportion:

$$360° : 59'\ 8{\cdot}33'' :: 24^h : x \text{ the quantity required,}$$
$$\therefore x = 3^m\ 56{\cdot}555^s,$$
$$\therefore 24 \text{ mean solar hours} = 24^h\ 3^m\ 56{\cdot}555^s \text{ sidereal hours.}$$

The quantity $3^m\ 56{\cdot}555^s$, the excess of a mean solar day over a sidereal day, is called the *acceleration of sidereal on mean solar time.*

Given the length of a sidereal day $= 24$ hours; to express its length in mean solar time.

In a sidereal day the meridian of any place revolves through $360°$; in a mean solar day the same meridian revolves through $360°\ 59'\ 8{\cdot}33''$ (art. 45).

Therefore the length of a sidereal day may be found in mean solar time by this proportion:

$$24 \text{ sidereal hours} : 24 \text{ mean solar hours} :: 360° : 360°\ 59'\ 8{\cdot}33'',$$
$$\therefore 24 \text{ sidereal hours} = \frac{24 \times 360°}{360°\ 59'\ 8{\cdot}33''} = 23^h\ 56^m\ 4{\cdot}0922^s \text{ mean solar hours.}$$

Formulæ for converting any portion of sidereal time into mean solar time, and the converse.

Let S = any interval of absolute time or duration expressed in sidereal time.
M = the same interval expressed in mean solar time.

Then, since the number of hours, minutes, &c., in a given interval must be inversely as the length of one of them, we have

$$\frac{M}{S}=\frac{360}{360^\circ\ 59'\ 8{\cdot}33''}=\cdot9972695667,$$

$$\text{and } \frac{S}{M}=\frac{360^\circ\ 59'\ 8{\cdot}33''}{360}=1{\cdot}002737909,$$

$$\therefore M= \cdot997269\times S=S-\cdot002731\ S \quad .\ .\ .\ .\ (1)$$

$$\text{and } S=1{\cdot}002738\times M=M+\cdot002738\ M \quad .\ .\ .\ .\ (2)$$

By means of these formulæ we may convert any portion of sidereal time into mean solar time, and the converse, as in the following examples.

70. Express 12 sidereal hours in mean solar time.

By formula (1), $M= S -\cdot002731\ S$, and $S=12$

$$\therefore M=12-\cdot002731\times 12=11^h\ 58^m\ 2{\cdot}02^s,$$

or 12 sidereal hours $=11^h\ 58^m\ 2{\cdot}02^s$ mean solar hours.

71. Express $11^h\ 58^m\ 2{\cdot}02^s$ mean solar time in sidereal time.

By formula (2), $S=M+\cdot002738\ M$,

$M=11^h\ 58^m\ 2{\cdot}02^s \qquad =11{\cdot}967228+\cdot002738\times 11{\cdot}967228$

$=11{\cdot}967228 \qquad =11{\cdot}999994=12$ hours nearly;

$\therefore 11^h\ 58^m\ 2{\cdot}02^s$ mean solar hours $=12$ sidereal hours.

EXAMPLES FOR PRACTICE.

72. Express 16 sidereal hours in mean solar time.

Ans. $15^h\ 57^m\ 22{\cdot}694^s$ mean solar time.

73. Express 14 mean solar hours in sidereal time.

Ans. $14^h\ 2^m\ 17{\cdot}995^s$ sidereal time.

(5.) Construction of "tables of time-equivalents."

By means of formulæ (1) and (2) the tables of time-equivalents given in the *Nautical Almanac* and in most collections of nautical tables may be computed.

These tables are used for readily converting intervals of sidereal time into equivalent intervals of mean solar time, and the converse; a few examples will show this.

74. Convert $8^h\ 43^m\ 51{\cdot}42^s$ sidereal time into mean solar time. By table

8^h (sidereal time)	. . .	7^h	58^m	$41{\cdot}36^s$	mean solar time.
43^m ,,	. . .		42	52·96	,,
51^s ,,	. . .			50·86	,,
$0{\cdot}42^s$,,	. . .			·42	,,
$\therefore 8^h\ 43^m\ 51{\cdot}42^s$ sidereal time $=$		8	42	25·6	,,

75. Convert $8^h\ 42^m\ 25{\cdot}6^s$ mean solar time into sidereal time. By table,

8^h (mean time)	8^h	1^m	$18{\cdot}85^s$	sidereal time.
42^m ,, ...		42	6·90	,,
25^s ,, ...			25·07	,,
0·6 ,, ...			·60	,,
∴ $8^h\ 42^m\ 25{\cdot}6^s$ mean time =	8	43	51·42	,,

EXAMPLES FOR PRACTICE.

76. Convert $10^h\ 10^m\ 30^s$ sidereal time into mean time.

Ans. $10^h\ 8^m\ 49{\cdot}984^s$.

77. Convert $10^h\ 10^m\ 30^s$ mean solar time into sidereal time.

Ans. $10^h\ 12^m\ 10{\cdot}29^s$.

PROBLEM V.

Given sidereal time at any instant, to find mean time at the same instant.

Let PQ represent the celestial meridian, AQ the celestial equator, A the first point of Aries, and m the mean sun.

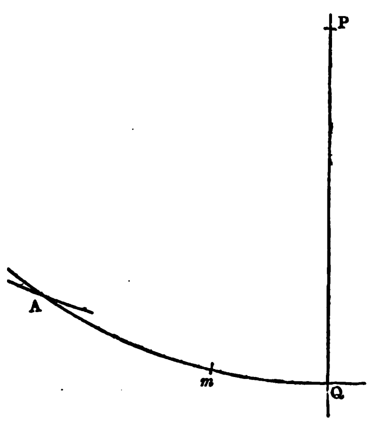
Then QA = sidereal time, Qm = mean time;

and Am = right ascension of the mean sun.

By the fig., Qm = QA − Am,

or mean time = sidereal time − right ascension of mean sun (adding 24 hours if necessary. See Prob. II.).

Hence it appears, that to find mean time we have only to subtract the right ascension of the mean sun for that instant of mean time from the given sidereal time, and the result will be mean time at that instant. But how shall we find the right ascension of the mean sun for that instant of mean time, since in the *Nautical Almanac* the right ascension of the mean sun is only recorded for every day at mean noon at Greenwich; and as we know not the time elapsed from mean noon at Greenwich, we cannot proportion for the change in the right ascension of the mean sun due to that elapsed time? We may proceed as follows. Let x = mean time required. Subtracting from sidereal time the right ascension of the mean sun at mean noon at the place, we get an approximate value of the mean time. Let this first approximation be denoted by t.



Now this time t is manifestly too great by the motion of the mean sun, expressed in time in the interval x; and this quantity is equal to $\cdot 0027379 \times x$, since the motion of the mean sun in the equator in 24 hours $= 3^m\ 56\cdot 555^s = \cdot 0027379$ of a day.

$$\therefore x = t - \cdot 0027379\, x.$$

$$\text{or } 1\cdot 0027379\, x = t, \therefore x = \cdot 9972696\, t.$$

And this coefficient of t is the same as the factor which is used to reduce an interval of sidereal time into mean time (p. 26). Hence it appears, that to find the mean time x it will be sufficient to correct the approximate time t (obtained by using the right ascension of the mean sun at noon) as if we were about to reduce sidereal time into mean time; *i.e.* we must subtract from t the acceleration of sidereal on mean time for the interval t, which quantity may be taken out of the Table of Time Equivalents.

78. Given sidereal time $= 3^h\ 40^m\ 45^s$, and the right ascension of the mean sun at mean noon at the place $= 12^h\ 35^m\ 24\cdot 14^s$; required mean time.

Mean time = sidereal time − RA mean sun.

Sidereal time	3^h	40^m	$45\cdot 00^s$ $(+\ 24^h)$
RA mean sun at noon	12	35	24·14
Mean time nearly	15	5	20·86 $= t$.
Cor. from table, 15^h .. 2^m $27\cdot 85^s$			
5^m $\cdot 82$			
21^s $\cdot 05$		2	28·72
Mean time	15	2	52·14

This result, however, is not quite correct, although nearer the truth than the quantity t; for we ought to have entered the table with the *correct* mean time, instead of the approximate time, $15^h\ 5^m\ 20\cdot 86^s$. A nearer approximation, however, may now be got by repeating the work, using the last estimated mean time, $15^h\ 2^m\ 52\cdot 14^s$, instead of $15^h\ 5^m\ 20\cdot 86^s$: thus,

Cor. for 15^h ..	2^m		$27\cdot 85^s$
	2^m		·33
	$52\cdot 14^s$		·14
		2	28·32
	15	5	20·86
Mean time =	15	2	52·54

If we approximate a third time, by using this last result instead of $15^h\ 2^m\ 52\cdot 14^s$, we shall find no difference in the correction; we may therefore conclude that the correct mean time is $15^h\ 2^m\ 52\cdot 54^s$.

In almost every problem in Nautical Astronomy in which are used

quantities taken from the *Nautical Almanac*, we shall find the results obtained are only approximate values of the quantities sought; and this arises from using an approximate time (called a Greenwich date) instead of the correct Greenwich time, which is seldom known. If the object of the problem is to find the correct time, we can make a second approximation, similar to the one above; but this, in the practical problems of Nautical Astronomy, is very seldom required.

EXAMPLES FOR PRACTICE.

79. Given sidereal time $= 12^h\ 10^m\ 10^s$, and the right ascension of the mean sun at mean noon at the place $= 1^h\ 42^m\ 14{\cdot}5^s$; required correct mean time. *Ans.* $10^h\ 26^m\ 12{\cdot}4^s$.

80. Given sidereal time $= 6^h\ 32^m\ 40{\cdot}5^s$, and the right ascension of the mean sun at mean noon $= 7^h\ 37^m\ 42{\cdot}4^s$; required correct mean time. *Ans.* $22^h\ 51^m\ 12{\cdot}7^s$.

PROBLEM VI.

To find at what time any heavenly body will pass a given meridian.

Let X be the given heavenly body on the meridian PQ, A the first point of Aries, and m the mean sun.

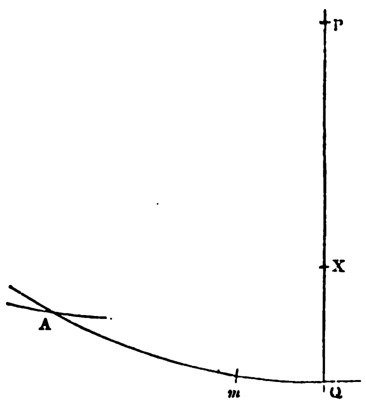
Then $\text{Q}m =$ mean time required,

$\text{A}m =$ right ascension of the mean sun at that time,

$\text{AQ} =$ star's right ascension;

and by the figure, $\text{Q}m = \text{AQ} - \text{A}m$,

or mean time $=$ star's RA $-$ RA mean sun.



From which expression the mean time of the star's transit may be found, as in the last problem.

First. Let the given meridian be that of Greenwich, for which the quantities in the *Nautical Almanac* are calculated.

81. Find at what time Antares passed the meridian of Greenwich on October 3, 1846; the star's right ascension being at that time $16^h\ 20^m\ 1^s$, and the right ascension of the mean sun at mean noon at Greenwich $12^h\ 47^m\ 13{\cdot}8^s$.

Mean time = star's RA — RA mean sun.

First approximation.

RA		16^h	20^m	$1{\cdot}0^s$
RA mean sun at noon .		12	47	13·8
Mean time nearly . . .		3	32	47·20
Cor. 3^h . . .	$29{\cdot}57^s$			
32^m . . .	5·25			
47^s	·13			34·95
Mean time more nearly .		3	32	12·25

Second approximation.

Proceed now as in the last problem, or more simply thus:

Cor. for $34{\cdot}95^s$			$0{\cdot}09^s$
1st approximation . . .	3^h	32^m	$12{\cdot}25^s$
Cor. mean time	3	32	12·34

EXAMPLES FOR PRACTICE.

82. Find at what time α Canis Majoris passed the meridian of Greenwich; the star's RA being $6^h\ 38^m\ 52{\cdot}2^s$, and the right ascension of mean sun at Greenwich mean noon being $11^h\ 6^m\ 2{\cdot}3^s$. Construct the figure, to show the positions of the first point of Aries and the mean sun with respect to the meridian. *Ans.* $19^h\ 29^m\ 38^s$ nearly.

83. Find at what time α Aquilæ passed the meridian of Greenwich, having given the right ascension of the star = $19^h\ 43^m\ 51{\cdot}5^s$, and the right ascension of the mean sun at Greenwich mean noon $0^h\ 6^m\ 40{\cdot}4^s$; and construct the figure. *Ans.* $19^h\ 33^m\ 58{\cdot}3^s$.

84. Find at what time α Leonis will pass the meridian of Greenwich, when its right ascension is $10^h\ 0^m\ 49{\cdot}76^s$, and the right ascension of the mean sun at Greenwich mean noon = $4^h\ 32^m\ 4{\cdot}68^s$; and construct the figure. *Ans.* $5^h\ 27^m\ 51{\cdot}24^s$.

Second. When the calculations are made for any other meridian than that of Greenwich, we must take into consideration the change of the mean sun's place corresponding to the difference of longitude between the two places. In practice, the correction of the RA of mean sun on this account is made in the Greenwich date, which is in fact the longitude in time applied to ship time.

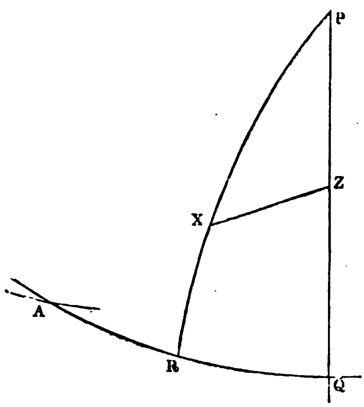
PROBLEM VII.

Given the altitude and declination of a heavenly body, and the latitude of the place of observation, to calculate the hour-angle of the heavenly body.

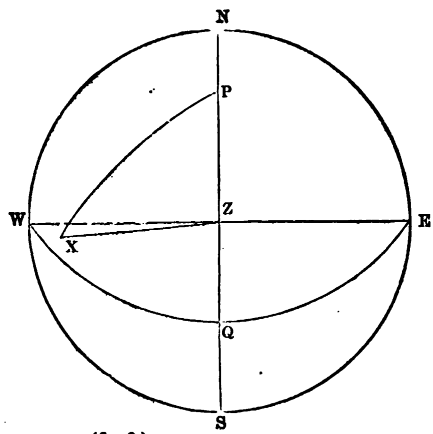
Let PZQ be the celestial meridian, P the pole, Z the zenith of spectator, and X the place of the heavenly body. Let AQ be the celestial equator; and through X draw PXR a circle of declination, and ZX a circle of altitude.

Then, in the spherical triangle ZPX, the three sides are given, to calculate P the hour-angle (*Trigonometry*, Part I., Rule VIII.)

For the polar distance PX = 90° − decl.;
zenith distance ZX = 90° − alt.;
and colatitude PZ = 90° − lat.;



(fig. 1.)



(fig. 2.)

85. Calculate the hour-angle of a heavenly body X, west of meridian, having given the latitude of observer=48° 40′ 45″ N., the star's declination =8° 25′ 45″ N., and star's altitude=13° 57′ 45″; and construct the figure.

Construction. (Fig. 2.)

Let NWSE represent the horizon, NZS the celestial meridian, WZE the prime vertical. Take NP=latitude; then P will represent the north pole of the heavens, since the altitude of the pole=latitude of spectator (for lat. ZQ = 90 − PZ and NP = 90 − PZ, ∴ ZQ = NP, the altitude of pole, see Ex. 8, p. 11), and PZ is the colatitude. Take PQ = 90°; then Q is a point in the celestial equator: through E and W, the east and west points, and Q, draw the circle WQE; this will represent the celestial equator (p. 6). Let X be the place of the heavenly body at the time of the observation (estimated according to its altitude and declination); and through X draw XP, a circle of declination, and XZ a circle of altitude. Then, in the spherical triangle ZPX, are given PX, the polar distance (=90°−decl.)=81° 34′ 15″, ZX the zenith distance (=90°−alt.)=76° 2′ 15″, and PZ the colat. (=90°−lat.)=41° 19′ 15″; to calculate ZPX, the hour-angle.

The hour-angle may be calculated either by the table of haversines, or by using only the common table of sines, &c., as follows. (See Rule VIII. in *Trigonometry*, Part I.)

BY HAVERSINES.

PX ..	81° 34′ 15″
PZ ..	41 19 15
	40 15 0
XZ ..	76 2 15
S ..	116 17 15
D ..	35 47 15
log. cosec. PX ..	0·004717
„ „ PZ ..	0·180275
$\frac{1}{2}$ log. hav. S ..	6·929099
$\frac{1}{2}$ log. hav. D ..	4·487496
log. hav. ZPX ..	9·601587
∴ ZPX=	5^h 13^m 39^s

BY THE COMMON RULE
(using only the table of log. sines, &c.).

PX ..	81° 34′ 15″
PZ ..	41 19 15
	40 15 0
ZX ..	76 2 15
S ..	116 17 15
D ..	35 47 15
$\frac{1}{2}$S ..	58 8 37
$\frac{1}{2}$D ..	17 53 37
log. cosec. PX ..	0·004717
„ „ PZ ..	0·180275
„ sin. $\frac{1}{2}$ S ..	9·929100
„ sin. $\frac{1}{2}$ D ..	9·487496
	2)19·601588
„ sin. $\frac{1}{2}$ ZPX ..	9·800794
$\frac{1}{2}$ ZPX ..	2^h 36^m $49{\cdot}5^s$
	2
∴ HOUR-ANGLE ZPX=	5 13 39

This latter method of finding the hour-angle is the one commonly adopted, since we require only such tables as Riddle's or Norie's. The concise and very superior method by means of haversines can only be used by those who have Inman's tables.

EXAMPLES FOR PRACTICE.

86. Find the hour-angle of a heavenly body, west of meridian, having given the latitude=47° 20′ N., the declination=11° 24′ 24″ N., and the altitude=42° 33′ 9″; and construct the figure.

Ans. Hour-angle=2^h 27^m 51^s.

87. Find the hour-angle of a heavenly body, east of meridian, having given the latitude=56° 10′ N., the declination=33° 11′ 44″ N., and the altitude=59° 3′ 59″; and construct the figure.

Ans. Hour-angle=2^h 0^m 30^s.

88. Find the hour-angle of a heavenly body, east of meridian, having given the latitude=50° 48′ 0″ N., the declination=14° 28′ 47″ S., and the altitude 3° 13′ 0″; and construct the figure.

Ans. Hour-angle=4^h 23 38^s.

PROBLEM VIII.

Given the hour-angle of the sun; to find ship mean time.

First. Let the sun be west of meridian. Let P be the pole, Z the zenith, and X the place of the sun. Then (fig. 1),

Apparent time=hour-angle ZPX, or arc QR;

and if the equation of time be *subtractive* from apparent time (and this is known from the *Nautical Almanac*), let m be the mean sun;

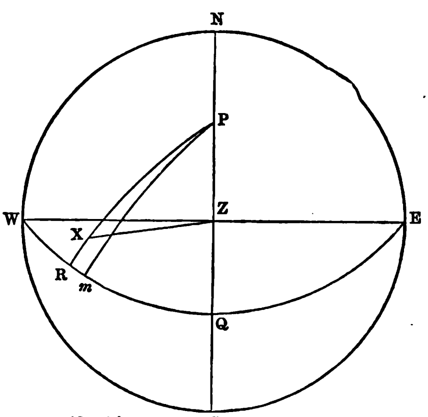
Then Qm=mean time required.

By the figure, Qm=QR—Rm,

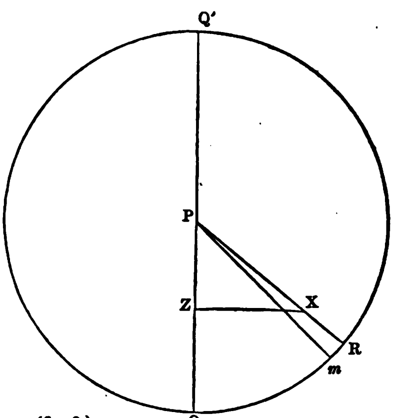
or mean time=sun's hour-angle—equation of time.

If the equation of time is additive to apparent time, then the mean sun m is in advance of the true sun X, and m should be placed in the figure on the other side of R. In this case mean time = sun's hour-angle + equation of time.

The proper sign to be used is always given in the *Nautical Almanac* with the equation of time.



(fig. 1.)



(fig. 2.)

Next. Let the sun be east of meridian. It will be more convenient to project the figure on the plane of the celestial equator. Let therefore P be the pole, Z the zenith, and X the place of the sun. Then (fig. 2),

Apparent time=arc QQ'R=24^h—hour-angle ZPX;

and if the equation of time is additive to apparent time, let m be the mean sun; then the arc QQ'm=mean time required.

And by the figure, QQ'm=QQ'R+Rm,

or mean time=24^h—sun's hour-angle+eq. of time.

EXAMPLES FOR PRACTICE.

89. Given the sun's hour-angle=$3^h\ 42^m\ 10^s$, west of meridian, and the equation of time=$14^m\ 10^s$ additive to apparent time; required mean time, and construct figure decl. 0°. *Ans.* Mean time=$3^h\ 56^m\ 20^s$.

90. Given the sun's hour-angle$=6^h\ 2^m\ 20^s$, east of meridian, and the equation of time$=3^m\ 42^s$, subtractive from apparent time; required mean time. Construct figure, decl. 10° N. *Ans.* Mean time$=5^h\ 53^m\ 58^s$ A.M.

PROBLEM IX.

Given the hour-angle of any other heavenly body, as a star; to find ship mean time.

We have to prove the following rule, viz.

(1.) When the star is *west* of meridian,

Mean time=star's hour-angle+star's right ascension
—right ascension of mean sun.

(2.) When the star is *east* of meridian,

Mean time$=24^h$—star's hour-angle+star's right ascension
—right ascension of mean sun.

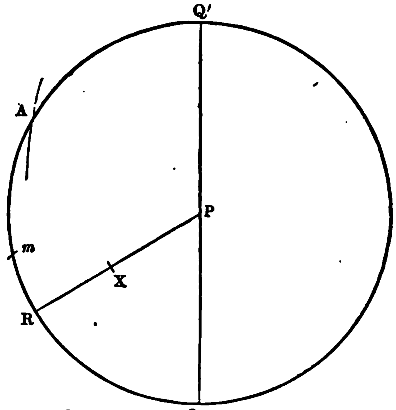
To do this we must show that these equations are true for all positions of the star, the mean sun, and first point of Aries, with respect to each other. The figures in this problem are projected on the plane of the equator.

First. Suppose the heavenly body to be *west* of meridian.

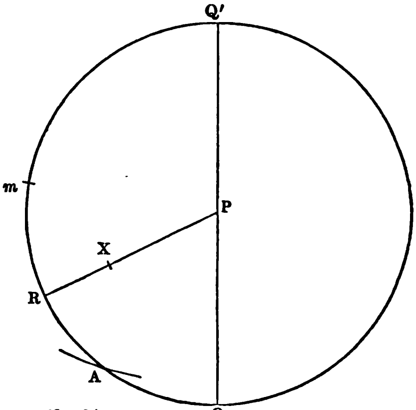
(*a.*) Let P be the pole, and X a heavenly body; and let A, the first point of Aries, and m, the mean sun, be situated with respect to X and to each other as in fig. 1.

Then QR = star's hour-angle, Am = right ascen. of mean sun.
AR = star's right ascen. and Qm = mean time required.

By the figure, Qm = QR + AR − Am,
or mean time = hour-angle + star's RA − RA mean sun.



(fig. 1.)



(fig. 2.)

(*b.*) Let the relative positions of A, m, and X, be as represented in fig. 2.

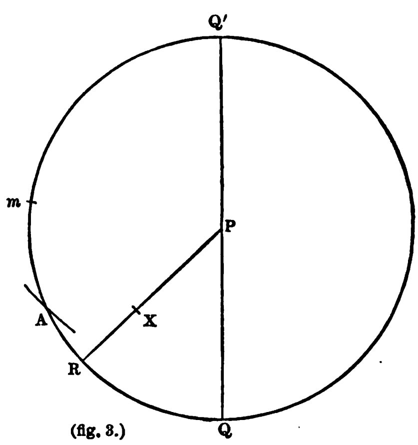
Then QR = star's hour-angle, $\quad$ Am = 24^h — RA mean sun,
AR = 24^h — star's right ascen. $\quad$ and Qm = mean time required.

By the figure, Qm=QR—AR+Am,
or mean time=star's hour-angle—(24^h—star's RA)+24^h—RA mean sun
=hour-angle+star's RA—RA mean sun.

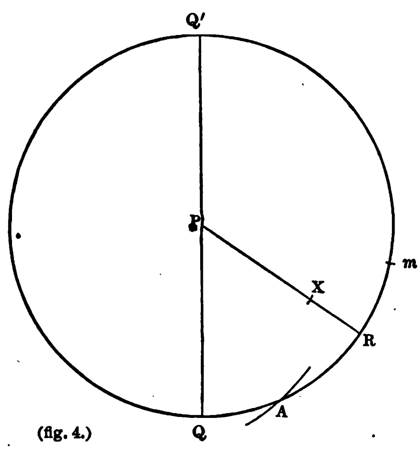
(*c.*) Let the relative positions of A, m, and X, be as represented in fig. 3.
Then QR = star's hour-angle, $\quad$ Am = 24^h — RA mean sun,
AR = star's right ascen. $\quad$ and Qm = mean time required.

Then, by the figure, Qm = QR + AR + Am,
or mean time = star's hour-angle + star's RA + 24^h — RA mean sun,
the same as before, by rejecting 24 hours. And the same result will be obtained for every other position of A, m, and X, with respect to each other : therefore, when the body is *west* of meridian,

Mean time = star's hour-angle + star's RA — RA mean sun
(rejecting, if necessary, 24 hours).



(fig. 3.)



(fig. 4.)

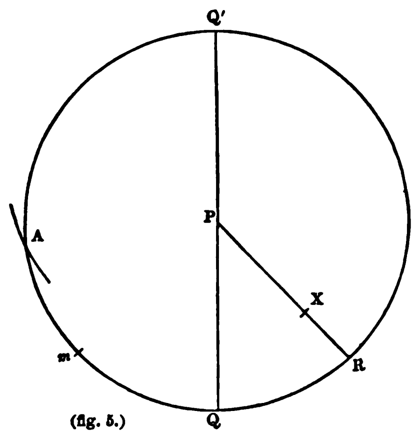
Next. Suppose the heavenly body to be *east* of meridian.

(*d.*) Let the relative positions of A, m, and X, be as represented in fig. 4.
Then QR = star's hour-angle, $\quad$ Am = right ascen. of mean sun,
AR = star's right ascen. $\quad$ and Qm = 24^h — mean time.

Then, by fig., Qm = QR + Am — AR,
or 24^h — mean time = hour-angle + RA mean sun — star's RA ;
∴ mean time = 24^h — hour-angle + star's RA — RA mean sun.

(*e.*) Let the relative positions of A, m, and X, be as represented in fig. 5.

Then QR = hour-angle, AR = star's RA,
Am = RA of mean sun,
and Qm = mean time required.



(fig. 5.)

By the figure, Qm = AR — Am — QR,
or mean time = star's RA — RA mean sun — hour angle.

This expression may be brought into the required form by adding and subtracting 24 hours; thus,

Mean time = 24^h — hour-angle + star's RA — RA mean sun — 24^h

the same as before, by rejecting 24 hours; and the same result may be obtained for any other positions of A, m, and X, with respect to each other. Therefore, when the body is *east* of meridian,

Mean time = 24^h — hour-angle + star's RA — RA mean sun
(rejecting or adding 24 hours, if necessary).

This form is adopted because, if the table of haversines is used to find the hour-angle, the quantity, 24^h — hour-angle, may be taken from the bottom of the page *by inspection.*

We have then in all cases this convenient rule: "add the star's right ascension to the angle taken from the table of haversines (remembering when the heavenly body is west of meridian to take the angle from the top, and when east, from the bottom), and from the result subtract the right ascension of the mean sun; the remainder will be ship mean time required (adding or rejecting 24 hours, if necessary)."

EXAMPLES FOR PRACTICE.

91. Given the hour-angle of α Aquilæ (west of meridian = $2^h\ 37^m\ 23^s$, star's right ascension = $19^h\ 43^m\ 52^s$, and right ascension of mean sun = $9^h\ 51^m\ 38^s$; find ship mean time, and construct the figure; star's declination 9° N. *Ans.* Mean time = $0^h\ 29^m\ 37^s$ A.M.

92. Given the hour-angle of α Cygni (east of meridian) = $4^h\ 1^m\ 35^s$, star's right ascension = $20^h\ 36^m\ 36^s$, and right ascension of mean sun = $9^h\ 17^m\ 7^s$; find ship mean time, and construct the figure, star's declination being = 45° N. *Ans.* Mean time = $7^h\ 17^m\ 54^s$.

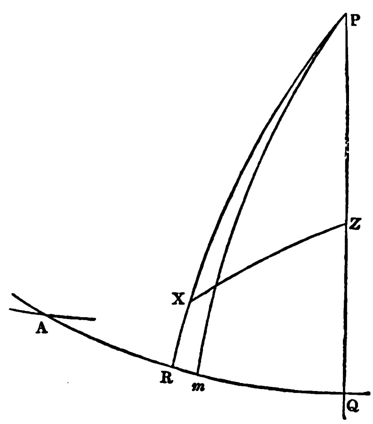
93. Given the hour-angle of β Geminorum = $1^h\ 15^m\ 57^s$ east of meridian, star's right ascension = $7^h\ 36^m\ 35^s$, and right ascension of mean sun = $22^h\ 34^m\ 43^s$; find ship mean time, and construct the figure; star's declination, 28° N. *Ans.* Mean time = $7^h\ 45^m\ 55^s$.

PROBLEM X.

Given the altitude of a heavenly body, and the time shown by a chronometer at the instant of observation; to determine the error of the chronometer on mean time at the place.

(1.) Let the heavenly body observed be the sun.

Let PZQ represent the celestial meridian, P the pole, Z the zenith, and X the place of the sun west of meridian, m the mean sun in the equator AQ; through m draw the circle of declination Pm, and through X draw the circle of declination PR and circle of altitude ZX. Then Qm represents mean time at the place where the observation was taken; the difference between mean time and the time shown by the chronometer at the instant of observation is the error of the chronometer on mean time at the place.



To find the value of Qm, we must calculate the hour-angle ZPX, which is in this case also apparent time, and subtract therefrom the angle XPm, the equation of time; the remainder, namely the angle ZPm, or arc Qm, will be the mean time required.

To compute the hour-angle ZPX. In the spherical triangle ZPX are given PZ = colat. of the place, PX = polar distance, or codeclination, taken out of the *Nautical Almanac*, and ZX = zenith distance, or co-altitude found from observation; to find the hour-angle ZPX = arc QR.

The equation of time Rm is taken out of the *Nautical Almanac*, as also the declination, and corrected for the time elapsed from noon by the common rule. (See any Practical Treatise on Navigation.)

Then Qm = QR − Rm,

or mean time = hour-angle − equation of time.

Let T = time shown by the chronometer at the instant of observation; then

$$\text{Error of chronometer} = \text{mean time} \sim \text{T}.$$

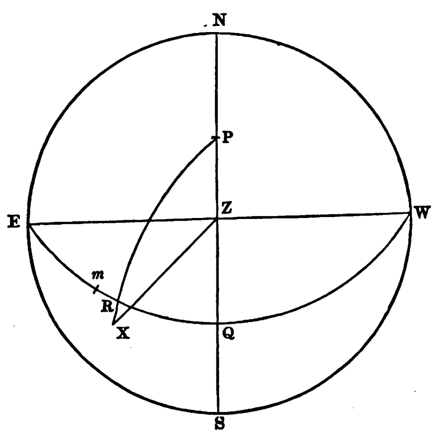
The position of the mean sun m with respect to the true sun X, that is, whether it is in advance or behind it, is known from the sign affixed to the equation of time in the *Nautical Almanac;* in the former case, we must add the equation of time instead of subtracting it, as in the figure.

TO FIND ERROR OF CHRONOMETER BY ALTITUDE OF THE SUN (WEST OF MERIDIAN).

94. Given the altitude of the sun $= 15° \ 10' \ 59''$ (west of meridian), the the latitude of observer $= 50° \ 48'$ N., the sun's declination $= 9° \ 1' \ 17''$ S., and the equation of time $= 14^m \ 25{\cdot}2^s$ (additive to apparent time); to find the error of chronometer on mean time at the place. The chronometer showed at the instant of observation $3^h \ 7^m \ 49{\cdot}8^s$.

Construction.

Let P be the pole, Z the zenith, and X the place of the sun, west of meridian and south of the equator. Through X draw PX, a circle of declination, and ZX, a circle of altitude. Then, in the triangle ZPX, the three sides are given, namely the polar distance PX $= 99° \ 1' \ 17''$, the zenith distance ZX $= 74° \ 49' \ 1''$, and the colatitude PZ $= 39° \ 12' \ 0''$; to compute the hour-angle ZPX, or arc QR, which is also apparent time (p. 16). Again, since the equation of time is additive to apparent time, let m be the position of the mean sun; then Rm represents the equation of time. To arc QR add Rm: the sum Qm will be mean time at the instant of observation; the difference between which and the time shown by the chronometer is the *error* of the chronometer on mean time at the place.



Calculation (1) *by haversines,* (2) *or by sines, &c.*

(Rule VIII., *Trigonometry,* Part I.)

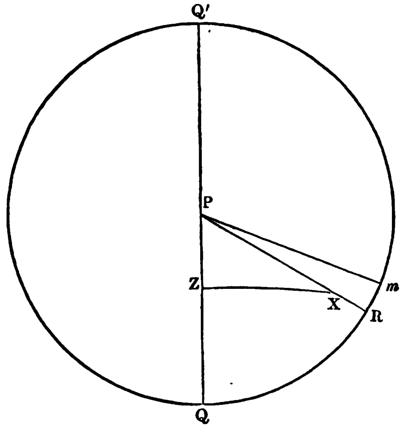
	°	′	″		°	′	″	
PX =	99	1	17	... 0·005405	99	1	17	... 0·005405
PZ =	39	12	0	... 0·199263	39	12	0	... 0·199263
	59	49	17	4·965043	59	49	17	9·965045
ZX =	74	49	1	4·115578	74	49	1	9·115578
	134	38	18	9·285289	134	38	18	19·285291
	14	59	44	$3^h \ 28^m \ 25^s$ app. time,	14	59	44	9·642645
				14 25·2 = Rm	67	19	9	1 44 12·5
∴ mean time =				3 42 50·2	7	29	52	2
Chronometer showed				3 7 49·8	∴ app. time =			3 28 25
∴ error of chronom. =				35 0·4 slow.				

TO FIND ERROR OF CHRONOMETER BY ALTITUDE OF THE SUN (EAST OF MERIDIAN).

95. Find the error of a chronometer on ship mean time, having given the latitude of observer $= 50° \ 48'$ N., the sun's altitude $= 39° \ 29' \ 18''$ (east of meridian), the declination $= 17° \ 33' \ 10''$ N., and the equation of time $=$ $3^m \ 48{\cdot}7^s$ (to be subtracted from apparent time); the chronometer showing at the instant of observation $8^h \ 26^m \ 59{\cdot}7^s$.

Construction.

Let the figure in this case be projected on the plane of the equator. Let P be the pole, Z the zenith, and X the place of the sun east of the meridian QQ′. Through X draw the circle of declination PR, and circle of altitude ZX. Then, in the spherical triangle ZPX, are given the three sides, to calculate the hour-angle ZPX, or arc QR, which measures it. Subtract this arc QR from 24 hours; the remainder is the arc QQ′R, which measures ship apparent time. Since the equation of time is subtractive from apparent time, let m be the position of the mean sun. From the arc QQ′R subtract Rm, the equation of time; the remainder, namely the arc QQ′m, represents ship mean time required: the difference between which and the time shown by the chronometer (remembering to add 12 hours to the chronometer time, if by so doing the difference between the times is made less) will be the error of the chronometer on mean time at the place.



Calculation of 24^h *— hour-angle* (see Rule p. 36).

PX...72°	26′	50″	......0·020705	
PZ...39	12	0	......0·199263	
33	14	50	4·824491	
ZX...50	30	42	4·176355	
83	45	32	9·220814	
17	15	52	$20^h \ 47^m \ 29^s = 24^h -$ hour-angle $=$ QQ′R,	
			$3 \ \ 48{\cdot}7 =$ Rm,	
			$20 \ \ 43 \ \ 40{\cdot}3 =$ QQ′m $=$ mean time.	
Chronom. $+ 12^h$...			$20 \ \ 26 \ \ 59{\cdot}7$	
∴ error of chronom. $=$			$16 \ \ 40{\cdot}6$ slow.	

If the common table of sines, &c., is used, and the body is east of meridian, as in this example, the hour-angle will be found as follows:

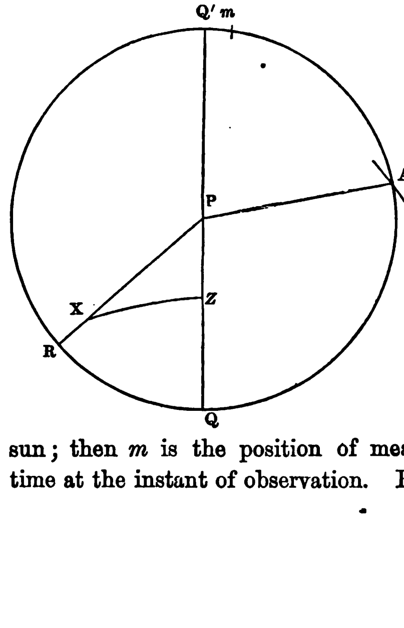
PX...72°	26′	50″......	0·020705	
PZ...39	12	0	0·199263	
33	14	50	9·824491	
ZX...50	30	42	9·176355	
83	45	32	19·220814	
17	15	52	9·610407	
41	52	46	1 36 15	
8	37.	56	2	
			3 12 30	= hour angle,
			24	
			20 47 30	= QQ′R.

TO FIND ERROR OF CHRONOMETER BY ALTITUDE OF A STAR OR MOON (WEST OF MERIDIAN).

96. Find the error of a chronometer on ship mean time, having given the latitude of observer=50° 48′ N., the altitude of Arcturus=44° 55′ 42″ (west of meridian), its declination=20° 0′ 15″ N., right ascension=14^h 8^m $30·5^s$, and right ascension of mean sun=4^h 48^m $7·5^s$; the chronometer showing at the instant of observation 0^h 14^m $22·3^s$.

Before constructing the figure, it will be better to compute the hour-angle, in order to estimate more correctly the position of the heavenly body with respect to the meridian : thus (p. 32),

Pol. dist. ....69°	59′	45″......	0·027026	
Colat. 39	12	0	0·199263	
30	47	45	4·788699	
Zen. dist. 45	4	18	4·094305	
75	52	3	9·109293	
14	16	33	∴ 2^h 48^m 8^s	=hour-angle.



Construction.

Let P be the pole, Z the zenith, and QPQ′ the celestial meridian. On the celestial equator take QR = 2^h 48^m 8^s, the hour-angle of the star; then the star is on the circle of declination PR, at a point X = 20° 0′ 15″ north of the equator. From R measure RQ′A = 14^h 8^m $50·5^s$, the star's right ascension; then A is the position of the first point of Aries. Again, from A measure Am = 4^h 48^m $7·5^s$, the right ascension of the mean sun; then m is the position of mean sun, and therefore QQ′m = mean time at the instant of observation. By Prob. IX., namely,

Mean time = hour-angle + star's RA — RA mean sun.

Thus hour-angle......	2^h	48^m	8^s
star's RA...............	14	8	30·5
	16	56	38·5
RA mean sun	4	48	7·5
∴ ship mean time =	12	8	31·0
and chronometer + 12^h =	12	14	22·3
∴ error of chronometer =		5	51·3 fast

TO FIND ERROR OF CHRONOMETER BY ALTITUDE OF A STAR OR MOON (EAST OF MERIDIAN).

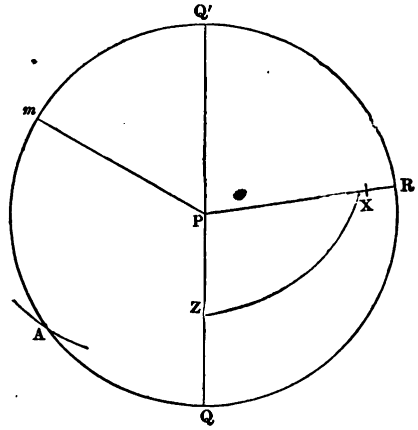
97. Find the error of a chronometer on ship mean time, having given the latitude of the place = 49° 57′ N., the altitude of Regulus = 8° 4′ 18″ (east of meridian), its declination equal 12° 39′ 49″ N., right ascension = 10^h 0^m 46^s, and the right ascension of the mean sun = 19^h 45^m 8^s; the chronometer showing at the instant of observation 8^h 2^m 10^s.

Before constructing the figure, compute the hour-angle, as in the last example, in order to estimate nearly the position of the heavenly body with respect to the meridian.

To compute hour-angle, or rather 24^h — hour-angle (see Rule, p. 36).

Pol. dist. ...77°	20^m	11″......	0·010693	
Colat. 40	3	0	0·191481	
37	17	11	4·935803	
Zen. dist....81	55	42	4·579546	
119	12	53	9·717523	
44	38	31	∴ 17^h 50^m 0^s = 24^h — hour-angle.	

Construction.



Let P be the pole, Z the zenith, and QPQ′ the celestial meridian. On the celestial equator take QQ′R = 17^h 50^m = 24^h — the star's hour-angle; then the star is on the circle of declination PR, at a point X = 12° 39′ 49″ north of the equator. From R measure RQA = 10^h 0^m 46^s, the star's right ascension; then A is the position of the first point of Aries. Again, from A measure ARm = 19^h 45^m 8^s, the right ascension of mean sun; then m is the position of the mean sun, and Qm measures mean time at the instant of observation.

Mean time may be computed from the formula (p. 34).

Mean time = (24^h — star's hour-angle) + star's RA — RA mean sun.

24^h — hour angle......	17^h	50^m	0^s
star's RA	10	0	46
	27	50	46
RA mean sun	19	45	8
∴ ship mean time =	8	5	38
Chronometer =	8	2	10
∴ error of chronometer =		3	28 slow.

EXAMPLES FOR PRACTICE.

98. Find the error of a chronometer on ship mean time, having given the latitude of the observer = 53° N., the sun's altitude equal = 23° 17′ 20″ (west of meridian), the declination = 2° 0′ 30″ N., and the equation of time = 6^m 58^s, to be added to apparent time; the chronometer showing at the instant of observation 3^h 24^m 46^s. *Ans.* Chronometer slow, 11^m 37^s.

99. Find the error of a chronometer on mean time at the place of observation, having given the latitude of the observer = 47° 30′ N., the sun's altitude = 20° 11′ 45″ (west of meridian), the declination = 20° 1′ 30″ N., and the equation of time = 3^m 45^s, to be subtractive from apparent time; the chronometer showing at the instant of observation 5^h 10^m 32^s.
Ans. 0^h 12^m 6^s slow.

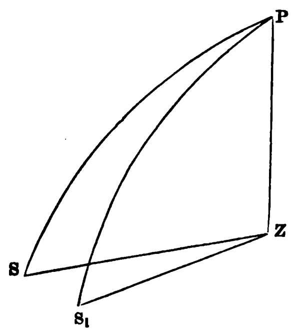
100. Find the error of a chronometer on mean time at the place of observation, having given the latitude of the observer = 50° 48′ N., the sun's altitude = 3° 4′ 15″ (east of meridian), the declination = 14° 35′ 20″ S., and the equation of time = 14^m 30^s, to be added to apparent time; the chronometer showing at the instant of observation 8^h 51^m 20^s. *Ans.* 1^h 1^m 16^s fast.

101. Find the error of a chronometer on mean time at the place of observation, having given the latitude of the observer = 49° 57′ N., the altitude of α Aquilæ (Altair), 37° 0′ 30″ (west of meridian), right ascension = 19^h 43^m 51^s, declination—8° 29′ 43″ N., and the right ascension of mean sun = 9^h 51^m 8^s; the chronometer showing at the instant of observation 11^h 42^m 17^s. *Ans.* 0^h 49^m 41^s slow.

102. Find the error of a chronometer on mean time at the place of observation, having given the latitude of the observer = 48° 50′ N., the altitude of α Cygni = 49° 33′ (east of meridian), right ascension = 20^h 36^m 36^s, declination = 44° 46′ 22″ N., and the right ascension of the mean sun = 9^h 15^m 39^s; the chronometer showing at the instant of observation 5^h 2^m 13^s.
Ans. 2^h 17^m 7^s slow.

103. Find the error of a chronometer on mean time at the place of observation, having given the latitude of the observer = 48° 50′ N., the altitude of Arcturus (east of meridian) = 47° 22′ 50″, right ascension = 14^h 9^m 10^s, declination = 19° 55′ 27″ N., and the right ascension of the mean sun = 1^h 50^m 8^s; the chronometer showing at the instant of observation 10^h 40^m 30^s. *Ans.* 0^h 58^m 24^s fast.

104. As it is impossible to observe at sea the altitude of a heavenly body with perfect accuracy, we will now show under what circumstances a small error committed in taking the altitude will produce the least error in the hour-angle computed from it. The problem about to be investigated will prove this very important fact, that the error in the hour-angle, and therefore in the mean time deduced from it, will be the least for a given error in the altitude *when the heavenly body is on the prime vertical*, that is, when its bearing is 8 points from the meridian. It is for this reason, namely that the unavoidable error of observation should produce the least effect on the time calculated from it, we are directed always to take our observations for time when the heavenly body bears as nearly east or west as possible. When we are able (and this may sometimes happen) to observe the altitude with very great exactness, this restriction need not be made, and we may take our observation for determining the time or hour-angle when the heavenly body is close to the meridian.



PROBLEM XI.

Given an error in the altitude, to find the corresponding error in the hour-angle.

Let P be the pole, Z the zenith, and S the true place of the heavenly body. Let S_1 be the supposed place, as determined by observation.

Let SZ the true zenith distance $=z$,
ZPS the true hour-angle $=h$;
S_1Z the erroneous zenith distance $=z_1$,
and ZPS_1 the hour-angle computed from it $=h_1$.
Then $h-h_1=$ error in the hour-angle, corresponding to
$z-z_1=$ the error in the altitude.

Let colat. $PZ=c$, and pol. dist. PS or $PS_1=p$.

$$\text{In triangle SPZ, } \cos. h=\frac{\cos. z-\cos. c\,.\cos. p}{\sin. c\,.\sin. p}\ (\textit{Trig.}\ \text{Art. } 55)$$

$$\text{,, } \quad S_1PZ,\ \cos. h_1=\frac{\cos. z_1-\cos. c\,.\cos. p}{\sin. c\,.\sin. p},$$

$$\therefore \cos. h-\cos. h_1=\frac{\cos. z-\cos. z_1}{\sin. c\,.\sin. p};\ \text{or,}$$

$$\sin. \tfrac{1}{2}(h+h_1)\,.\sin. \tfrac{1}{2}(h-h_1)=\frac{\sin. \tfrac{1}{2}(z+z_1)\,.\sin. \tfrac{1}{2}(z-z_1)}{\sin. c\,.\sin. p}.$$

Now, since $h-h_1$ and $z-z_1$ are small, $\frac{1}{2}(h+h_1)=h$, and $\frac{1}{2}(z+z_1)=z$ nearly: also $\sin. \frac{1}{2}(h-h_1)$ and $\sin. \frac{1}{2}(z-z_1)$ may be replaced by $\frac{1}{2}(h-h_1)$ and $\frac{1}{2}(z-z_1)$; since the sine of an arc is nearly equal to the arc itself when the angle is small (*Trig.* p. 101). Making these substitutions, we have

$$\sin. h \,.\, \tfrac{1}{2}(h-h_1)=\frac{\sin. z}{\sin. c \,.\, \sin. p} \,.\, \tfrac{1}{2}(z-z_1) \ldots (1)$$

Let $A = PZS$, the azimuth or bearing of the body; then, in triangle ZPS,

$$\frac{\sin. h}{\sin. A}=\frac{\sin. z.}{\sin. p} \text{ (\textit{Trigonometry}, Art 55);}$$

$$\text{or } \sin. h=\frac{\sin. A \,.\, \sin. z}{\sin. p}.$$

Substituting this value of sin. h in (1), we have

$$\frac{\sin. A \,.\, \sin. z}{\sin p} \,.\, (h-h_1)=\frac{\sin. z}{\sin. c \,.\, \sin. p} \,.\, (z-z_1)$$

$$\therefore h-h_1=\frac{1}{\sin. c \,.\, \sin. A} \,.\, (z-z_1)$$

$$\text{or error in hour-angle}=\frac{1}{\cos. \text{lat.} \,.\, \sin. A} \,.\, \text{error in altitude.}$$

From this expression, it is seen that the error in the hour-angle is the least for a given error in the altitude when the sin. azimuth, and therefore the azimuth itself, is the greatest; that is, when the heavenly body is due east or west.

The following examples will more clearly show this.

105. Find the error in the hour-angle corresponding to an error of 4 minutes in the altitude, taken at a place in latitude 50° 48′ N.

First. Supposing the bearing of the body was S. b. E.

Second. Supposing the bearing of the body was due east.

$$h-h_1=\frac{1}{\cos. \text{lat.} \,.\, \sin. A} \,.\, (z-z_1)$$

$$=\text{sec. lat.} \,.\, \text{cosec. } A \,.\, (z-z_1) \ldots \text{ in arc}$$

$$=\tfrac{1}{15} \text{ sec. lat.} \,.\, \text{cosec. } A \,.\, (z-z_1) \ldots \text{ in time.}$$

1. Bearing S. b. E.		2. Bearing East.	
log. sec. 50° 48′	0·199263		0·199263
„ cosec. 11° 15′	0·709764	log. cosec. 90°	0·000000
„ 4	0.602060	„ 4	0·602060
	1·511087		0·801323
„ 15	1·176091	„ 15	1·176091
„ $h-h_1$	0.334996	„ $h-h_1$	$\bar{1}$·625232
$\therefore h-h_1=2{\cdot}16^m=2^m\ 9{\cdot}7^s$		$\therefore h-h_1=0{\cdot}422^m=25{\cdot}3^s$.	

From these results it appears that the error in the hour-angle in one case is $2^m\ 9{\cdot}7^s$, and in the other $25{\cdot}3^s$., for the same error of 4 minutes in the altitude.

106. Find the error in the hour-angle corresponding to an error of two minutes in the altitude, at a place in latitude 2° 30′ N.

First. Supposing the bearing of the body was S. 15° E.

Second. Supposing the bearing of the body was S. 105° E.

Ans. Error at 1st bearing, $30{\cdot}9^s$.
„ 2d „ $8{\cdot}3^s$.

The more accurate methods of finding the error of a chronometer, by equal altitudes and by transit observations, will be given in Chapter VII.

TO FIND THE MEAN DAILY RATE OF A CHRONOMETER.

Assuming that the chronometer has gone nearly equably during the interval between two observations taken a few days apart, we may easily obtain its *mean daily rate* by taking the difference between the errors at the two observations, and dividing it by the number of intermediate days: but suppose the chronometer has (what not unseldom happens) an accelerated rate; then the above method of obtaining its rate will give very inaccurate results. A formula for determining the error of a chronometer at any given time, having an increasing or decreasing rate, will be found investigated (among other very useful information relating to chronometers) in the valuable publications of Admiral Shadwell.

CHAPTER III.

INVESTIGATION OF RULES FOR DETERMINING THE LATITUDE.

Latitude by meridian altitudes above and below pole.

PROBLEM XII.

Given the meridian altitudes of a heavenly body above and below the pole; to determine the latitude.

Let NWSE represent the horizon, P the pole, and Z the zenith; then NZS is the celestial meridian. Let xx_1d be a parallel of declination described by a heavenly body about the pole P, and x and x_1 its places when on the meridian; then (fig. 1)

$x\text{N}$=star's meridian altitude above the pole.

$x_1\text{N}$=star's meridian altitude below the pole.

Also PN=altitude of pole=latitude (p. 31).

and $\text{P}x=\text{P}x_1$=star's polar distance.

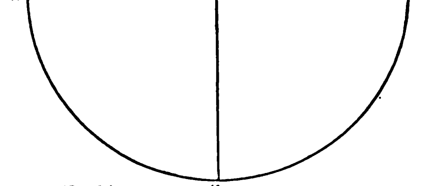
$$\text{By the fig., } \text{PN}=x\text{N}-x\text{P},$$

$$\text{and } \text{PN}=x_1\text{N}+x_1\text{P},$$

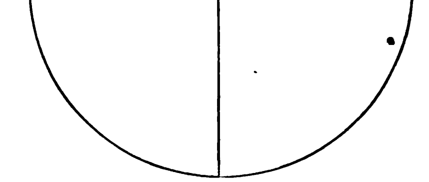
$$\therefore \text{ adding, } 2\text{PN}=x\text{N}+x_1\text{N} \;\; \text{.. since } x\text{P}-x_1\text{P},$$

$$\text{or } \text{PN}=\tfrac{1}{2}(x\text{N}+x_1\text{N}),$$

$$\therefore \text{ latitude}=\tfrac{1}{2}\{\text{alt. above pole}+\text{alt. below pole}\}.$$



(fig. 1.)



(fig. 2.)

107. Given the meridian altitude of a heavenly body above the north pole=70° 42′, and its meridian altitude below the pole=35° 18′. Construct a figure, and find by calculation the latitude (fig. 2).

Let NWSE represent the horizon, and NZS the celestial meridian. On NZ take NX$=70° 42'$, and N$X_1=35° 18'$; bisect XX_1 in P, then P is the north pole and PN the altitude of the pole=latitude required.

$$\therefore \text{ by above formula, lat.} = \frac{70° 42' + 35° 18'}{2} = 53° \text{ N.}$$

EXAMPLES FOR PRACTICE.

108. The meridian altitude of a heavenly body above the north pole was 82° 10′, and its meridian altitude below the pole was 40° 30′. Construct a figure, and find by calculation the latitude. *Ans.* Lat. 61° 20′ N.

109. The meridian altitude of a heavenly body above the south pole was 56° 42′ 10″, and the meridian altitude below the pole was 6° 45′ 32″. Construct a figure, and find by calculation the latitude.

Ans. Lat 31° 43′ 51″ S.

110. The meridian altitude of a heavenly body above the south pole was 60° 0′ 0″, and the meridian altitude below the pole was 45° 0′ 0″. Construct a figure, and find by calculation the latitude.

Ans. Lat. 52° 30′ 0″ S.

Latitude by meridian altitude below pole.

PROBLEM XIII.

Given the meridian altitude of a heavenly body below the pole, and its declination; to find the latitude.

Let NWSE represent the horizon, NZS the celestial meridian, P the pole of the heavens, and X a heavenly body on the meridian below the pole (fig. 1).

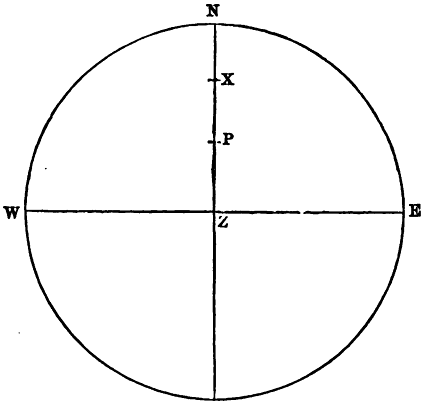
Then XN=meridian alt. of heavenly body,

PX=polar distance$=90°-$decl.

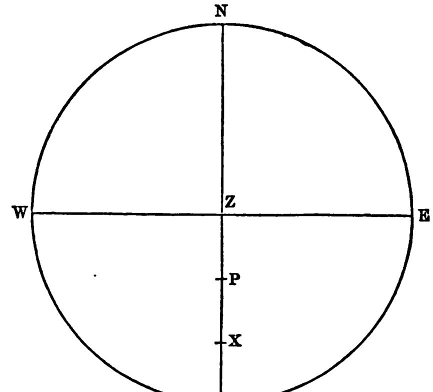
and PN=alt. of pole=latitude.

But by the figure, PN=XN+XP,

$\therefore$ latitude=merid. alt.$+90°-$decl.



(fig. 1.)



(fig. 2.)

111. Given the meridian altitude of α Centauri under the south pole $= 27^\circ\ 25'\ 58''$, and its declination $= 60^\circ\ 14'\ 40''$ S. Construct a figure, and find by calculation the latitude (fig. 2).

Let NWSE represent the horizon, and NZS the celestial meridian. In SZ take $SX = 27^\circ\ 25'\ 58''$; then X is the place of the star under the south pole. Take $XP = 29^\circ\ 45'\ 20''$, the star's south polar distance; then P is the south pole, and PS = alt. of south pole = lat. required.

$$\text{By formula, lat.} = \text{mer. alt.} + 90^\circ - \text{decl.}$$
$$\therefore \text{lat.} = 27^\circ\ 25'\ 58'' + 90^\circ - 60^\circ\ 14'\ 40''$$
$$= 57^\circ\ 11'\ 18''\ \text{S.}$$

EXAMPLES FOR PRACTICE.

112. Given the meridian altitude of a heavenly body below the north pole $= 32^\circ\ 42'\ 10''$, and its declination $= 54^\circ\ 42'$ N. Construct a figure, and find by calculation the latitude. *Ans.* Lat. $= 68^\circ\ 0'\ 10''$ N.

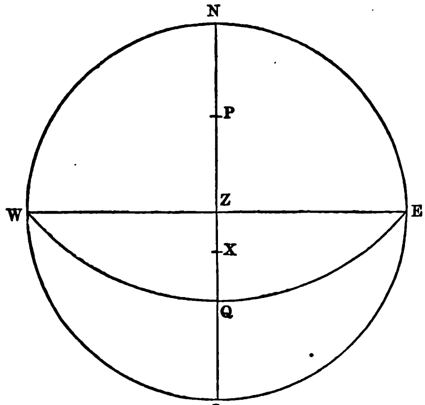
113. Given the meridian altitude of a heavenly body below the north pole $= 5^\circ\ 42'\ 15''$, and its declination $= 45^\circ\ 23'\ 12''$ N. Construct a figure, and find by calculation the latitude. *Ans.* Lat. $= 50^\circ\ 19'\ 3''$ N.

114. Given the meridian altitude of a heavenly body under the south pole $= 10^\circ\ 14'\ 17''$, and its declination $= 70^\circ\ 41'\ 15''$ S. Construct the figure, and find by calculation the latitude. *Ans.* $29^\circ\ 33'\ 2''$ S.

Latitude by meridian altitude above pole.

PROBLEM XIV.

Given the meridian altitude of a heavenly body above the pole, its declination, and the bearing of the zenith from the body; to find the latitude.

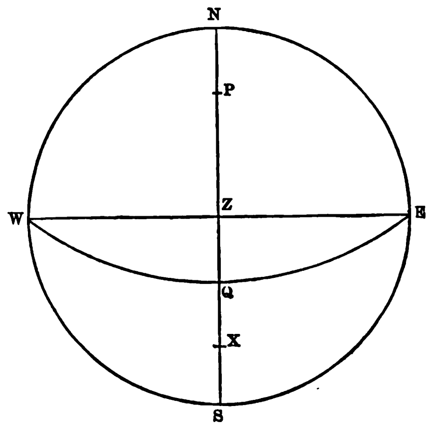


First. When the bearing of zenith from the body and declination have the same name.

Let NWSE represent the horizon, NZS the celestial meridian, and WZE the prime vertical. Let X be the place of a heavenly body on the meridian, zenith being north of the body, and XQ its declination north; through Q draw the great circle WQE, then WQE represents the celestial equator, and ZQ the latitude required. Since XS the altitude is given, therefore $XZ = 90^\circ - XS$ the zenith distance is known, and XQ its declination, can also be found from the *Nautical Almanac*.

By the fig., $ZQ = XZ + XQ$, or latitude = zenith dist. + decl.

Second. When the bearing of zenith from the body and declination have different names.



Let NWSE represent the horizon, NZS the celestial meridian, and WZE the prime vertical. Let X be the place of a heavenly body on the meridian, zenith being north of the body, and XQ its declination south; through Q draw the great circle WQE, then WQE represents the celestial equator, and ZQ is the latitude required. Since XS the altitude is given, therefore XZ=90°—XS the zenith distance is known, and its declination, XQ, is also given.

By the fig., ZQ=XZ—XQ, or latitude=zenith dist.—decl.

By constructing figures to suit different positions of the heavenly body with respect to the equator and zenith, we shall see that generally

latitude=mer. zen. dist. $\pm$ decl.

The sum, when bearing of zenith and the declination have the same name.

The difference, when bearing of zenith and the declination have different names.

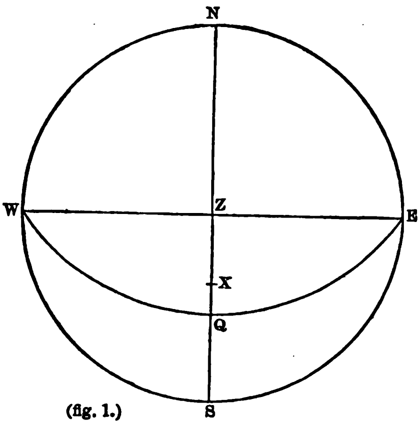
115. Given the meridian altitude of a heavenly body = 56° 10′ 15″, zenith north of the body, and its declination=15° 22′ 10″ N. Construct a figure, and find by calculation the latitude.

Let NWSE represent the horizon, NZS the celestial meridian, and WZE the prime vertical. Take SX = 56° 10′ 15″, and XQ = 15° 22′ 10″, and through Q draw the great circle WQE; then WQE is the celestial equator, and ZQ=latitude required.

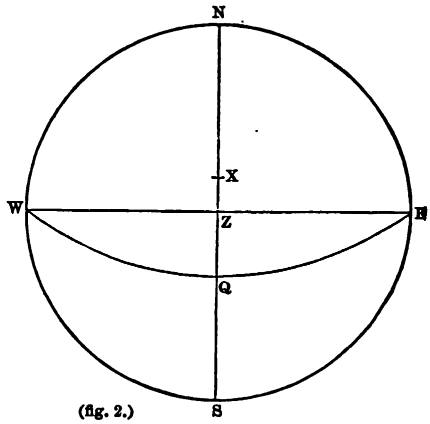
By the figure (fig. 1, next page), ZQ=ZX+XQ
=merid. zen. dist. + decl.

Calculation.

	90°		
	56	10′	15″=SX
	33	49	45 =ZX
	15	22	10 =XQ
∴ lat. =	49	11	55 N=ZQ



(fig. 1.)



(fig. 2.)

116. Given the meridian altitude of a heavenly body $= 72° 42' 15''$, zenith south of the body, and its declination $= 47° 32' 14''$ N. Construct a figure, and find by calculation the latitude (fig. 2).

Let NWSE represent the horizon, NZS the celestial meridian, and WZE the prime vertical. Take NX $= 72° 42' 15''$, and XQ $= 47° 32' 14''$, and through Q draw the great circle WQE; then WQE is the celestial equator, and ZQ = latitude required.

By the figure, ZQ = XQ − XZ,
or latitude = mer. zen. dist. ∼ decl.

Calculation.

90°			
72	42'	15''	
17	17	45	= XZ
47	32	14	= XQ
30	14	29	= ZQ

The latitude is evidently north by the figure.

EXAMPLES FOR PRACTICE.

117. Given the meridian altitude of a heavenly body $= 32° 42' 15''$, zenith north of the body, and its declination $= 10° 14' 32''$ N. Construct a figure, and find by calculation the latitude. *Ans.* Lat. $= 67° 32' 17''$ N.

118. Given the meridian altitude of a heavenly body $= 72° 48' 10''$, zenith north of the body, and its declination $= 25° 36' 42''$ S. Construct a figure, and find by calculation the latitude. *Ans.* Lat. $= 8° 24' 52''$ S.

119. Given the meridian altitude of a heavenly body $= 65° 35' 10''$, zenith south of the body, and its declination $= 6° 42' 10''$ S. Construct a figure, and find by calculation the latitude. *Ans.* Lat. $= 31° 7' 0''$ S.

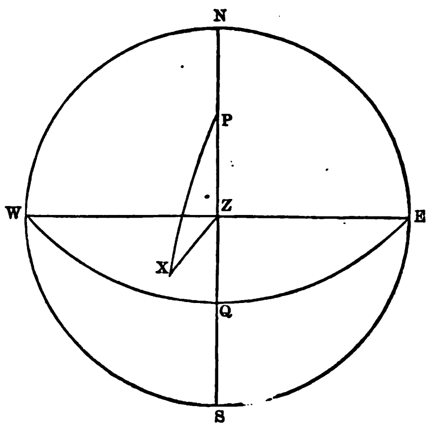
120. Given the meridian altitude of a heavenly body$=72° 44' 0''$, zenith south of the body, and its declination$=22° 42' 5''$ N. Construct a figure, and find by calculation the latitude. *Ans.* Lat.$=5° 26' 5''$ N.

Latitude by altitude near the meridian, ABOVE *pole.*

FIRST METHOD, *using estimated latitude.*

PROBLEM XV.

Given the altitude of a heavenly body *near the meridian*, above the pole, its hour-angle and declination, and the estimated latitude of the observer; to find the correct latitude.



Let X be a heavenly body, P the pole, and Z the zenith.

Let hour-angle $\text{ZPX}=h$,

latitude $\text{ZQ}=l$,

zenith dist. $\text{ZX}=z$,

decl. of $\text{X}=d$.

Then polar distance $\text{PX}=90°-d$, when the lat. and decl. have the same name, and polar distance$=90°+d$, when of different names.

In triangle ZPX, $\cos. \text{ZPX}=\dfrac{\cos. \text{ZX}-\cos. \text{PX} \,.\, \cos. \text{PZ}}{\sin. \text{PX} \,.\, \sin. \text{PZ}}$

(*Trigonometry*, Part II. art. 55),

or $\cos. h=\dfrac{\cos. z-\sin. l \,.\, \sin. d}{\cos. l \,.\, \cos. d}$, when l and d have same name,

or $\cos. h=\dfrac{\cos. z+\sin. l \,.\, \sin. d}{\cos. l \,.\, \cos. d}$, when l and d have different names.

In the first case, $\cos. l \,.\, \cos. d \,.\, \cos. h=\cos. z-\sin. l \,.\, \sin. d$

$$\therefore \cos. z-\sin. l \,.\, \sin. d=\cos. l \,.\, \cos. d \,.\, \cos. h$$
$$=\cos. l \,.\, \cos. d \,.\, (1-\text{vers.}\, h)$$
$$=\cos. l \,.\, \cos. d-\cos. l \,.\, \cos. d \,.\, \text{vers.}\, h$$
$$\therefore \cos. z+\cos. l \,.\, \cos. d \,.\, \text{vers.}\, h=\cos. l \,.\, \cos. d+\sin. l \,.\, \sin. d$$
$$=\cos. (l\sim d)=1-\text{vers.}\, (l\sim d)$$
$$\therefore \text{vers.}\, (l\sim d)=1-\cos. z-\cos. l \,.\, \cos. d \,.\, \text{vers.}\, h$$
$$=\text{vers.}\, z-\cos. l \,.\, \cos. d \,.\, \text{vers.}\, h.$$

In the second case, we shall find in a similar manner that

$$\text{vers.}\, (l+d)=\text{vers.}\, z-\cos. l \,.\, \cos. d \,.\, \text{vers.}\, h.$$

But $l \sim d$, or $l+d$, is the meridian zenith distance of the heavenly body at some place in the same latitude l as the spectator at the instant when its declination was d. Denote this *meridian* zenith distance by z. Then

$$\text{vers. z} = \text{vers. } z - \cos. l \,.\, \cos. d \,.\, \text{vers. } h.$$

The meridian zenith distance being thus found, corresponding to the declination d for the time of observation, the problem is thus reduced to finding the latitude from the meridian altitude of a heavenly body and its declination; the rest of the steps will therefore be the same as in Problem XIV.

To simplify the formula,

$$\text{vers. z} = \text{vers. } z - \cos. l \cos. d \,.\, \text{vers. } h,$$ by adapting it to the table of versines.

Assume $\text{vers. } \theta = \cos. l \,.\, \cos. d \,.\, \text{vers. } h$

$$= 2 \cos. l \cos. d \,.\, \text{hav. } h.$$ In logarithms,

$$\therefore \log. \text{vers. } \theta - 6 = \cdot 301030 + \log. \cos. l + \log. \cos. d + \log. \text{hav. } h - 30$$

(*Trig.* Part I. art. 31),

$$\therefore \log. \text{vers. } \theta = 6 \cdot 301030 + \log. \cos. l + \log. \cos. d + \log. \text{hav. } h - 30.$$

From which vers. θ may be found.

Then $\text{vers. z} = \text{vers. } z - \text{vers. } \theta$.

From which the meridian zenith distance z is found; and thence by last problem the latitude, for

$$\text{latitude} = \text{meridian zenith distance} \underset{\sim}{+} \text{decl.}$$

In the above expression for finding the latitude, the latitude itself is involved. Now, as this quantity is only known approximately, it may be sometimes necessary to repeat one part of the operation, using the latitude last found, which will be nearer the truth than the given estimated latitude; and if the result thus determined differs much from the last one used, to make a further calculation until the latitude deduced does not differ from the one last found: it is seldom, however, necessary to repeat the operation more than once. This will be seen in the following example, where the approximation has been carried beyond what was requisite, in order to show the practical utility of the method; and that with an error of one degree in the latitude a sufficiently correct result will be obtained without even a second approximation.

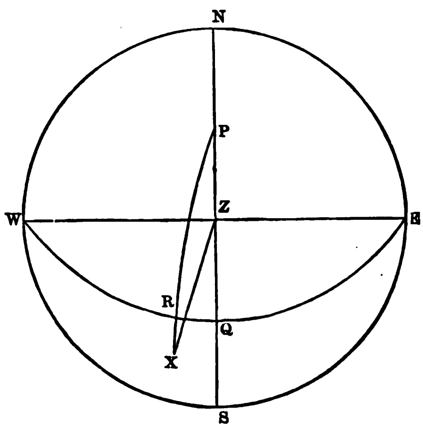
121. Given the altitude of a heavenly body$=27° \ 0'$, zenith north of the body, its declination$=11° \ 29' \ 8''$ S., and hour-angle*$=0^h \ 43^m \ 25^s$, the estimated latitude being 50° N. Construct a figure, and find by calculation the true latitude.

* The hour-angle of a star is easily deduced from the formulæ in Problem IX. p. 34, viz.

Mean time=star's hour-angle+star's RA−RA mean sun,

∴ hour-angle=mean time+RA mean sun−star's RA.

Let NWSE represent the horizon, P the pole, Z the zenith, and X the heavenly body near the meridian. Then we have given declination XR$=11° 29' 8''$ S.$=d$, zenith dist. ZX$=63° 0'=z$, hour-angle ZPX$=$ $0^h 43^m 25^s=h$, and estimated latitude$=90°-$PZ$=50°=l$; to find the latitude correctly.



By Calculation.

vers $\theta=2$ hav. h . cos. d . cos. l,

vers. mer. zen. dist. $=$vers. zen. dist. $-$vers. θ,

latitude$=$mer. zen. dist.$-$decl.

		2d approximation.	3d approximation.
const. log.	6·301030		
log. cos. d	9·991215		
„ hav. h	7·951588		
	4·243833	4·243833	4·243833
„ cos. l	9·808067	9·800853	9·800747
„ vers. θ	4·051900	4·044686	4·044580
∴ vers. θ =	11269	11084	11081
vers. z=	546009	546009	546009
∴ vers. mer. zen. dist.=	534740	534925	534928
∴ mer. zen. dist.=	62° 16′ 23″ N.	62° 17′ 6″ N.	62° 17′ 6″ N.
declination=	11 29 8 S.	11 29 8 S.	11 29 8 S.
∴ latitude=	50 47 15 N.	50 47 58 N.	50 47 58 N.

As the last two results come out the same, we see that the approximation is carried far enough. When the latitude is known within a few minutes of the truth, a second approximation will seldom be necessary. Thus, in the above example, if we had supposed the estimated latitude to be 50° 28′ N., which we know from the above result is 20′ wrong, and had worked with this, we should have found our *first* result to be 50° 47′ 40″ N., and this is only 18″ less than the truth; showing in this case that an error of 20 minutes in the latitude used in the calculation will not produce any important error in the latitude deduced from it.

EXAMPLE FOR PRACTICE.

122. Given the altitude of a heavenly body$=25° 59' 9''$, zenith north of the body, its declination$=13° 12' 41''$ S., and hour-angle$=0^h 2^m 17^s$, the

estimated latitude being 50° N. Construct a figure, and find by calculation the true latitude. *Ans.* 1st approximation, lat. $=50^\circ\ 48'\ 3''$ N.
2d " $=50\ \ 48\ \ 3$ N.

Latitude by altitude near meridian, ABOVE *pole.*

SECOND METHOD, *without using estimated latitude.*

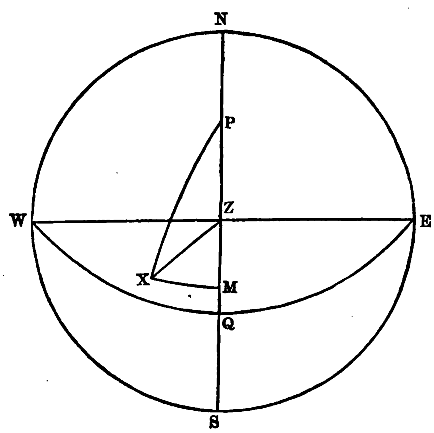
A *direct* method of finding the latitude from an observation near the meridian, in which the latitude by account is not required, may be obtained from the following investigation:

PROBLEM XVI.

Given the altitude of a heavenly body off the meridian above the pole, its hour-angle and declination; to find the latitude.

Let NWSE represent the horizon, NZS the celestial meridian, and X a heavenly body near the meridian, P the pole, and Z the zenith.

Let hour-angle ZPX$=h$, polar distance PX$=p$,
zenith distance ZX$=90^\circ-a$, a being the altitude.



From X drop a perpendicular XM upon the meridian.

Let PM$=x$, ZM$=y$, and XM$=z$.

Then the colatitude PZ$=x-y$, when the perpendicular XM does not fall between P and Z, as in the figure; and colatitude PZ$=x+y$, when the perpendicular falls between P and Z: this will be seen by constructing a figure. (The position of perpendicular XM with respect to the points P and Z may in most cases be easily determined by the observer when the altitude is taken.)

1. To find PM or x. In right-angled triangle PMX,

$$\cos. h=\cot. p\ .\ \tan. x,\ \therefore\ \tan. x=\tan. p\ .\ \cos. h \ldots . (1)$$

2. To find ZM or y. In right-angled triangle XPM,

$$\cos. p=\cos. z\ .\ \cos. x,$$

ZXM, $\cos.$ YZ or $\sin. a=\cos. y\ .\ \cos. z$;

dividing, to eliminate $\cos. z$, we have $\dfrac{\cos. p}{\sin. a}=\dfrac{\cos. x}{\cos. y}$,

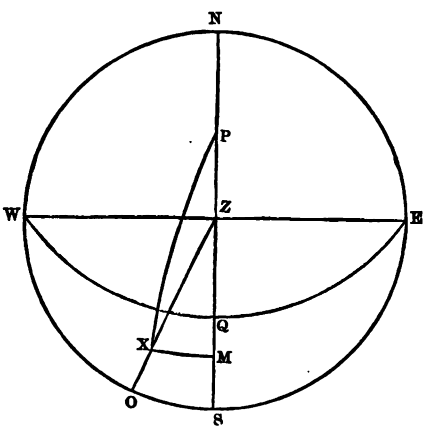
$$\therefore\ \cos. y\ .\ \cos. p=\cos. x\ .\ \sin. a,$$

$$\text{or } \cos. y=\sec. p\ .\ \cos. x\ .\ \sin. a\ .\ .\ (2)$$

Formulæ (1) and (2) will determine x and y; and thence the colatitude $=x \pm y$ is known.

In finding the latitude by formulæ (1) and (2), it will be necessary to attend to the magnitude of the polar distance; for tan. x will be positive or negative according as the polar distance is less or greater than 90°; but the value of tan. x, and also that of cos. y, are readily found in the usual manner by putting the proper signs, + or −, over the given quantities, as directed in the Rule, Art. 31, *Trigonometry*, Part I.

123. Given the altitude of a heavenly body=27° 0′, zenith north of the body, its decl. = 11° 29′ 8″ S., and hour-angle = $0^h\ 43^m\ 25^s$. Construct a figure, and find by calculation the latitude.



Let NWSE represent the horizon, NZS the celestial meridian, and XO the altitude of the heavenly body = 27° 0′. Take XP = 101° 29′ 8″, the polar distance; then P is the north pole. Let PQ=90°, and through Q draw the celestial equator WQE; then PZ=colatitude to be determined. Let fall the perpendicular XM upon the meridian.

By Formulæ (1) *and* (2).

$$\overset{-}{\underline{\text{tan. } x}} = \overset{+}{\text{cos. } h} \cdot \overset{-}{\text{tan. } p},$$

$$\overset{+}{\underline{\text{cos. } y}} = \overset{-}{\text{sec. } p} \cdot \overset{-}{\text{cos. } x} \cdot \overset{+}{\text{sin. } a},$$

where x=PM, y=ZM, and ∴ colat. PZ=$x-y$.

To find x.		To find y.	
log. cos. h	9·992160		
„ tan. p	0·692098	log. sec. p	0.700883
∴ „ tan x	10·684258	„ cos. x	9·306634
Arc	78° 18′ 40″	„ sin. a	9·657047
	180	„ cos. y	9·664564
∴ x=	101 41 20		∴ y=62° 29′ 20″

$$\begin{array}{rrrl} x= & 101° & 41' & 20'' \\ y= & 62 & 29 & 20 \\ \hline x-y= & 39 & 12 & 0 \\ & 90 & & \\ \hline \therefore \text{lat.}= & 50 & 48 & 0 \text{ N.} \end{array}$$

EXAMPLES FOR PRACTICE.

124. Given the altitude of a heavenly body=50° 52′ 29″, zenith north of the body, its decl.=11° 44′ 58″ N., and hour-angle=$0^h\ 12^m\ 4^s$. Construct a figure, and find by calculation the latitude. *Ans.* 50° 47′ 45″ N.

125. Given the altitude of a heavenly body=46° 29′ 15″, zenith north of the body, its decl.=7° 51′ 53″ N., and hour-angle=$0^h\ 33^m\ 51^s$. Construct a figure, and find by calculation the latitude. *Ans.* 50° 48′ 30″ N.

126. Given the altitude of a heavenly body=45° 42′ 30″, zenith south of the body, its declination=7° 51′ 0″ S., and hour-angle=$0^h\ 22^m\ 14^s$. Construct a figure, and find by calculation the latitude. *Ans.* 51° 55′ 0″ S.

Since ship mean time, from which the hour-angle has to be deduced (p. 52, note), is generally only approximately known at sea, let us now inquire under what circumstances an error in the ship time will produce the least error in the latitude determined from it. The investigation will be similar to the one in p. 44; and we shall be able to show that an error in the ship time will produce a less error in the latitude the smaller the bearing of the body is from the meridian. It is for this reason that we have assumed in the preceding problems that the altitude for determining the latitude is taken *near* noon. It is evident, however, that when the time is known very nearly, that is, within a few seconds of the truth, we need not be restricted to taking the altitudes near the meridian; but the above rule may be extended to observations taken when the bearing of the heavenly body is 3 or 4 or even more points from the meridian.

PROBLEM XVII.

Given a small error in the hour-angle of a heavenly body or time at the ship; to find the corresponding error in the latitude deduced from it (fig. next page).

Let SPZ$=h$, the true hour-angle,
$\text{S}_1\text{PZ}=h_1$, the erroneous hour-angle,
SZ$=z$, the true zenith distance,
$\text{S}_1\text{Z}=z_1$, the erroneous zenith distance,
$\text{PS}=\text{PS}_1=p$, the polar distance,
PZ$=c$, the colatitude.

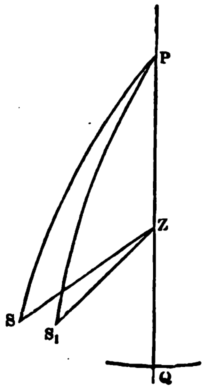
In triangle SPZ, $\cos. h=\dfrac{\cos. z-\cos. c \,.\, \cos. p}{\sin. c \,.\, \sin. p}$,

„ S_1PZ, $\cos. h_1=\dfrac{\cos. z_1-\cos. c \,.\, \cos. p}{\sin. c \,.\, \sin. p}$,

$\therefore \cos. h-\cos. h_1=\dfrac{\cos. z-\cos. z_1}{\sin. c \,.\, \sin. p}$,

or $\sin. \frac{1}{2}(h+h_1) \,.\, \sin \frac{1}{2}(h-h_1)=\dfrac{\sin. \frac{1}{2}(z+z_1) \,.\, \sin. \frac{1}{2}(z-z_1)}{\sin. c \,.\, \sin. p}$.

But since $h=h_1$ nearly, and $z=z_1$ nearly, we may assume $h=\frac{1}{2}(h+h_1)$, $z=\frac{1}{2}(z+z_1)$, $\frac{1}{2}(h-h_1)=\sin.\ \frac{1}{2}(h-h_1)$, and $\frac{1}{2}(z-z_1)=\sin.\ \frac{1}{2}(z-z_1)$.



Making these substitutions and cancelling,

$$\therefore \sin.\ h\ .\ (h-h_1)=\frac{\sin.\ z}{\sin.\ c\ .\ \sin.\ p}\ .\ (z-z_1).$$

Let the azimuth PZS be denoted by A; then, in triangle ZPS, $\frac{\sin.\ h}{\sin.\ A}=\frac{\sin.\ z}{\sin.\ p}$; $\therefore \sin.\ h=\frac{\sin.\ A\ .\ \sin.\ z}{\sin.\ p}$.

Substituting this value of sin. h, we have

$$\frac{\sin.\ A\ .\ \sin.\ z}{\sin.\ p}\ .\ (h-h_1)=\frac{\sin.\ z}{\sin.\ c\ .\ \sin\ p}\ .\ (z-z_1);$$

$$\therefore z-z_1=\sin.\ c\ .\ \sin.\ A\ .\ (h-h_1),$$

or error in the zenith distance=cos. lat.. sin. azimuth. error in hour-angle.

But if we assume the heavenly body to be near the meridian, the zenith distance observed is nearly equal to the meridian zenith distance (=lat. $\pm$ decl.). Therefore an error in the observed zenith distance will produce the same error (nearly) in the latitude, or $z-z_1$=error in latitude nearly.

Hence error in lat.=cos. lat. sin. azimuth. error in hour-angle nearly.

From this formula it appears that the error in the latitude produced by a small error in the time or hour-angle will increase as the azimuth or bearing of the heavenly body increases: hence the rule always directs the observation to be taken as near to the meridian as possible when the time at the ship is uncertain within a minute or two. The necessity for this restriction will be clearly shown by means of an example.

127. Find the error in the latitude corresponding to an error of 3 minutes (=45′ in arc) in the hour-angle at a place in lat. 50° 48′ N.

First. When the bearing of body is 1 point.

Second. When the bearing of body is 1°.

Error= cos. l . sin. bearing $(h-h_1)$.

1.		2.	
log. cos. l	9·800737	log. cos. l	9·800737
„ sin. 11° 15′	9·290236	„ sin. 1°	8·241855
„ 45	1·653213	„ 45	1·653213
„ error	0·744186	„ error	$\bar{1}$·695805
	∴ error=5·5′=5′ 30″		∴ error=0·5′=30″

We thus see that in this case the error in the latitude is eleven times greater in one observation than in the other for the same error in the ship time. But if the time is known within a second or two of the truth, the observation may be taken at a considerable distance from the meridian without producing any error in the latitude on that account. The following altitudes,

taken at the Naval College (lat 50° 48′ 3″ N.), will show this, the time of the observations being known to be correct within a few seconds.

EXAMPLES FOR PRACTICE.

128. Given the altitude of a heavenly body=45° 31′ 26″, zenith north of the body, its decl.=8° 14′ 1″ N., and hour-angle=$1^h\ 2^m\ 8^s$. Construct a figure, and find by calculation the latitude. *Ans.* Lat.=50° 48′ 15″ N.

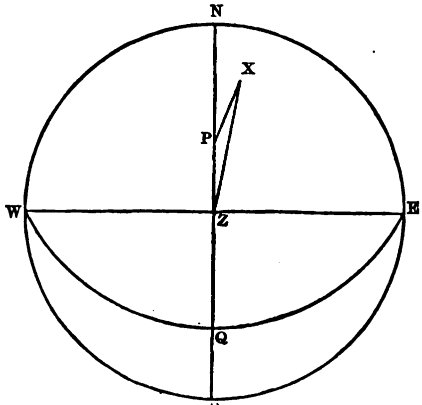
129. Given the altitude of a heavenly body=30° 23′ 29″, zenith north of the body, its decl.=2° 27′ 35″ S., and hour-angle=$2^h\ 5^m\ 28^s$. Construct a figure, and find by calculation the latitude. *Ans.* Lat.=50° 47′ 59″ N.

130. Given the altitude of a heavenly body=35° 4′ 7″, zenith north of the body, its decl.=10° 54′ 26″ N., and hour-angle=$3^h\ 5^m\ 36^s$. Construct a figure, and find by calculation the latitude. *Ans.* Lat.=50° 48′ 30″ N.

Latitude by altitude near the meridian, BELOW *pole.*

PROBLEM XVIII.

Given the altitude of a heavenly body near the meridian *below the pole,* its hour-angle and declination, and the estimated latitude of the observer; to find the true latitude.



Let NWSE represent the horizon, NZS the celestial meridian, P the pole, Z the zenith, and X a heavenly body near the meridian below the pole.

Let hour-angle ZPX=h, zenith distance XZ=z,
latitude PN=l, declination of X=d.

In the spherical triangle ZPX, $\cos. h = \frac{\cos. z - \sin. l \,.\, \sin. d}{\cos. l \,.\, \cos. d}$,

$$\therefore \cos. d \,.\, \cos. l \,.\, \cos. h = \cos. z - \sin. l \,.\, \sin. d,$$

$$\text{or} - \cos. d \,.\, \cos. l \,.\, \cos. (12^h - h) = \cos. z - \sin. l \,.\, \sin. d,$$

$$\therefore \cos. z - \sin. l . \sin. d = -\cos. d . \cos. l . \{1 - \text{vers.}\ (12 \sim h)\}$$
$$= -\cos. d . \cos. l + \cos. d . \cos. l . \text{vers.}\ (12 \sim h),$$
$$\therefore \cos. z + \cos. d . \cos. l - \sin. d . \sin. l = \cos. d . \cos. l . \text{vers.}\ (12 - h),$$
$$\text{or } \cos. z + \cos. (d+l) = \cos. d . \cos. l . \text{vers.}\ (12 - h),$$
$$\text{or} - \cos. (180^\circ - z) + \cos. (d+l) = \cos. d . \cos. l . \text{vers.}\ (12 - h),$$
$$\therefore \cos. (d+l) = \cos. (180^\circ - z) + \cos. d . \cos. l . \text{vers.} (1 \sim h) .. (1)$$

But $l+d = 90^\circ - \text{colat.} + 90^\circ - \text{pol. dist.} = 180^\circ - (\text{colat.} + \text{pol. dist.})$
$= 180^\circ - \text{a mer. zen. dist.} = 180^\circ - (90^\circ - \text{a mer. alt.})$
$= 90^\circ + \text{a mer. alt.}$

and $180^\circ - z = 180^\circ - (90^\circ - \text{alt.}) = 90^\circ + \text{alt.}$

Making these substitutions in (1), we have

$$\cos. (90^\circ + \text{mer. alt.}) = \cos. (90^\circ + \text{alt.}) + \cos. d . \cos. l . \text{vers.}\ (12 \sim h).$$
$$\text{or vers.}\ (90^\circ + \text{mer. alt.}) = \text{vers.}\ (90^\circ + \text{alt.}) - \cos. d . \cos. l . \text{vers.}\ (12 \sim h.$$

From which formula the meridian altitude, increased by 90°, of the heavenly body at some place in latitude l at the instant when its declination was d, may be found; and the observation is thus reduced to that of finding the latitude by a *meridian* altitude under the pole.

To adapt to logarithms the formula

$$\text{vers.}\ (90^\circ + \text{mer. alt.}) = \text{vers.}\ (90^\circ + \text{alt.}) - \cos. d . \cos. l . \text{vers.}\ (12 \sim h),$$

Let $\text{vers.}\ \theta = \cos. l . \cos. d . \text{vers.}\ (12 \sim h)$,

$\therefore \text{vers.}\ \theta = 2 \cos. l . \cos. d . \text{hav.}\ (12 \sim h)$,

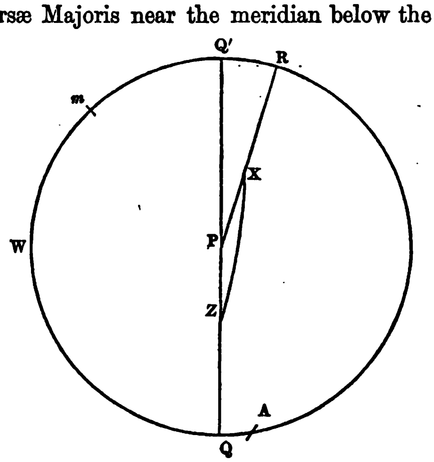
or $\log. \text{vers.}\ \theta = 6{\cdot}301030 + \log. \cos. l + \log. \cos. d + \log. \text{hav.}\ (12 \sim h) - 30$,

from which expression vers. θ may be found.

Then $\text{vers.}\ (90^\circ + \text{mer. alt.}) = \text{vers.}\ (90^\circ + \text{alt.}) - \text{vers.}\ \theta$, which determines the value of $90^\circ + \text{mer. alt.}$

From this we find the latitude as in the rule for finding the latitude under the pole, viz. $\text{Lat.} = 90^\circ + \text{mer. alt.} - \text{decl.}$ .. (p. 47).

131. Given the altitude of α Ursæ Majoris near the meridian below the pole $= 26^\circ\ 10'\ 0''$ at $9^h\ 2^m$ P.M. mean time at the place, the estimated latitude being $53^\circ\ 0'$ N.; also the following data from the *Nautical Almanac:* star's right ascension $= 10^h\ 54^m\ 55^s$, star's declination $= 62^\circ\ 30'\ 40''$ N., right ascension of mean sun $= 14^h\ 54^m\ 21^s$. Construct a figure, and find by calculation the true latitude.



Construction.

Let QWQ′ represent the celestial equator, QPQ′ the celestial

meridian, and P the pole. Take Qm=9^h 2^m; then m is the place of the mean sun. From m measure mQ′A=14^h 54^m 21^s; then A is the place of the first point of Aries. Again, from A take AR=10^h 54^m 55^s; and draw PR, a circle of declination passing through the place of the heavenly body. Let RX=62° 30′ 52″=star's declination, and QZ=53° 0′=estimated latitude; and through Z draw ZX, a circle of altitude: then in the spherical triangle ZPX are given PX the polar distance, ZX the zenith distance, and ZPX the hour-angle, with ZQ the estimated latitude; to find the correct latitude.

By Formulæ.

Hour-angle=mean time + RA mean sun − star's RA (p. 52, note)

$$\text{vers. } \theta = \cos. l \,.\, \cos. d \,.\, \text{hav. } (12 \sim h) \;\ldots\ldots\ldots\ldots (1)$$

$$\text{vers. } (90° + \text{mer. alt.}) = \text{vers. } (90° + \text{alt.}) - \text{vers. } \theta \;\ldots\ldots\ldots\ldots (2)$$

$$\text{and latitude} = 90° + \text{alt.} - \text{decl.} \;\ldots\ldots\ldots\ldots (3)$$

To find $12 \sim h$.

	h	m	s		
Mean time.........	9	2	0		6·301030
RA mean sun......	14	54	21	cos. l	9·779463
RA meridian	23	56	21	cos. d	9·664243
Star's RA	10	54	55	hav. $(12 \sim h)$	8·251781
Hour-angle	13	1	26		3·996517
∴ $12 \sim h$=	1	1	26		

vers. θ..................	9920
90° + alt. = 116° 10′ vers...	1440984
vers. (90° + mer. alt.)...	1431064
∴ 90° + mer. alt. =	115° 32′ 6″
decl. =	62 30 40
∴ latitude =	53 1 26 N.

Latitude by altitude of POLE STAR.

PROBLEM XIX.

Given the altitude of the pole star off the meridian, and the tabular correction; to find the latitude.

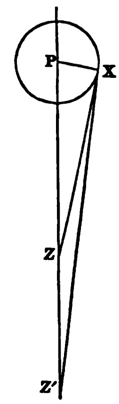
The bearing of α Ursæ Minoris (the pole star), which lies within 2° of the north pole, is very small in any latitude below 70° N., so that an error of a few minutes in the hour-angle is of little consequence (p. 57) when the latitude is to be determined approximately.

The rules deduced from the preceding problems for finding the latitude by an altitude near the meridian, should be applied when the latitude is required to a greater degree of accuracy than is necessary for the common purposes of navigation; but the latitude can be obtained sufficiently near for practical purposes, and with little calculation, by applying to the altitude of the pole star a *correction*, which has been formed into a table, and called the "correction of the pole star."

This correction may be calculated in the following manner.

Investigation of the tabular "correction of the pole star."

Let P be the pole, X the place of the pole star, Z the zenith of a place, and Z′ the zenith of another place a few degrees from the former. The difference of the zenith distance of the star and the colatitude will be nearly the same at both places; that is, PZ − ZX = PZ′ − Z′X nearly, since the polar distance PX is small. Call this difference d.



Then d = PZ − ZX = colat. − zen. dist.

= 90° − lat. − 90° + alt. = alt. − lat.

∴ latitude = alt. − d.

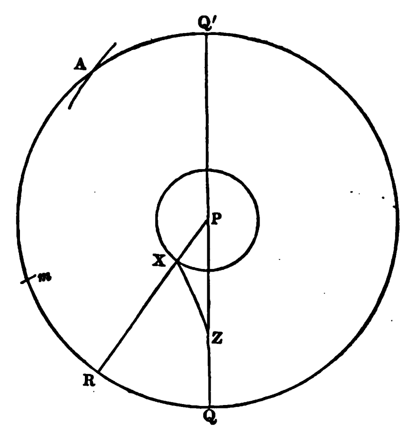
The correction d is subtractive in the present case: but it is seen by the figure that d is additive when the position of X is such that the zenith distance is greater than the colatitude, therefore, generally,

Latitude = $d\pm$altitude of pole star $\pm$ tab. corr. d.

If, therefore, we calculate the value of d for certain latitudes, as for 20°, 30°, &c., and for certain times, as for every 10 minutes of the right ascension of the meridian,—a quantity that depends for its value on ship mean time (since right ascension of meridian = ship mean time + right ascension of mean sun [p. 24]),—and tabulate the results, then the latitude of a place near any of the assumed latitudes recorded in the table is readily found by simply applying the correction d, with its proper sign, to the altitude of the heavenly body.

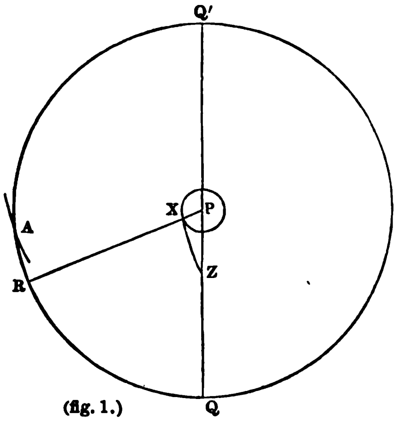
(8.) The value of d for any assumed latitude and time may be computed as follows:

Let QAQ′ represent the celestial equator, QQ′ the celestial meridian, P the pole, and Z the zenith of a place in a given latitude ZQ. Let Qm = given mean time, and suppose X the place of the star at that instant, and A the first point of Aries. Through X draw the circle of declination PR, and circle of altitude ZX. Then d = PZ − ZX, and ZX is unknown. We have therefore to compute ZX for the assumed latitude ZQ and time Qm.

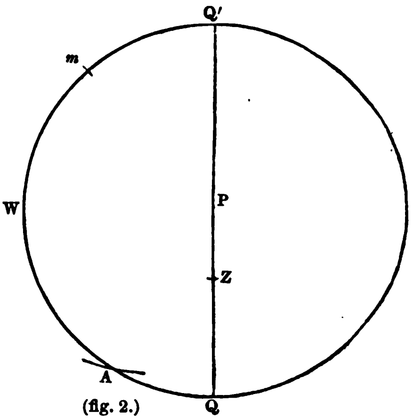


In the spherical triangle ZPX, are given PX, the star's polar distance, PZ the colatitude, and the hour-angle ZPX (since hour-angle = mean time + RA

mean sun − star's RA [note, p. 52]); to find ZX (*Spherical Trigonometry,* Rule IX.): then d is known, since d=PZ − ZX.



(fig. 1.)



(fig. 2.)

Given the right ascension of the meridian = $6^h\ 10^m$; to compute the correction, the assumed latitude being 50° N., the star's RA $1^h\ 7^m\ 7^s$, and declination 88° 32′ 50″ .. (fig. 1).

Star's hour-angle=RA mer. − star's RA.

RA mer. QA=	6^h	10^m	0^s	......const. log.	6·301030
Star's RA, AR=	1	7	7	......log. sin. colat..........	9·808067
∴ star's hour-angle ZPX=	5	2	23	„ sin. pol. dist. ...	8·404360
PZ=	40°	0′	0″	„ hav. HA	9·575969
PX=	1	27	10		4·089426
	38	32	50 vers.		12287
					217904
					230191 vers.
				∴ ZX=39° 39′ 49″	
				and PZ=40 0 0	
				∴ d= 20 11	

∴ the correction for the pole star, when the right ascension of meridian is $6^h\ 10^m$, is 20′ 11″ subtractive.

In the same manner may be found the corrections for right ascension of meridian 10^m, 20^m, up to $24^h\ 0^m$, and for different assumed latitudes. These results constitute the table in Inman's nautical tables.

132. Dec. 4, 1857, at $9^h\ 40^m$ P.M., mean time nearly, the true altitude of the pole star was 54° 38′ 25″. Construct a figure, and find by pole-star table the latitude; by *Nautical Almanac* corrected RA mean sun being $16^h\ 55^m$.

Construction (fig. 2).

Let QWQ′ represent the celestial equator, QQ′ the celestial meridian, and

P the pole. Take $Qm = 9^h\ 40^m$; then m is the mean sun. From m measure $mQ'A = 16^h\ 55^m$; then A is the first point of Aries, and AQ=right ascension of meridian, the argument of the table: or RA mer. may be found by formula (p. 24), thus,

RA mer. = mean time + RA mean sun.

RA mean sun	16^h	55^m	Tr. alt.	$54°$	$38'$	$25''$
Mean time	9	40	Cor.	1	20	36—
RA mer.	2	35	∴ Latitude=	53	17	49 N.

Entering the table with $2^h\ 35^m$ at side and $50°$ at top, the required correction is $1°\ 20'\ 36''$—.

EXAMPLES.

133. Feb. 10, 1857, at $11^h\ 40^m$ P.M. mean time, given the altitude of the pole star $= 60°\ 3'\ 19''$. Construct the figure, and find by table the latitude. By *Nautical Almanac* corrected RA mean sun $= 21^h\ 24^m\ 34^s$.

Ans. $60°\ 51'$ N.

134. January 20, 1857, at $9^h\ 40^m$ P.M. mean time, given the altitude of the pole star $= 31°\ 33'\ 19''$. Construct the figure, and find by table the latitude. By *Nautical Almanac* corrected RA mean sun $= 20^h\ 0^m\ 29^s$.

Ans. $31°\ 1'$ N.

135. October 4, 1857, at $11^h\ 30^m$ P.M. mean time, given the altitude of the pole star $= 62°\ 2'\ 15''$. Construct a figure, and find by table the latitude. By *Nautical Almanac* corrected RA mean sun $= 12^h\ 53^m\ 49^s$.

Ans. $60°\ 32'$ N.

136. May 10, 1857, at $11^h\ 20^m$ P.M. mean time, given the altitude of the pole star $= 52°\ 17'\ 54''$. Construct a figure, and find by table the latitude. By *Nautical Almanac* corrected RA mean sun $= 3^h\ 14^m\ 18^s$.

Ans. $53°\ 39'\ 0''$ N.

Problems XVI. and XVIII. applied to finding the latitude by an altitude of the pole star.

The last problem enables us to find a near approximate latitude from an altitude of the pole star, the reduction to the meridian being made by means of a table; but it is evident that if we consider the pole star as an ordinary star, we may make use of Problems XV. or XVI. to find the latitude from its altitude near the meridian above the pole, and Problem XVIII. to find the latitude from its altitude near the meridian below the pole; by which problems the latitude can be calculated from the altitude of the pole star without the aid of a table, and with a greater degree of accuracy.

In the two examples following, let the latitude be found from the altitude of the pole star by means of Problems XV. or XVI. and XVIII.

137. Given the altitude of pole star near the meridian above the pole $= 54°\ 38'\ 25''$, zenith south of the body, at $9^h\ 40^m$ P.M. mean time at the place,

in estimated latitude 53° 18′ N. Construct a figure, and find by Problem XV. or XVI. the correct latitude.

Data from *Nautical Almanac:* RA mean sun=$16^h\ 55^m\ 7^s$, star's RA= $1^h\ 7^m\ 7^s$, star's decl.=88° 32′ 50″ N. *Ans.* Latitude=53° 17′ 32″ N.

138. Given the altitude of pole star near the meridian under the pole= 38° 17′ 11″, at $10^h\ 20^m$ P.M. mean time at the ship, in latitude by account 39° 27′ N. Construct a figure, and find by Problem XVIII. the correct latitude.

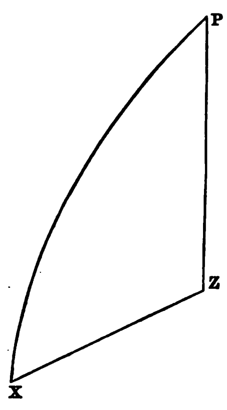
Data from *Nautical Almanac:* RA mean sun=$5^h\ 16^m\ 8^s$, star's RA= $1^h\ 7^m\ 7^s$, star's decl.=88° 32′ 50″ N. *Ans.* Latitude=39° 26′ 53″ N.

Latitude by altitudes taken within a few minutes of each other.

PROBLEM XX.

Given the altitudes of a heavenly body, taken within a few minutes of each other; to find the latitude.

When the altitudes can be taken *very accurately,* the following method of determining the latitude appears to give results sufficiently near for practical purposes; and as the observations are taken within a short time of each other, it possesses on this account an advantage over the common rule for double altitude, which requires a considerable interval to elapse between the two observations.



Let the change in the altitudes, and the corresponding change in the time between the observations, be both very accurately noted. Then, from these contemporary increments of time and altitude, the angle of position PXZ may be calculated very nearly by means of the formula about to be investigated. We shall then have given in the triangle PZX the polar distance PX, the zenith distance ZX (calculated for the mean or middle time between the two observations), and the included angle PXZ; to find the colatitude PZ, and this is readily done by the common rule in Trigonometry (*Trig.* Part I. Rule IX.).

Formula for computing the angle of position PXZ from the contemporary increments of time and altitude.

Let S and S_1 (fig. p. 65) be the places of the heavenly body when their altitudes were taken.

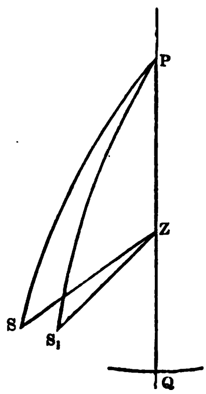
h=hour-angle SPZ, z=zenith distance ZS,

h_1=hour-angle S_1PZ, z_1=zenith distance ZS_1,

l=latitude and d=declination for the middle time between the observations.

Then $h-h_1$=difference of observed times,

$z-z_1$=corresponding difference of altitudes.



In triangle PSZ, $\cos. h = \dfrac{\cos. z - \sin. d \,.\, \sin. l}{\cos. d \,.\, \cos. l}$,

" PS_1Z, $\cos. h_1 = \dfrac{\cos. z_1 - \sin. d \,.\, \sin. l}{\cos. d \,.\, \cos. l}$,

$\therefore \cos. h - \cos. h_1 = \dfrac{\cos. z - \cos. z_1}{\cos. d \,.\, \cos. l}$;

or $2 \sin. \frac{1}{2}(h+h_1) \,.\, \sin. \frac{1}{2}(h-h_1)$

$$= \frac{2 \sin. \frac{1}{2}(z+z_1) \,.\, \sin. \frac{1}{2}(z-z_1)}{\cos. d \,.\, \cos. l}$$

or $\sin. h \,.\, (h-h_1) = \dfrac{\sin. z}{\cos. d \,.\, \cos. l} \,.\, (z-z_1)$

(by substituting h for $\frac{1}{2}(h+h_1)$, z for $\frac{1}{2}(z+z_1)$, and $\frac{1}{2}(h-h_1)$ and $\frac{1}{2}(z-z_1)$ for their sines, and cancelling): in which expression z is the zenith distance and d the declination corresponding to the middle time h between the observations (fig. p. 64).

$$\therefore \frac{\sin. h \,.\, \cos. l}{\sin. z} = \frac{z-z_1}{h-h_1} \,.\, \frac{1}{\cos. d}. \quad \text{But } \frac{\sin. PXZ}{\sin. h} = \frac{\cos. l}{\sin. z}$$

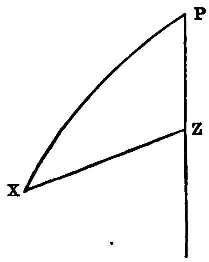
$$\therefore \sin. PXZ = \frac{\sin. h \,.\, \cos. l}{\sin. z}; \text{ whence } \sin. PXZ = \frac{z-z_1}{h-h_1} \,.\, \frac{1}{\cos. d} \text{ in time,}$$

$$\text{or } \sin. PXZ = \frac{1}{15} \,.\, \frac{z-z_1}{h-h_1} \,.\, \frac{1}{\sin. \text{pol. dist.}} \text{ in arc,}$$

$$\therefore \text{cosec. angle of position } PXZ = \frac{15\,(h-h_1)}{z-z_1} \,.\, \sin. \text{pol. dist.}$$

From this expression the angle PXZ may be computed, if the contemporary quantities $h-h_1$ and $z-z_1$ have been correctly noted. Knowing, then, the polar distance, zenith distance, and included angle X, in the triangle PZX, we can compute the colatitude PZ by the common rule in *Spherical Trigonometry* (Part I. Rule IX.) for finding the third side, having given two sides and the included angle.

139. The following observations were taken at the Royal Naval College (lat. 50° 48′ 3″ N.). At $1^h\ 42^m\ 53{\cdot}7^s$ the true zenith distance of the sun's center was found to be 45° 7′ 37″, at $1^h\ 52^m\ 8{\cdot}6^s$ it was 45° 55′ 4″, the polar distance at the middle time was 80° 35′ 19″; to find the latitude.



Formula adapted for calculation.

$$\text{cosec. } PXZ = \frac{15(h-h_1)}{z-z_1} \,.\, \sin. p,$$

$$\text{vers. colat.} = \text{vers.}(p-z) + 2 \sin. p \,.\, \sin. z \text{ hav. } PXZ$$
$$= \text{vers.}(p-z) + \text{vers. } H.$$

Assuming vers. $H = 2 \sin. p \,.\, \sin. z \,.\, \text{hav. } PXZ$
(see *Trig.* Part II., the proof of Rule IX.),

where p=pol. dist. PX, z=zen. dist. ZX, at the middle time between the two observations.

h ......$1^h\ 52^m\ 8\cdot6^s$	z ......$45°\ 55'\ 4''$	......z ...$45°\ 55'\ 4''$	
h_1......$1\ 42\ 53\cdot7$	z_1......$45\ 7\ 37$	......z_1...$45\ 7\ 37$	
$9\ 14\cdot9$	$47\ 27$	$91\ 2\ 41$	
60	60	$z=45\ 31\ 20$	
$h-h_1=\ 554\cdot9$	$z-z_1=\ 2847$	$p=80\ 35\ 19$	
		$p-z=35\ 3\ 59$	

To find angle of position PXZ.

log. 15 ....$1\cdot176091$	const. log. ....$6\cdot301030$
„ $h-h_1$ ......$2\cdot744215$	log. sin. p ....$9\cdot994113$
„ sin. p.........$9\cdot994113$	„ sin. z............$9\cdot853397$
$13\cdot914419$	„ hav. PXZ ......$8\cdot491433$
„ $z-z_1$.........$3\cdot454387$	„ vers. H ....$4\cdot639973$
„ cosec. PXZ . $10\cdot460032$	∴ vers. H= 43649
∴ PXZ=$20°\ 17'$	vers. $(p\sim z)$= 181516
	vers. colat. 225165
	∴ colat.=$39°\ 12'$
	and latitude=$50\ 48$ N.

EXAMPLES FOR PRACTICE.

140. August 13, 1858, A.M., the following observations were taken at the Royal Naval College (lat. 50° 48′ N.).

At $9^h\ 58^m\ 45\cdot6^s$ A.M., the true zenith distance of the sun's center was 44° 35′ 59″, at $10^h\ 9^m\ 44\cdot0^s$ it was 43° 20′ 57″; required the latitude (corrected polar distance being 75° 16′ 15″). *Ans.* 50° 50′ N.

141. August 13, 1858, A.M., at $10^h\ 2^m\ 19\cdot9^s$ A.M., the true zenith distance of the sun's center was 44° 10′ 56″, at $10^h\ 17'\ 26\cdot2^s$ it was 42° 30′ 55″; required the latitude (corrected polar distance being 75° 16′ 24″). *Ans.* 50° 51′ N.

142. August 13, 1858, A.M., at $10^h\ 21^m\ 37\cdot5^s$ A.M., the true zenith distance of the sun's center was 42° 3′ 24″, at $10^h\ 31^m\ 41\cdot0^s$ it was 41° 3′ 23″; required the latitude (corrected polar distance being 75° 16′ 33″). *Ans.* 50° 48′ N.

From the great difficulty of observing at sea the altitude of a heavenly body correctly, the above method of determining the latitude is probably of little practical use. It can, however, be adopted with considerable advantage *on shore*, where the altitudes may be taken with great accuracy by means of an artificial horizon.

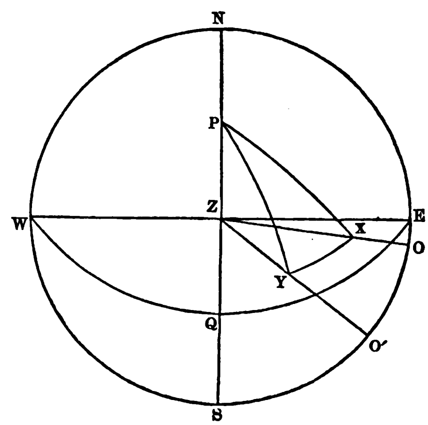
THE DOUBLE ALTITUDE.

The important rule for finding the latitude from two altitudes of the same or two heavenly bodies, observed at the same or at different times, may be more clearly explained if we deal with each case separately.

PROBLEM XXI.

Given the altitudes of the sun, or any other heavenly body, observed at two different times, and the elapsed time between the two observations; to find the latitude.

Let NWSE represent the horizon, NZS the celestial meridian, P the pole, and Z the zenith of the spectator. Let X and Y be the places of the heavenly body when its altitudes were observed. Draw the circles of altitude ZO and ZO′, and circles of declination PX and PY, through X and Y. Draw also XY, the arc of a great circle.



First. Suppose the arc XY, drawn through the two places of the heavenly body, not to pass, when produced, between the zenith and pole, as in the figure.

In the two spherical triangles PXY and ZXY, there are given the two polar distances PX and PY (from the *Nautical Almanac*), the angle XPY (the interval between the observations, which will be given sufficiently near if noted by a chronometer whose rate is known), and the two zenith distances ZX and ZY (from the observed altitudes); to find the colatitude PZ, and thence the latitude.

(1.) In the spherical triangle PXY, are given the polar distances PX and PY, and included angle XPY; to calculate XY. Call this arc 1.

(2.) In the spherical triangle PXY, are given the polar distances PX and PY, and arc 1, just found; to calculate the angle PXY, the angle at the place whose bearing is the greater. Call this angle PXY arc 2.

(3.) In the spherical triangle ZXY, are given the zenith distances ZX and ZY, and arc 1; to find the angle ZXY, the angle at the greater bearing. Call this angle ZXY arc 3.

(4.) Subtract arc 3 from arc 2; the remainder is angle PXZ, or arc 4.

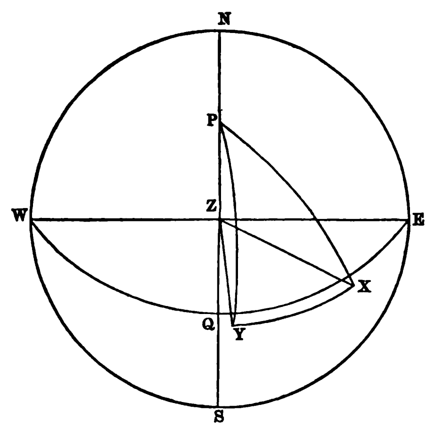
(5.) *Lastly.* In the triangle PXZ, are given PX and ZX, the polar dist. and zenith dist. at the greater bearing, and the included angle PXZ, or arc 4, to calculate PZ, the colatitude; and thence ZQ, the latitude, is known.

This rule for finding the latitude, although long, is direct, and free from any ambiguity of cases, and has also the great advantage of being a simple application of two of the common rules of Spherical Trigonometry.

143. In latitude by account 50° N., at $8^h\ 58^m\ 28^s$ A.M., the sun's altitude was 25° 3′ 47″, and declination 2° 58′ 35″ S.; and at $11^h\ 11^m\ 43^s$ A.M. the sun's altitude was 35° 39′ 28″, and declination 2° 56′ 30″ S. Construct a figure, and find by calculation the true latitude.

Construction.

Let NWSE represent the horizon, NZS the celestial meridian; then, the latitude being about 50° N., take NP=50°, and P will be the north pole of the heavens: take PQ=90°, and through W, Q, E, draw the celestial equator WQE. Since the sun has south declination, and is on the east of meridian, let X be the place of the sun at the first observation, and Y its place at the second; draw the circles of declination PX and PY, and circles of altitude ZX and ZY, and through X and Y draw the great circle XY: then the colatitude PZ may be found as follows.



(1.) In the spherical triangle PXY, are given PX=90°+decl.=92° 58′ 35″, PY=90°+decl.=92° 56′ 30″ and included angle XPY=$2^h\ 13^m\ 15^s$; to find XY=33° 16′ 0″.

(2.) In the spherical triangle PXY, are given PX=92° 58′ 35″, PY=92° 56′ 30″, and arc XY=33° 16′ 0″; to find angle PXY=90° 49′ 45″.

(3.) In the spherical triangle ZXY, are given ZX=90°−alt.=64° 56′ 13″, XY=33° 16′ 0″, and ZY=90°−alt.=54° 20′ 32″; to find the angle ZXY=62° 35′ 15″. Hence the angle PXZ (the *difference* between PXY and ZXY)=28° 14′ 30″.

(4.) In the spherical triangle PXZ, are given PX=92° 58′ 35″, ZX=64° 56′ 13″, and included angle PXZ=28° 14′ 30″; to find the colatitude PZ=39° 12′ 2″. Hence the latitude ZQ=50° 47′ 58″ N.

Calculation.

(1.) To find arc XY (*Trig.* Part I. Rule IX.).

PX......	92°	58′	35″......	6·301030	
PY......	92	56	30	9·999414	
Diff. ...		2	5	9·999427	
				8·914644	
			log. vers.	5·214515	∴ vers.=163875
					vers. diff... 000
			∴ XY=33° 16′ 0″		vers. XY...163875.

(2.) To find angle PXY.

92°	58′	35″......	0·000586
33	16	0	0·260795
59	42	35	4·987511
92	56	30	4·456316
152	39	5	9·705208
33	13	55.∴PXY=90° 49′ 45″.	

(3.) To find angle ZXY.

64°	56′	13″......	0·042946
33	16	0	0·260795
31	40	13	4·833834
54	20	32	4·293478
86	0	45	9·431053
22	40	19	∴ ZXY=62° 35′ 15″.

To find angle PXZ.

PXY	90°	49′	45″
ZXY	62	35	15
∴ PXZ=	28	14	30

(4.) To find arc PZ, the colatitude.

	92°	58′	35″......	6·301030
	64	56	13	9·999414
Diff. ...	28	2	22	9·957054
				8·774664
				5·032162 ∴ vers.=107686

vers. diff....117376

vers. colat....225062

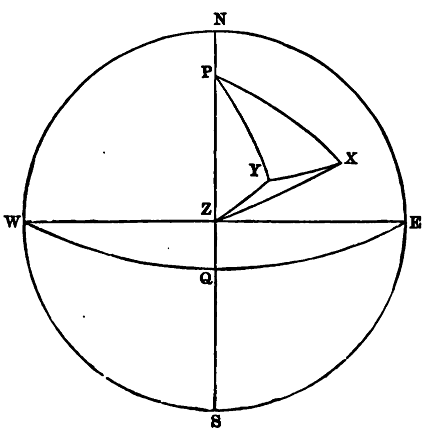
∴ colatitude...39° 12′ 2″

and latitude=50 47 58 N.

EXAMPLE FOR PRACTICE.

144. In latitude by account 50° N., at 11^h 23^m 22^s A.M., as shown by chronometer, the sun's altitude was 32° 41′ 45″, and declination 5° 23′ 9″ S.; and at 3^h 19^m 51^s P.M., as shown by chronometer, the sun's altitude was 21° 25′ 22″, and declination 5° 19′ 17″ S. Construct a figure, and find by calculation the true latitude.

Ans. Latitude=50° 47′ 48″ N.



Second. Suppose the arc XY, drawn through the two places of the same or two heavenly bodies, to pass (produced if necessary) *between* the zenith and the pole, as in the annexed figure.

Let X and Y be the two places of the heavenly body when the altitudes are taken. Through X and Y draw XY, the arc of a great circle, which, if produced, will evidently pass between P and Z, the pole and zenith.

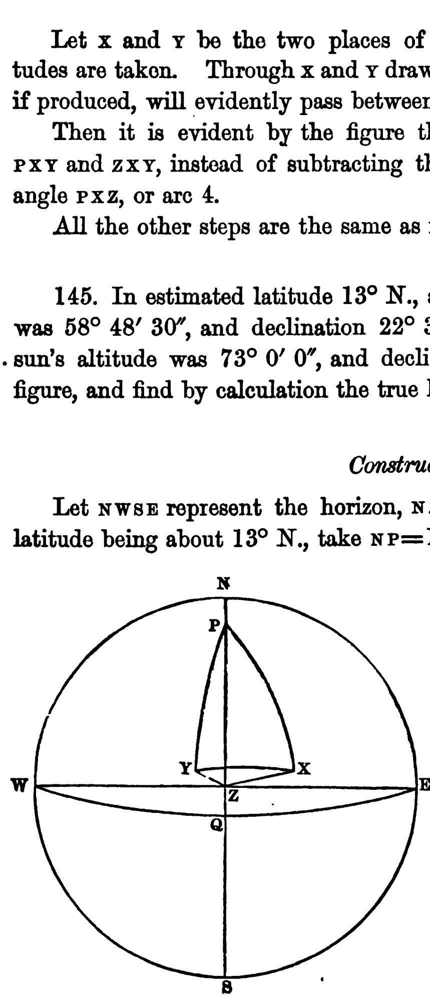
Then it is evident by the figure that we must *add* together the angles PXY and ZXY, instead of subtracting them, as in the first case, to find the angle PXZ, or arc 4.

All the other steps are the same as in the preceding example.

145. In estimated latitude 13° N., at $9^h\ 54^m\ 25^s$ A.M., the sun's altitude was 58° 48′ 30″, and declination 22° 35′ 30″ N.; and at $1^h\ 0^m\ 5^s$ P.M. the sun's altitude was 73° 0′ 0″, and declination 22° 34′ 40″ N. Construct a figure, and find by calculation the true latitude.

Construction.

Let NWSE represent the horizon, NZS the celestial meridian; then, the latitude being about 13° N., take NP=13°, and P will be the north pole of the heavens: take PQ=90°, and through W, Q, E, draw the celestial equator WQE. Let X be the place of the sun at the A.M. observation, and Y its place at the P.M. observation (estimated as near as possible by means of the altitudes and declination). Through X and Y draw the circles of declination PX and PY, and circles of altitude ZX and ZY, and join XY. It is evident by the figure that the arc XY passes between the zenith and pole. Then the colatitude PZ may be found as follows:



(1.) In the spherical triangle PXY, are given PX=90°—decl.=67° 24′ 30″, PY=90°—decl.=67° 25′ 20″, and angle XPY (=sum of hour-angles)= $3^h\ 5^m\ 40^s$; to find XY=42° 40′ 26″.

(2.) In the spherical triangle PXY, are given PX=67° 24′ 30″, PY=67° 25′ 20″, and arc XY=42° 40′ 26″; to find angle PXY=80° 40′.

(3.) In the spherical triangle ZXY, are given ZX=90°—alt.=31° 11′ 30″, ZY=90°—alt.=17° 0′ 0″, and XY=42° 40′ 26″; to find the angle ZXY= 21° 9′ 45″.

Hence the angle ZXP (the *sum* of PXY and ZXY)=101° 49′ 45″.

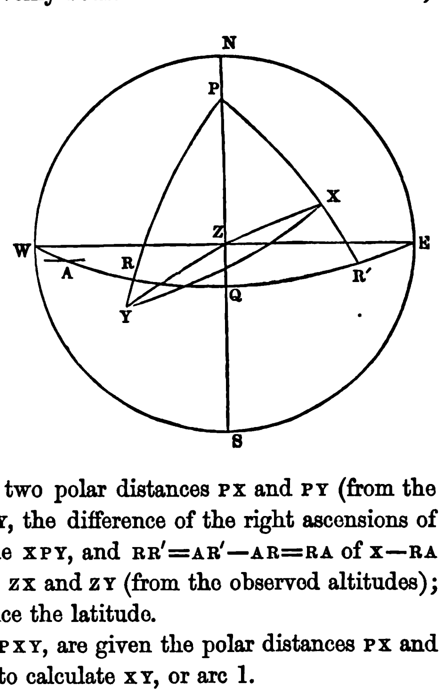
(4.) In the spherical triangle PXZ, are given PX=67° 24′ 30″, ZX= 31° 11′ 30″, and included angle ZXP=101° 49′ 45″; to find the colatitude ZP=76° 40′ 2″. Hence the latitude=13° 19′ 58″ N.

EXAMPLE FOR PRACTICE.

146. In latitude by account 9° 19′ N., at 9^h 45^m 12^s A.M., the sun's altitude was 67° 14′ 10″, and decl. 22° 13′ 14″ N.; and at 1^h 10^m 17^s P.M., the sun's altitude was 79° 8′ 30″, and decl. 22° 14′ 0″ N. Construct a figure, and find by calculation the true latitude. *Ans.* Lat.=9° 20′ N.

PROBLEM XXII.

Given the altitudes of two heavenly bodies observed at the same time; to find the latitude.



Let NWSE represent the horizon, NZS the celestial meridian, P the pole, Z the zenith, WQE the celestial equator, and A the first point of Aries. Let X and Y be the places of the two heavenly bodies whose altitudes are observed. Draw the circles of altitude ZX and ZY, and circles of declination PX and PY, through X and Y; draw also XY, the arc of a great circle connecting the two bodies.

In the two spherical triangles PXY and ZXY, there are given the two polar distances PX and PY (from the *Nautical Almanac*), the angle XPY, the difference of the right ascensions of X and Y (for RR′ measures the angle XPY, and RR′=AR′—AR=RA of X—RA of Y), and the two zenith distances ZX and ZY (from the observed altitudes); to find the colatitude PZ, and thence the latitude.

(1.) In the spherical triangle PXY, are given the polar distances PX and PY, and the included angle XPY; to calculate XY, or arc 1.

(2.) In the spherical triangle PXY, are given the polar distances PX, PY, and arc XY or arc 1; to calculate the angle PXY, the angle at the heavenly body whose bearing is the greater. Call this angle arc 2.

(3.) In the spherical triangle ZXY, are given the zenith distances ZX and ZY, and arc 1 or XY; to find the angle ZXQ, the angle at the greater bearing. Call this angle arc 3.

(4.) When the arc XY does not pass between the zenith and the pole, subtract arc 3 from arc 2, the remainder is angle PXZ. Call this arc 4. When the arc XY does pass between the zenith and the pole, add arc 3 to arc 2; the sum will be angle PXZ, or arc 4.

(5.) *Lastly*. In the spherical triangle PXZ, are given PX and XZ, the polar distance and zenith distance at the greater bearing, and the included

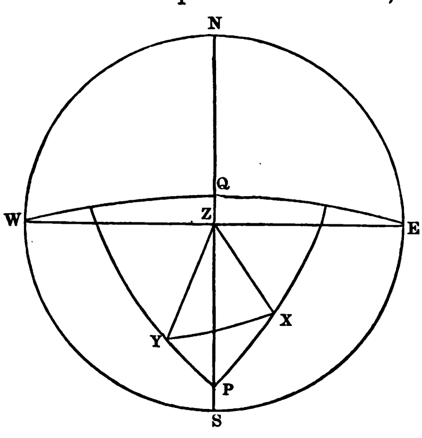
angle PXZ, or arc 4, to calculate PZ the colatitude; and thence ZQ, the latitude, is known.

147. In latitude by account 14° 30′ S., the altitude of Canopus was 47° 30′, bearing about S.E. b. S.; and at the same time the altitude of Achernar was 33° 40′ 20″, bearing about S.S.W.: declination of Canopus $=52° 36′ 40″$ S., RA$=6^h 20^m 47^s$; declination of Achernar$=57° 57′ 20″$ S., RA$=1^h 32^m 37^s$. Required the true latitude.

NOTE. It was seen, when the altitudes were taken, that a line joining the two stars would pass *between* the zenith and pole.

Construction.

Let NWSE represent the horizon, NZS the celestial meridian; then, the latitude being about 14° S., take SP$=14°$, and P will be the south pole of the heavens: take PQ$=90°$, and through W, Q, E, draw the celestial equator WQE. Since the declination of Canopus is about 50° S., and its bearing S.E. b. S., let X be the place of Canopus; and since the declination of Achernar is about 57° S., and its bearing S.S.W., let Y be the place of Achernar. Draw the circles of declination PX and PY, and circles of altitude ZX and ZY, and through the stars X and Y draw the great circle XY; then the colatitude PZ may be found as follows:



(1.) In the spherical triangle PXY, are given PX$=90°-$decl.$=37° 23′ 20″$, PY$=90°-$decl.$=32° 2′ 40″$, and included angle XPY (=diff. of right ascension of stars)$=4^h 48^m 20^s$; to find XY$=39° 24′ 48″$.

(2.) In the spherical triangle PXY, are given PX$=37° 23′ 20″$, PY$=32° 2′ 40″$, and arc XY$=39° 24′ 48″$; to find angle PXY$=52° 40′ 30″$.

(3.) In the spherical triangle ZXY, are given ZX$=90°-$alt.$=42° 30′$, ZY$=90°-$alt.$=56° 19′ 40″$, and XY$=39° 24′ 48″$; to find the angle ZXY$=92° 1′ 30″$.

Hence the angle ZXP (the *sum* of PXY and ZXY)$=144° 42′ 0″$.

(4.) In the spherical triangle PXZ, are given PX$=37° 23′ 20″$, ZX$=42° 30′ 0″$, and included angle ZXP$=144° 42′ 0″$; to find the colatitude ZP$=75° 27′ 46″$.

Hence the latitude ZQ$=14° 32′ 14″$ S.

EXAMPLES FOR PRACTICE.

148. In latitude by account 41° N., the altitude of α Andromedæ was 73° 14′, bearing about S. b. E.; and at the same time the altitude of α Tauri was 18° 27′ 30″, bearing about East:

Decl. of α Andromedæ...28° 18′ 24″ N.......RA...0h 1m 2s
„ α Tauri ....16 13 21 N.......RA...4 27 47

Required the true latitude. *Ans.* Latitude=41° 26′ N.

149. In latitude by account 36° 30′ N., the altitude of Sirius was 27° 46′ 3″, bearing about S.W.; and at the same time the altitude of Spica was 12° 49′ 44″, bearing about E.S.E.

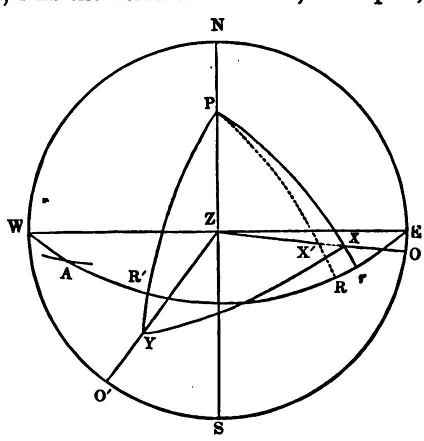
Decl. of Spica.........10° 25′ 26″ S......RA...13h 17m 45s
„ Sirius ......16 31 29 S......RA... 6 38 53

Required the true latitude. *Ans.* Latitude=36° 34′ N.

PROBLEM XXIII.

Given the altitudes of two heavenly bodies observed at different times, and the elapsed time; to find the latitude.

Let NWSE represent the horizon, NZS the celestial meridian, P the pole, and Z the zenith. Let X and Y be the two places of the heavenly bodies when their altitudes XO and YO′ were observed at different times.



First. Suppose the altitude of X to be first taken, and that the altitude of Y was taken after an interval, during which the body X had moved from X to X′; then X′PY is the difference of the right ascension of the two bodies, XPX′ is the elapsed time between the observations, and XPY is the polar angle between the two places of the heavenly bodies when their altitudes were taken. This polar angle is the one to be used to determine the several arcs, 1, 2, &c., and finally the latitude, as in the preceding cases. The polar angle XPY is found by the following practical rule: "Add elapsed time (expressed in sidereal time) to the right ascension of the heavenly body first observed, and take the difference between the sum and the right ascension of the heavenly body last observed; the remainder will be the polar angle required (subtracted from 24 hours, if greater than 12 hours)." This rule may be proved as follows:

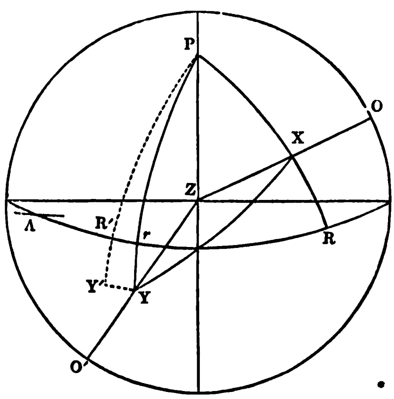
In the fig., let A be the first point of Aries; then AR=right ascension

of X′, AR′=right ascension of Y, and XPX′ or arc rR=elapsed time. Also R′r=polar angle XPY required.

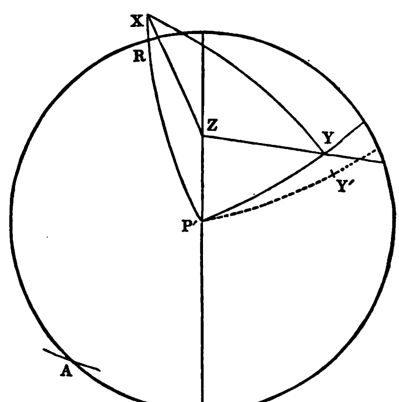
To find the polar angle. R′r=Rr+AR−AR′,
or polar angle=elapsed time+RA of body first observed−RA of body last observed; which is the rule.

Second. Suppose the altitude of Y (fig. 1) first taken, and that the altitude of X was not taken till after an interval, during which Y had moved to Y′; then XPY is the polar angle between the two places of the heavenly bodies when their altitudes were taken.

To find the polar angle XPY. Rr=AR−(R′r+AR′),
or polar angle=RA of body last observed−(elapsed time+RA of body first observed); which is the rule.



(fig. 1.)



(fig. 2.)

150. In latitude by account 47° 54′ S., the following observations of the two stars Regulus and Antares were made at different times; to determine the true latitude (fig. 2):

	True alt.	Decl.	RA.	Bearing.
Regulus	...27° 3′ 23″	...12° 50′ 32″ N.	... 9h 58m 46·6s	...N. 30° W.
Antares	...37 58 51	...26 1 18 S.	...16 18 23·1	... S. 80 E.

The altitude of Antares was taken 1h 1m 19·5s after that of Regulus, as measured by a chronometer.

Project the figure on the plane of the celestial equator, in order to exhibit more clearly the place of the first point of Aries. Let P′ be the south pole and Z the zenith of the observer, X the place of Regulus when its altitude was taken, and Y the place of Antares at that time, and Y the place of Antares when its altitude was taken; the positions of X and Y with respect to the meridian PZ being determined from their estimated bearings.

Draw the circles of declination P′X, P′Y, and circles of altitude ZX and ZY, and join XY.

(1.) In the triangle P′XY, are given P′X$=102°\ 50'\ 32''$, P′Y$=63°\ 58'\ 42''$, and angle XP′Y$=$AP′X$-($AP′Y′$+$YP′Y′$)=5^h\ 18^m\ 17{\cdot}0^s$; to find XY$=86°\ 29'\ 50''$.

(2.) In the triangle P′XY, are given P′X$=102°\ 50'\ 32''$, P′Y$=63°\ 58'\ 42''$, and XY$=86°\ 29'\ 50''$; to P′YX$=106°\ 7'\ 20''$.

(3.) In the triangle ZXY, are given ZX$=62°\ 56'\ 37''$, ZY$=52°\ 1'\ 9''$, and XY$=86°\ 29'\ 50''$; to find ZYX$=57°\ 58'\ 0''$.

Hence the angle PYZ$=48°\ 9'\ 20''$.

(4.) In triangle P′ZY, are given P′Y, ZY, and P′YZ; to find P′Z$=42°\ 3'\ 16''$, the colatitude.

Hence the latitude$=47°\ 56'\ 44''$ S.

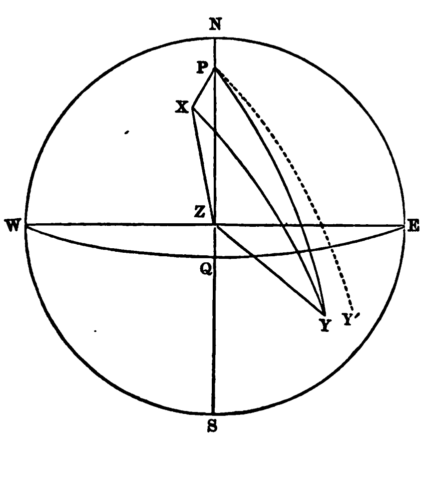
151. In latitude by account 14° 14′ N., the following observations of the two stars Dubhe and Antares were made at different times; to determine the true latitude:

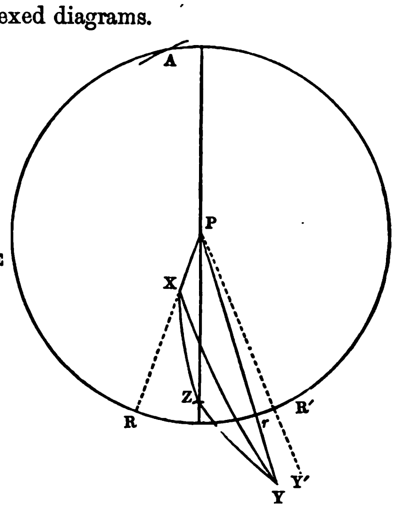
	Alt.	Decl.	R.A.	Bearing.
Dubhe......	41° 1′ 2″...	62° 43′ 10″ N....	$10^h\ 52^m\ 31{\cdot}6^s$...	N. 10° W.
Antares ...	15 52 39 ...	26 1 18 S. ...	16 18 23·1 ...	S. 48 E.

The altitude of Antares was taken at $30^m\ 6{\cdot}4^s$ after that of Dubhe, as measured by a chronometer.

Construction.

The figure may be projected on the plane of the horizon or the plane of the equator, as may be seen in the annexed diagrams.





Let P be the north pole, Z the zenith, and X the place of Dubhe when its altitude was observed, and Y′ the place of Antares at that time, and Y the place of Antares when its altitude was taken; the positions of X and Y

with respect to the meridian PZ being determined from their estimated bearings. Complete the figure as in the last example.

Then the polar angle $XPY = AR' - (AR + rR') = 4^h\ 55^m\ 45{\cdot}1^s$.

(1.) In triangle PXY, find	$XY = 106°$	$3'$	$32''$
(2.) „ PXY, „	$PYX = 27$	17	30
(3.) „ ZXY, „	$ZYX = 37$	38	15
Hence the angle	$PYZ = 64$	55	45
(4.) In triangle PZY, find	$PZ = 75$	44	42

$\therefore$ lat. $= 14°\ 15'\ 18''$ N.

152. In latitude by account 51° N., the following observations of the two stars α Bootis and α Aquilæ were made at different times:

	Alt.	Decl.	RA.	Bearing.
α Bootis	22° 49′ 19″	19° 55′ 16″ N.	$14^h\ 9^m\ 12^s$	W.N.W.
α Aquilæ	47 27 59	8 29 55 N.	19 43 54	S.b.W.

The altitude of α Aquilæ was taken $10^m\ 52^s$ after that of α Bootis. Construct a figure, and calculate the true latitude.

Ans. Lat. = 50° 57′ 30″ N.

Sumner's rule for finding the latitude.

A method of finding the latitude, that deserves the attention of the seaman, is the one called Sumner's method. When one altitude only can be taken, the place of the observer may be assumed to be on a line, which is found as follows. With the estimated latitude of the ship, the altitude and declination, and by means of the chronometer, showing Greenwich mean time, calculate the longitude (see Probl. XXIV. XXV.). Mark the spot on the chart corresponding to this latitude and longitude. Assume another latitude a few degrees different from the former, and find the longitude as before. Mark the spot on the chart corresponding to this latitude and longitude. Join the two spots thus found, and the place of the ship will be on or very near to the line, or the line produced. If another altitude could be taken an hour or two afterwards, and two spots determined in like manner, the line joining them will intersect the other line on the chart in or near to the true place of the ship, and thus the position of the ship will be found very nearly. The estimated latitudes should, if possible, be taken one greater and one less than the true, and within a few degrees of each other; a small diagram or chart bounded by the two parallels of latitude passing through the estimated latitudes may be easily constructed, and the place of the ship indicated thereon by the intersection of the two lines found as above. This method is more fully described in Mr. John Riddle's *Navigation.*

CHAPTER IV.

INVESTIGATION OF RULES FOR DETERMINING THE LONGITUDE, VARIATION OF THE COMPASS, ETC.

Longitude by chronometer.

153. The rate of a chronometer, and its original error on Greenwich mean time, being given, the mean time at Greenwich at any moment is readily found, and therefore can be known at that instant when the altitude of a heavenly body is taken for finding mean time at the ship (see Problems VIII. and IX.). The altitude for determining ship mean time should be taken when the heavenly body bears as nearly east or west as possible, since in that position of the body an error in the observation will produce the least error in the hour-angle, and therefore in the mean time deduced from it (see Problem XI., p. 43).

Ship mean time being thus found, the longitude is known, since

$$\text{longitude in time} = \text{Greenwich mean time} \sim \text{ship mean time}.$$

154. The principal operation, therefore, in finding the longitude by chronometer, is to calculate the ship mean time; and this may be done from the altitude of the sun, or any other heavenly body, observed when to the east or west of the meridian. We will consider these cases separately, and give an example and construction for each.

SUN CHRONOMETER.

PROBLEM XXIV.

To find the longitude from the altitude of the sun.

The Greenwich mean time is supposed to be known by means of the chronometer, the error and rate of which are given.

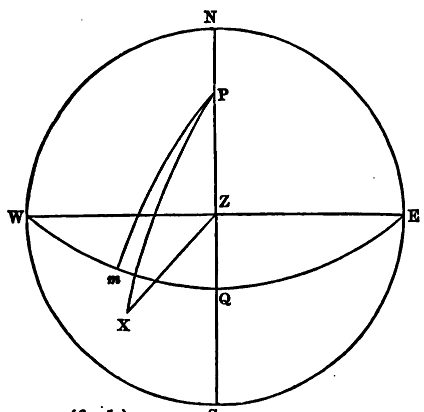
To find ship mean time.

First. When the sun is *west* of meridian (fig. 1, next page).

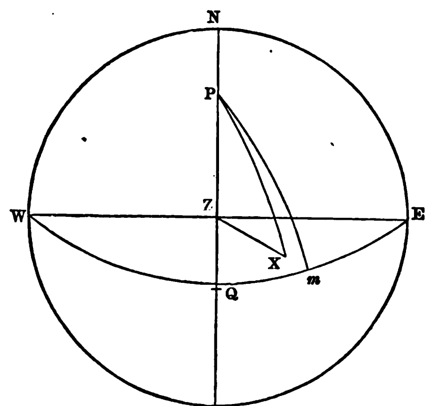
Let X be the place of the sun (west of meridian), m the place of the mean sun; and complete the figure as before. In the spherical triangle ZPX, the three sides are given, namely ZX the zenith distance, found by observation, PX the polar distance, from the *Nautical Almanac*, and PZ the

colatitude of the ship; from which the hour-angle ZPX may be computed, which is also in this case ship apparent time. Also the equation of time, namely the angle mPX, is known from the *Nautical Almanac*. Hence ship mean time, namely the angle ZPm, is found by applying ship apparent time ZPX to the equation of time mPX.

Then the difference between the time so deduced and the Greenwich mean time, as shown by chronometer (corrected for error and rate), is the longitude in time.



(fig. 1.)



(fig. 2.)

Second. When the sun is *east* of meridian (fig. 2).

Let X be the place of the sun (east of meridian), and m the mean sun. The hour-angle ZPX, as in the former case, may be computed, and thence the ship apparent time, namely $24^h -$ ZPX, obtained.* Then, by applying the equation of time mPX to apparent time, ship mean time, namely the angle $24^h -$ ZPm, is obtained.

The difference between the time so deduced and Greenwich mean time, found from its error and rate, is the longitude in time.

STAR CHRONOMETER.

PROBLEM XXV.

To find the longitude from the altitude of any other heavenly body, as a star or the moon.

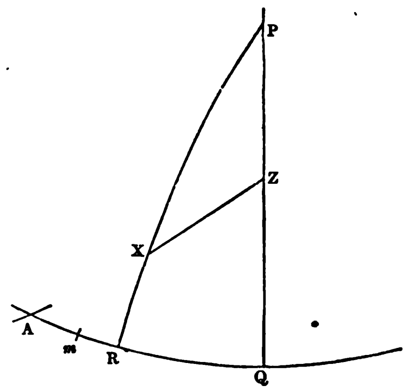
To find ship mean time.

Let X be the place of the heavenly body, AQ the equator, A the first point of Aries, and m the mean sun.

* This quantity, 24^h—hour-angle, may be readily found, if the table of haversines is used, by taking the quantity out from the bottom of the page instead of the top. Hence this practical rule: "West of meridian, take hour-angle from top; east of meridian, take hour-angle from bottom of table of haversines."

In the fig., Qm is the ship mean time required,
Am is the right ascension of mean sun,
AR is the star's right ascension,
and RQ measures the angle XPZ, the star's hour-angle.

Now Qm=RQ+AR−Am,
or ship mean time=star's hour-angle
+star's RA − RA mean sun.



The star's hour-angle ZPX is computed from knowing in the spherical triangle ZPX the colatitude PZ, the polar distance PX, and the zenith distance ZX, and thence ship mean time deduced.*

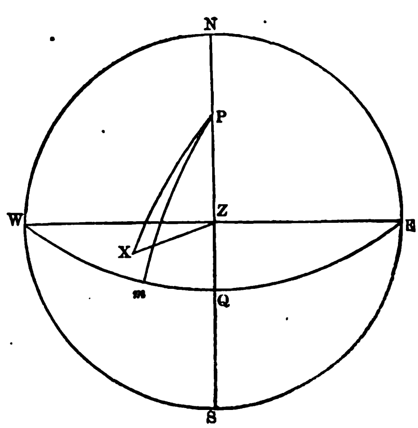
Star's RA, and RA of mean sun, are found in the *Nautical Almanac.*

Greenwich mean time, expressed astronomically, is found from the chronometer, corrected for error and rate.

EXAMPLES.

Longitude by sun's altitude. West of meridian.

155. April 21, about 2^h 30^m P.M., in latitude 46° 50′ N., given the true altitude of sun=42° 15′ 13″ (west of meridian), declination= 11° 44′ 28″ N., equation of time $=1^m$ 4^s, subtractive from apparent time, and the Greenwich mean time $= 8^h$ 35^m $16{\cdot}8^s$ A.M. ($=20^h$ 35^m $16{\cdot}8^s$, on April 20th); to find the longitude of ship.



Let X be the place of the sun, P the pole, Z the zenith, and m the mean sun. Then, in the spherical triangle ZPX, are given ZX =47° 44′ 47″, PX=78° 15′ 32″, and PZ=43° 10′ 0″; to find the angle ZPX, or ship apparent time.

* See Problem IX., where ship mean time is found from the star's hour-angle, &c., and all the cases considered.

Calculation of apparent time ZPX.

43°	10′	0″	..............0·164866			
78	15	32	..............0·009184			
35	5	32	4·820567			
47	44	47	4·042198			
82	50	19	9·036815			
12	39	15	∴ ZPX=	2^h	34^m	7^s
			and XPm=		1	4

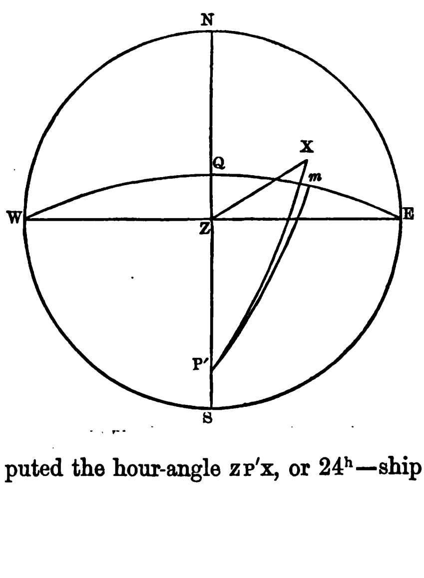
∴ ship mean time, April 21 = 2 33 3 = mQ
or ship mean time, April 20 = 26 33 3
Greenwich, April 20 = 20 35 17 (deduced from chronometer)
∴ longitude in time = 5 57 46 = 89° 26′ 30″ E.

NOTE. Instead of taking the difference between the polar distance and colatitude, the algebraic difference between the latitude and declination is generally used, and the secants taken out instead of the cosecants of PX and PZ; the logarithmic result will manifestly be the same.

156. May 8, about 4^h 0^m P.M., in latitude 40° N., given the altitude of the sun = 32° 25′ 27″ (west of meridian), declination = 17° 9′ 18″ N., equation of time = 3^m $42{\cdot}6^s$, subtractive from apparent time and Greenwich mean time = 7^h 2^m 3^s (deduced from chronometer). Construct a figure, and find by calculation the longitude. *Ans.* Longitude = 44° 42′ W.

Longitude by sun's altitude. East of meridian.

157. June 9, about 9^h 40^m A.M., in latitude 10° S., given the altitude of the sun = 42° 47′ 33″ (east of meridian), declination = 22° 55′ 26″ N., the equation of time = 1^m 12^s, subtractive from apparent time, and Greenwich mean time = 7^h 33^m 2^s A.M. (= 19^h 33^m 2^s on June 8); to find the longitude.



Let X be the place of the sun, P′ the south pole, Z the zenith, and m the mean sun. Then, in the spherical triangle ZP′X, the three sides are known, namely P′Z = 90° − latitude, and ZX = 90° − altitude, and P′X = 90° + declination; from which may be computed the hour-angle ZP′X, or 24^h − ship apparent time.

Calculation of apparent time.

10°	0′	0″	S.	0·006649
22	55	26	N.	0·035733
32	55	26		4·808669
47	12	27		4·094552
80	7	53		8·945603
14	17	1		

$\therefore\ 24^h - \text{ZPX} = 21^h\ 41^m\ 46^s$

and $\text{XP}m = \quad 1\ \ 12$

$\therefore\ 24^h - \text{ZP}m = 21\ \ 40\ \ 34$ = ship mean time on 8th.

Chronometer = $19\ \ 33\ \ \ 2$ = Greenwich mean time on 8th.

$\therefore$ long. in time = $2\ \ \ 7\ \ 32 = 31°\ 53'$ E.

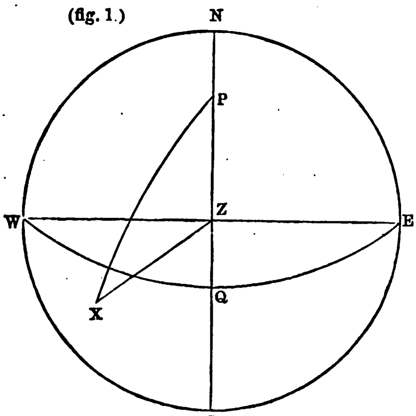
158. April 18, about $9^h\ 27^m$ A.M., in latitude 50° 48′ N., given the altitude of the sun = 38° 21′ 19″ (east of meridian), declination = 10° 55′ 29″ N.; the equation of time = $0^m\ 43{\cdot}6^s$, subtractive from apparent time, and the Greenwich mean time = $9^h\ 24^m\ 41{\cdot}8^s$ (A.M. at Greenwich). Construct a figure, and find by calculation the longitude.

Ans. Longitude = $4^m\ 25{\cdot}4^s$ W.

Longitude by star's altitude. West of meridian.

159. In latitude 30° 10′ N., given the altitude of Sirius = 30° 32′ 7″ (west of meridian), declination = 16° 31′ 19″ S., RA = $6^h\ 38^m\ 43^s$, RA mean sun = $23^h\ 40^m\ 53^s$, and the Greenwich mean time (deduced from chronometer) = $11^h\ 54^m\ 53^s$; to find the longitude.

(fig. 1.)



(fig. 2.)

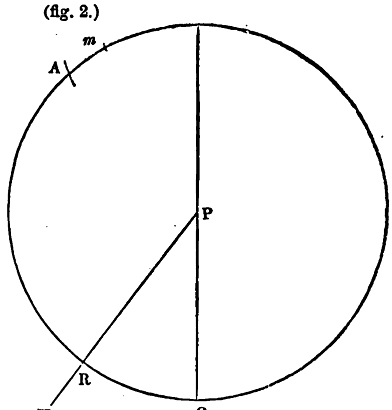


Fig. 1. Let X be the place of the star (west of meridian), and complete

the figure; then in the spherical triangle ZPX, the three sides are known, from which the star's hour-angle ZPX may be computed$=2^h\ 32^m\ 55^s$. (See below.)

Fig. 2. Take QR=star's hour-angle$=2^h\ 32^m\ 55^s$, RA=star's right ascension$=6^h\ 38^m\ 43^s$, and AQm=right ascension of mean sun$=23^h\ 40^m\ 53^s$. Then Qm=ship mean time to be found, as follows:

Calculation of hour-angle and ship mean time.

30°	10′	0″	N.	0·063201
16	31	19	S.	0·018310
46	41	19		4·902752
59	27	13		4·046026
106	8	32		9·030289
12	45	54		

∴ ZPX, or arc QR=	2^h	32^m	55^s	
AR=	6	38	43	
AQ=	9	11	38	$+24^h$
AQm=	23	40	53	
∴ Qm=	9	30	45	=ship mean time.
Chronometer=	11	54	53	=Greenwich mean time.
∴ longitude in time=	2	24	8	=36° 2′ W.

160. In latitude 48° 30′ N., given the altitude of Arcturus=38° 37′ 13″ (west of meridian), declination = 19° 56′ 42″ N., RA$=14^h\ 9^m\ 1^s$, RA mean sun$=7^h\ 13^m\ 42{\cdot}1^s$, and the Greenwich mean time$=8^h\ 20^m\ 10{\cdot}4^s$. Construct the figures, and find by calculation the longitude.

Ans. Longitude=32° 29′ 30″ E.

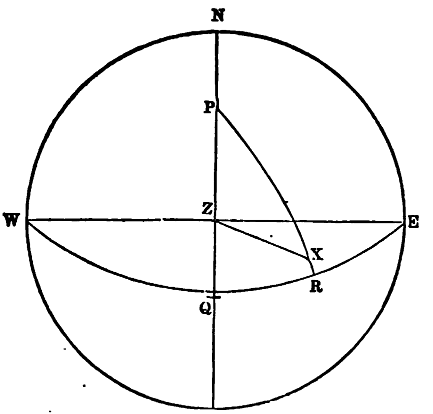
161. In latitude 30° 10′ N., given the altitude of Sirius=30° 32′ 47″ (west of meridian), declination=16° 31′ 19″ S., RA$=6^h\ 38^m\ 43^s$, RA mean sun$=23^h\ 40^m\ 55^s$, and the Greenwich mean time$=11^h\ 54^m\ 52{\cdot}7^s$. Construct the figures, and find by calculation the longitude.

Ans. Longitude=36° 2′ 30″ W.

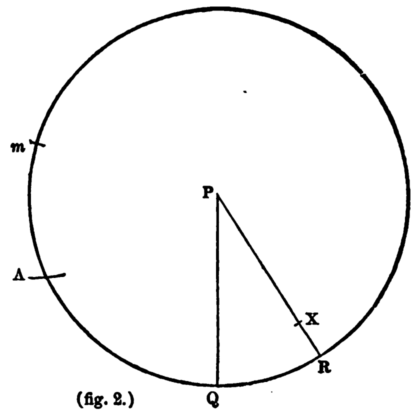
Longitude by star's altitude. East of meridian.

162. In latitude 41° 20′ N., given the altitude of β Canis Majoris= 42° 17′ 27″ (east of meridian), declination=5° 35′ 43″ N., RA$=7^h\ 31^m\ 40^s$, RA of mean sun$=21^h\ 21^m\ 30^s$, and the Greenwich mean time$=3^h\ 31^m\ 26^s$; to find the longitude.

Fig. 1 (page 83). Let X be the place of the star (east of meridian), and complete the figure; then, in the triangle ZPX, the three sides are given to compute the hour-angle ZPX$=2^h\ 21^m\ 56^s$; hence 24^h-ZPX$=21^h\ 38^m\ 4^s$.



(fig. 1.)



(fig. 2.)

Fig. 2. Take QR=star's hour-angle, RA=star's right ascension, and AQm =RA of mean sun. Then Qm=ship mean time to be found, as follows:

Calculation of 24^h—ZPX, *and ship mean time.*

41°	20′	0″ N.	..............0·124429				
5	35	43 N.	..............0·002075				
35	44	17	4·823167				
47	42	33	4·018181				
83	26	50	8·967852				
11	58	16	∴	21^h	38^m	4^s	$=24^h$—ZPX, or arc RAQ.
				7	41	40	=AR
				29	9	44	
				21	21	30	=AQm
			Ship mean time=	7	48	14	=Qm
			Greenwich mean time=	3	31	26	
			∴ longitude in time=	4	16	48	=64° 12′ 0″ E.

163. In latitude 50° 50′ N., given the altitude of α Cygni=49° 33′ 25′ (east of meridian), declination=44° 46′ 6″ N., RA=20^h 36^m $31{\cdot}3^s$, RA mean sun=11^h 32^m $46{\cdot}4^s$, and Greenwich mean time=7^h 19^m $51{\cdot}5^s$. Construct the figures, and find by calculation the longitude. *Ans.* 35° 22′ 0″ W.

LONGITUDE BY LUNAR OBSERVATIONS.

164. The moon in its motion round the earth is seen continually to change its distance from certain bright stars lying near its path. The angular distances of these heavenly bodies from the moon have been computed and recorded in the *Nautical Almanac* for intervals of every three hours, namely at 3, 6, 9, &c., o'clock at Greenwich. The quantities thus

inserted in the *Nautical Almanac* are the *true* distances; that is, the distances are supposed to be seen from the center of the earth. An observer at any place on the surface of the earth may also measure the angular distance between the moon and one of these stars by means of his sextant; but the distance so observed is the *apparent* distance, and before it can be compared with the true distances in the *Nautical Almanac*, in order to determine the mean time at Greenwich when the observation was taken, it must be corrected for the effects of parallax and refraction (see Chap. V.), so as to obtain the true distance of the two bodies at the moment of observation.

165. The formula about to be investigated will enable us to find the true distance from the apparent or observed distance. This is technically called *clearing the distance.* If the true distance thus computed is exactly the same as the one inserted in the *Nautical Almanac* for some given hour at Greenwich, we have found *Greenwich mean time* when the observation was taken. But this coincidence must rarely happen; the computed true distance will generally come out between two of the recorded distances: to obtain the Greenwich mean time corresponding to that distance a simple proportion only is necessary, as will be seen hereafter.

166. The *mean time at the place* at the instant the observed distance is taken can be readily found in the same manner as in the rule for finding the longitude by chronometer (p. 77), namely by observing the altitude of a heavenly body and computing its hour-angle, and thence ship mean time.

167. The method of finding the longitude by a lunar observation, therefore, divides itself into two distinct parts.

1. To find Greenwich mean time.
2. To find ship mean time.

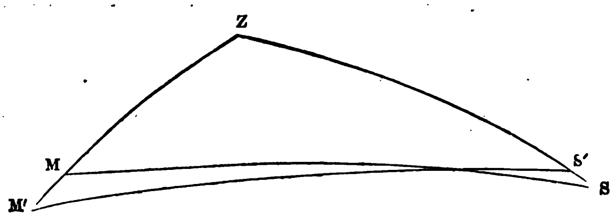
The former is deduced from the observed distance; the latter, in the usual way, from an observed altitude.

To clear the distance.

PROBLEM XXVI.

Given the apparent distance, and the apparent and true altitudes of the two bodies; to investigate a formula for computing the *true* distance.

Let S′ be the apparent place of the heavenly body, as the sun or star; then, in consequence of refraction and parallax, the former exceeding the



latter, its true place (Chap. V.) will be below S′, as S, in a vertical plane (if

we suppose the effect of parallax to take place in a plane passing through the true zenith Z, which it does very nearly). Let M′ be the apparent place of the moon; then its true place will be above M′, since the moon is depressed more by parallax than it is raised by refraction: let, therefore, its true place be supposed to be at M, in a plane passing through Z. Through M′ and S′ draw the great circle M′S′; this arc will be the *apparent* distance found by observation. Through M and S, the true places of the heavenly bodies, draw the arc MS. The arc MS is the *true distance* to be computed.

The true distance MS may be found by the common rules of Spherical Trigonometry (see seventh method of clearing the distance); but it will be more readily obtained by means of a special formula or rule, which will determine the true distance in *terms of the versines*, since the table of versines is calculated to the nearest second; and this will render it unnecessary to proportion for seconds, which must be done when the distance is expressed in terms of the sines, &c.

Investigation of formula for clearing the distance.

Let Z be the zenith of the observer; then, if we suppose the effect of parallax to take place in a vertical circle,* ZM′ and ZS are circles of altitude.

Let z=ZM, the *true* zenith distance of the moon,

z_1=ZS, the *true* zenith distance of the sun or star,

a=the apparent altitude of moon,

a_1=the apparent altitude of sun or star.

Then ZM′$=90°-a$, and ZS′$=90°-a_1$.

Let d=M′S′, the apparent distance of the centers of the two bodies,

and D=MS, the true distance of the centers;

it is required to investigate a formula for computing D.

In triangle ZMS, $\cos. \text{Z}=\dfrac{\cos. \text{D}-\cos. z\,.\,\cos. z_1}{\sin. z\,.\,\sin. z_1}$,

" ZM′S′, $\cos. \text{Z}=\dfrac{\cos. d-\sin. a\,.\,\sin. a_1}{\cos. a\,.\,\cos. a_1}$,

$$\therefore \frac{\cos. \text{D}-\cos. z\,.\,\cos. z_1}{\sin. z\,.\,\sin. z_1}=\frac{\cos. d-\sin. a\,.\,\sin. a_1}{\cos. a\,.\,\cos. a_1},$$

$$\therefore \frac{\cos. \text{D}-\cos. z\,.\,\cos. z_1}{\sin. z\,.\,\sin. z_1}+1=\frac{\cos. d-\sin. a\,.\,\sin. a_1}{\cos. a\,.\,\cos. a_1}+1,$$

$$\text{or } \frac{\cos. \text{D}-(\cos. z\,.\cos. z_1-\sin. z\,.\sin z_1)}{\sin. z\,.\,\sin. z_1}=\frac{\cos. d+\cos. a\,.\cos. a_1-\sin. a\,.\,\sin. a_1}{\cos. a\,.\,\cos. a_1};$$

$$\therefore \frac{\cos. \text{D}-\cos. (z+z_1)}{\sin. z\,.\,\sin. z_1}=\frac{\cos. d+\cos. (a+a_1)}{\cos. a\,.\,\cos. a_1};$$

$$\text{or } \cos. \text{D}-\cos. (z+z_1)=\{\cos. d+\cos. (a+a_1)\}\,.\,\frac{\sin. z\,.\,\sin. z_1}{\cos. a\,.\,\cos. a_1}.$$

Assume $\dfrac{\sin. z\,.\,\sin. z_1}{\cos. a\,.\,\cos. a_1}=2\cos. \text{A}$ (A being an auxiliary angle);

* This is not strictly correct (see Problem XXXVI.).

$$\therefore \text{cos. D} - \text{cos.}(z+z_1) = 2\ \text{cos. A}\ .\ \{\text{cos.}\ d + \text{cos.}\ (a+a_1)\}$$
$$= 2\ \text{cos. A}\ .\ \text{cos.}\ d + 2\ \text{cos. A}\ .\ \text{cos.}\ (a+a_1)$$
$$= \text{cos.}\ (d+\text{A}) + \text{cos.}\ (d \sim \text{A}) + \text{cos.}\ (a+a_1+\text{A}) + \text{cos.}\ (a+a_1 \sim \text{A})$$
$$\therefore 1 - \text{cos. D} = 1 - \text{cos.}\ (z+z_1) + 1 - \text{cos.}\ (d+\text{A}) + 1 - \text{cos.}\ (d \sim \text{A}) + 1 - \text{cos.}\ (a+a_1+\text{A}) + 1 - \text{cos.}\ (a+a_1 \sim \text{A}) - 4,$$
$$\therefore \text{vers. D} = \text{vers.}\ (z+z_1) + \text{vers.}\ (d+\text{A}) + \text{vers.}\ (d \sim \text{A}) + \text{vers.}\ (a+a_1+\text{A}) + \text{vers.}\ (a+a_1 \sim \text{A}) - 4.$$

But tabular versine $=1000000 \times$ versine (see *Trigonometry*, Part I. Art. 32). Reducing the formula to tabular versines and clearing of fractions, we have,

$$\left[\text{since vers. D} = \frac{\text{tab. vers. D}}{1000000},\ \text{vers.}\ (z+z_1) = \frac{\text{tab. vers.}\ (z+z_1)}{1000000},\ \&c.\right]$$

$$\text{tab. vers. D} = \text{tab. vers.}\ (z+z_1) + \text{tab. vers.}\ (d+\text{A}) + \text{tab. vers.}\ (d \sim \text{A}) + \text{tab. vers.}\ (a+a_1+\text{A}) + \text{tab. vers.}\ (a+a_1 \sim \text{A}) - 4000000\ ;$$

or, suppressing the word tabular, it being understood that the formula is expressed in terms of tabular versines, we have

$$\text{vers. D} = \text{vers.}\ (z+z_1) + \text{vers.}\ (d+\text{A}) + \text{vers.}\ (d \sim \text{A}) + \text{vers.}\ (a+a_1+\text{A}) + \text{vers.}\ (a+a_1 - \text{A}) - 4000000.$$

From which expression the true distance D may be computed, when the true and apparent altitudes and the apparent distance are known.

Before, however, this formula can be used, the value of the auxiliary angle A must be computed, and formed into a table; this has been done, and may be found in the Nautical Tables of Inman and others.

The auxiliary angle A may be computed as follows.

Construction of table of auxiliary angle A.

Let it be required to compute the value of A, when the zenith distance of the moon $z=32° \ 19' \ 50''$, the app. alt. of the moon $a=57° \ 11' \ 24''$, zenith distance of sun $z_1=55° \ 39' \ 46''$, and app. alt. of sun $a_1=34° \ 21' \ 32''$.

$$\text{Since } 2\ \text{cos. A} = \frac{\text{sin.}\ z\ .\ \text{sin.}\ z_1}{\text{cos.}\ a\ .\ \text{cos.}\ a_1} = \text{sin.}\ z\ .\ \text{sin.}\ z_1\ .\ \text{sec.}\ a\ .\ \text{sec.}\ a_1,$$

$\therefore$ in logarithms,

$$\text{log. cos. A} = \text{log. sin.}\ z + \text{log. sec.}\ a + \text{log. sin.}\ z_1 + \text{log. sec.}\ a_1 - 30{\cdot}301030.$$

Calculation.

log. sin. z	9·728190
„ sec. a	10·266120
„ sin. z_1	9·916839
„ sec. a_1	10·083273
	39·994422
	30·301030
„ cos. A	9·693392 . ∴ A=60° 25′ 16″.

And in the same manner may other values of A be computed for any given app. alt. of the moon, the argument of the table.

The true distance D being thus calculated by means of the preceding formula, or otherwise, the mean time at Greenwich corresponding thereto may be found as follows.

PROBLEM XXVII.

Given the true distance of the moon from some other heavenly body, to calculate the corresponding Greenwich mean time.

The Greenwich time corresponding to the true distance may be found by a simple proportion, thus:

As the difference of distances in the *Nautical Almanac* for the three hours between which the computed true distance lies : 3 hours : : difference between the distance in *Nautical Almanac* at the beginning of the 3 hours and the computed distance : t; t being the hours, minutes, and seconds elapsed from the beginning of the three hours.

The quantity t is, however, usually found by means of proportional logarithms, as follows:

Let d=difference of distances in 3 hours (taken from *Nautical Almanac*),
d'=difference between first distance taken out and calculated distance,
t=time elapsed from the hour opposite the first distance taken out.

Then $d : d' :: 3$ hours $: t$, $\therefore td = 3d'$;

$\therefore \log. t + \log. d = \log. 3^h + \log. d'$, or $\log. t = \log. 3^h - \log. d + \log. d'$;
$\therefore \log. 3^h - \log. t = -(\log. 3^h - \log. d) + \log. 3^h - \log. d'$,
or prop. log. t = prop. log. d' − prop. log. d (Chapter V.).

Then the time t thus found, added to the hour opposite the first distance in the *Nautical Almanac* taken out, will be the Greenwich mean time at the instant when the observation was taken.

168. Given the sun's true distance (calculated from an observed distance) $=73° 51' 58''$, and two distances taken out of the *Nautical Almanac*, namely at $6^h = 73° 23' 44''$, and at $9^h = 74° 54' 7''$; to find Greenwich mean time when the observation was taken.

True distance...	73°	51′	58″
At 6^h	73	23	44
	74	54	7
		28	14=d'
	1	30	23=d

First method, by proportion.

$1° 30' 23'' : 3^h :: 28' 14'' : t$,

$\therefore t = 0^h 56^m 14^s$

Hence Greenwich mean time $= 6^h 56^m 14^s$.

Second method, by proportional logarithms.

prop. log. t=prop. log. d'—prop. log. d.

prop. log. d'.........·80451

prop. log. d ....·29918

∴ prop. log. t .... 50533 ∴ $t=0^h\ 56^m\ 14^s$.

Time at first distance in *Nautical Almanac*.........6

∴ Greenwich mean time=6 56 14, when the sun's true distance was 73° 51′ 58″.

EXAMPLE FOR PRACTICE.

169. Given the sun's true distance (calculated from an observed distance) =110° 49′ 48″, and two distances taken out of the *Nautical Almanac*, namely at 0^h or Greenwich mean noon=110° 23′ 53″, and at 3^h=111° 47′ 28″; to find Greenwich mean time when the observation was taken: first, by proportion; second, by proportional logarithms.

Ans. Greenwich mean time=$0^h\ 55^m\ 49^s$.

(2.) TO FIND SHIP MEAN TIME.

The ship mean time must be calculated for the same instant as the distance was observed which determined the instant of Greenwich mean time. Ship time may therefore be obtained from an altitude of a heavenly body taken at that instant, or at some known interval before or after. It is obtained, in fact, precisely in the same manner as directed in Problem XXIV., for finding the longitude by chronometer.

LASTLY. TO FIND THE LONGITUDE IN TIME.

The difference between Greenwich mean time and ship mean time thus found is the LONGITUDE IN TIME of the ship.

The following sun-lunar, worked out in detail, will indicate the several steps in the rule for finding the longitude by a lunar observation. The method of determining the longitude by a star-lunar differs very little from that by sun-lunar; the construction of the figures, to suit the several cases that may occur, will present no difficulty to the student who is acquainted with the preceding problems.

EXAMPLE OF SUN-LUNAR. ALTITUDES OBSERVED.

170. Given the necessary observations and quantities taken out of the *Nautical Almanac*; to calculate the several parts of a lunar. These are;

(1.) The true distance between the sun and moon.

(2.) The Greenwich mean time corresponding to that distance.

(3.) The hour-angle of the moon or sun, selecting that body that is farthest from the meridian (Problem XI.).

(4.) The ship mean time deduced from that hour-angle.

(5.) The longitude in time.

(1.) *To calculate the true distance.*

(By formula, p. 86, and auxiliary angle A.)

Quantities required to be given by previous calculation are the following:

Moon's apparent altitude........................	20°	57′	24″	$=a$
Moon's true altitude..............................	21	47	26	$=90°-z$
Sun's apparent altitude	13	33	38	$=a_1$
Sun's true altitude	13	29	48	$=90°-z_1$
Apparent distance of sun and moon	88	23	12	$=d$

Auxiliary angle A......60° 10′ 43″.

Then, to find the true distance D, we have

$$\text{vers. } D = \text{vers. } (z+z_1) + \text{vers. } (d \sim A) + \text{vers. } (d \sim A) + \text{vers. } (a+a_1+A) + \text{vers. } (a+a_1 \sim A) - 4000000.$$

z	68°	12′	34″	
z_1	76	30	12	
$z+z_1=$	144	42	46	vers.
d	88	23	12	
A	60	10	43	
$d+A=$	148	33	55	„
$d-A=$	28	12	29	„
a	20	57	24	
a_1	13	33	38	
$a+a_1$	34	31	2	
A	60	10	43	
$a+a_1+A=$	94	41	45	„
$a+a_1\sim A=$	25	39	41	„

Calculation.

vers. $(z+z_1)$	1816138...130
„ $(d+A)$	1853096...140
„ $(d-A)$	0118696... 66
„ $(a+a_1+A)$ ...	1081649...217
„ $(a+a_1\sim A)$ ...	0098545... 85
	4968124
	638
	4968762
	4000000
vers. D	·0968762
	589...173

$\therefore$ D=88° 12′ 36″.

(2.) *To calculate Greenwich mean time.*

Quantities required to be given are:

True distance at time of observation	88°	12′	36″
Distance from *Nautical Almanac* at 21ʰ	88	50	57
„ „ „ 24ʰ	87	23	51

Then, to find Greenwich mean time, we have

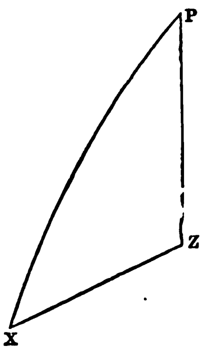
$$\text{prop. log. } t = \text{prop. log. } d' - \text{prop. log. } d.$$

True dist. at obs. ...	88°	12′	36″	prop. log. d'...67151
„ 21^h...	88	50	57	prop. log. d ...31525
„ 24^h...	87	23	51	∴ prop. log. t...35626
∴ d'=		38	21	∴ t= $1^h\ 19^m\ 15^s$ = { time elapsed since 21 o'clock.
d=	1	27	6	21

∴ Greenwich mean time=22 19 15

(3.) *To calculate the moon's hour-angle.* Quantities given are:

Moon's pol. dist....	84°	53′	19″,	found from *Nautical Almanac.*
„ zen. dist....	68	12	34	„ observation.
Colat. of place......	39	22	48	(moon west of meridian).

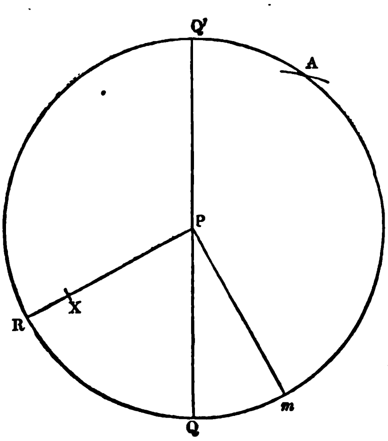


To find hour-angle ZPX.

84°	53′	19″	.... 0·001731
39	22	48	.... 0·197603
45	30	31	.... 4·922892
68	12	34	.... 4·294029
113	43	5	9·416255
22	42	3	∴ hour-angle=$4^h\ 5^m\ 40^s$.

(4.) *To calculate ship mean time.*
Quantities required to be given are:

Moon's hour-angle	4^h	5^m	40^s	
„ right ascension	11	17	1,	from *Nautical Almanac.*
RA of mean sun	17	8	29	„ „ „



To find ship mean time.

By formula, p. 34, or by fig. (see construction),

Ship mean time=moon's hour-angle +moon's RA−RA mean sun.

Hour-angle	4^h	5^m	40^s
Moon's RA	11	17	1
	15	22	41
Add	24		
	39	22	41
RA mean sun............	17	8	29
∴ ship mean time=	22	14	12

(5.) *To calculate the longitude.*

Greenwich mean time	22^h	19^m	15^s
Ship mean time	22	14	12
∴ long. in time=		5	3
or in degrees=	1°	15′	45″ west.

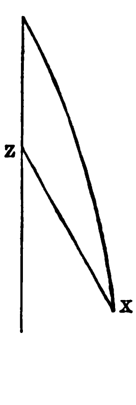
In the above example the ship mean time has been obtained from the *moon's* altitude, and this was done because that heavenly body was the more distant of the two from the meridian (see Problem XI.). If ship mean time is computed from the *sun's* altitude, we must proceed as follows.

(3.) *To calculate the sun's hour-angle.*

Quantities required to be given are:

Sun's pol. dist.......112°	45′	31″,	found from	*Nautical Almanac.*
„ zen. dist....... 76	30	12	„	observation.
Colat. of place 39	22	48		

The sun is east of meridian (known from observation).



To find hour-angle ZPX,
and thence 24^h—ZPX, or app. time.

112°	45′	31″	0·035201
39	22	48	0·197603
73	22	43	4·984825
76	30	12	3·435643
149	52	55	8·653272
3	7	29	

∴ 24^h—ZPX=22^h 22^m 1^s, or apparent time.

(4.) *To calculate ship mean time.*

Quantities required to be given are:

Ship app. time...22^h 22^m 1^s Equat. of time...0^h 7^m $49{\cdot}4^s$ sub.

By formula, p. 20:

Ship mean time=apparent time—equation of time.

Ship apparent time..................	22^h	22^m	1^s
Equation of time.....................		4	49·4
∴ ship mean time=	22	14	11·6

(5.) *To calculate the longitude.*

Greenwich mean time	22^h	19^m	15^s
Ship mean time	22	14	12
∴ long. in time=		5	3 W.
or long.=	1°	15′	45″ W.

EXAMPLE OF SUN-LUNAR. ALTITUDES OBSERVED.

171. Given the following quantities; to construct figures, and compute the longitude.

Moon's app. alt. ...	56°	16′	40″	True distances, from *Nautical Almanac.*			
„ true alt. ...	56	46	21	At 3^h	77°	13′	6″
Sun's app. alt.	34	15	42	„ 6^h	78	35	22
„ true alt.	34	14	24				
Apparent dist.	78	37	16	Sun's pol. dist.	95	58	30
Auxil. angle A	60	25	12	„ zen. dist.	55	45	36
				Colat.	55	0	0
				Sun west of meridian.			

Equation of time...11^m $42{\cdot}9^s$, additive to app. time.

NOTE. Ship time in this ex. is found from sun's altitude.

Ans. Longitude=36° 40′ W.

EXAMPLE OF STAR-LUNAR. ALTITUDES OBSERVED.

172. Given the following quantities; to construct figures, and compute the longitude.

Moon's app. alt. ...	34°	20′	3″	True distances, from *Nautical Almanac.*			
„ true alt. ...	35	6	46	At 9^h	96°	32′	41″
Star's app. alt.	32	57	59	„ 12^h	95	7	31
„ true alt.	32	56	30				
Apparent dist.	96	55	26	Star east of meridian.			
Auxil. angle A	60	18	2	Star's pol. dist.	81°	30′	55″
				„ zen. dist.	57	3	30
				Colat.	44	39	50
				Star's RA	19°	43′	11″
				RA mean sun	4	59	47·7

NOTE. Ship time is found from star's altitude.

Ans. Longitude=29° 17′ 15″ E.

LONGITUDE BY LUNAR. ALTITUDES CALCULATED.

When the altitudes are to be calculated, we must previously know the hour-angles of the heavenly bodies; and these are readily found when the time at the ship is given, which it is supposed to be in this case (note, p. 52). The following examples, and the constructions which accompany them, will sufficiently indicate the several steps to be taken for finding the longitude.

First example. Sun-lunar. Altitudes calculated.

(The distance cleared by means of *auxiliary angle A.*)

173. Dec. 8, 1857, at 10^h 14^m 12^s A.M., ship mean time, in lat 50° 37′ 12″ N., and long. by account 1° 6′ W., the observed distance of the nearest limbs of the sun and moon was 87° 51′ 30″. Construct the figure, and find by calculation the longitude.

Elements from *Nautical Almanac*, computed for the Greenwich date:

(1.) Sun's decl.=22° 45′ 31″ S.; (2.) sun's semi.=16′ 17″; (3.) eq. of time=7^m $49{\cdot}4^s$ (additive to mean time); (4.) RA mean sun=17^h 8^m $28{\cdot}8^s$; (5.) moon's RA=11^h 17^m 1^s; (6.) moon's decl.=5° 6′ 41″ N.; (7.) moon's semi. (aug.)=15′ 25·5″; (8.) moon's hor. parallax=56′ 15·4″; (9.) distances at 21^h=88° 50′ 57″, at 24^h=87° 23′ 51″.

The several parts of the lunar to be calculated are in order as follows:

(1.) The hour-angles of the sun and moon.

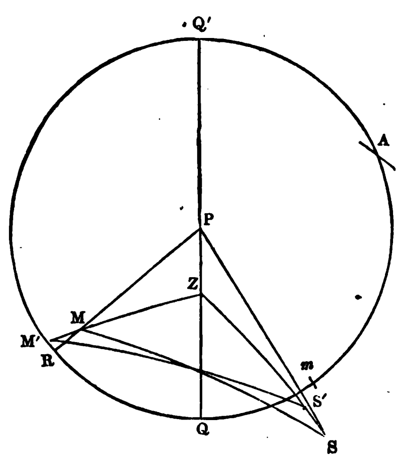
(2.) The true and apparent altitudes of sun and moon.

(3.) The true distance.

(4.) The Greenwich mean time. (5.) The longitude in time.

Construction of figure.

Let QRQ′ represent the celestial equator, QQ′ the celestial meridian, P the pole, and Z the zenith. Take QQ′m=22^h 14^m 12^s, then m is the place of the mean sun. Let S be the true place of the sun (decl. 22° 45′ 31″ S.), and through S draw ZS, a circle of altitude; then since the sun is raised more by refraction than depressed by parallax, its apparent place will be above S, as at S′, in same vertical circle ZS (supposing the earth to be a sphere): draw the circle of decl. PS. Again, take mQA=17^h 8^m $28{\cdot}8^s$, the right ascension of mean sun; then A is the place of the first point of Aries. To find the place of the moon, take AR=11^h 17^m 1^s, the moon's RA, and draw the circle of declination PR; then the moon is in PR. Let M be the true place of the moon (decl. 5° 6′ 41″ N.), and through M draw ZM, a circle of altitude; then, since the moon is depressed more by parallax than raised by refraction, its apparent place will be below M, as at M′, in the same vertical circle. Join M′S′ and MS; then M′S′ is the apparent distance of the two bodies (=87° 51′ 30″+sum of semidiameters)=88° 23′ 12″, and MS is the true distance to be computed.



(1.) To find the hour-angles ZPS and ZPM.

Sun's hour-angle ZPS.				Moon's hour-angle ZPM. (See note, p. 52.)			
Ship mean time QQ'm ...	22^h	14^m	12^s	RA mean sun ...	17^h	8^m	$28{\cdot}8^s$
Equation of time mPS ...		7	49·4	Ship mean time..	22	14	12·0
	22	22	1·4		39	22	40·8
	24			Moon's RA	11	17	1·0
∴ sun's hour-angle ZPS...	1	37	58·6		28	5	39·8
				∴ moon's hour-angle ZPM=	4	5	39·8

To find the sun's true zenith distance ZS.

In the spherical triangle ZPS, are given PZ the colatitude, PS the polar distance, and ZPS the hour-angle; to calculate ZS the true zenith distance= 76° 30′ 12″ (*Trig.* Rule IX.).

To find the moon's true zenith distance ZM.

In the spherical triangle ZPM, are given PZ the colatitude, PM the polar distance, and ZPM the hour-angle; to calculate ZM the true zenith distance =68° 12′ 34″.

(2.) To find the true and apparent altitudes of the sun.

Sun's true zenith dist.=	76°	30′	12″	=ZS
∴ sun's true alt.=	13	29	48	=90°−ZS
and by the tables, sun's cor. in alt.=		3	50	=SS′
∴ sun's apparent alt.=	13	33	38	=90°−ZS′

To find the true and apparent altitudes of the moon.

Moon's true zen. dist.=	68°	12′	34″	=ZM
∴ moon's true alt.=	21	47	26	=90°−ZM
and by the tables, moon's cor. in alt.=		50	2	=MM′ (see Chap. V.).
∴ moon's app. alt.=	20	57	24	
and by the tables, aux. angle A=	60	10	45	

Hence by the formula, p. 86, the true distance MS=88° 12′ 36″; and the Greenwich mean time corresponding to this distance=22^h 19^m 15^s (see p. 88), and the long. in time=0^h 5^m 3^s W., or 1° 15′ 45″ W.

174. *Second Example. Sun-lunar. Altitudes calculated.*

(The distance in this example is cleared by the common rules of Spherical Trigonometry; see the Seventh Method.)

Given the following quantities; to construct a figure, and calculate the longitude.

(1.) Lat.$=50^\circ$ $37'$ $30''$ N., sun's decl.$=22^\circ$ $42'$ $28''$ N., sun's hour-angle $=1^h\,23^m\,24^s$ (west of meridian); to calculate sun's true altitude$=57^\circ\,42'\,16''$.

(2.) Lat$=50^\circ$ $37'$ $30''$, moon's decl.$=11^\circ$ $43'$ $50''$ N., moon's hour-angle $=3^h\,6^m\,55^s$ (east of meridian); to calculate moon's true altitude$=35^\circ\,39'\,23''$.

(3.) Sun's correction in alt.$=32''+$, moon's correction in alt.$=45'\,8''-$; to find sun's app. alt.$=57^\circ$ $42'$ $48''$, and moon's apparent alt.$=34^\circ$ $54'$ $15''$.

(4.) Sun's app. zen. dist.$=32^\circ\,17'\,12''$, moon's app. zen. dist.$=55^\circ\,5'\,45''$, and apparent dist. of centers$=65^\circ$ $35'$ $17''$; to calculate the angle at zenith $=99^\circ$ $15'$ $15''$.

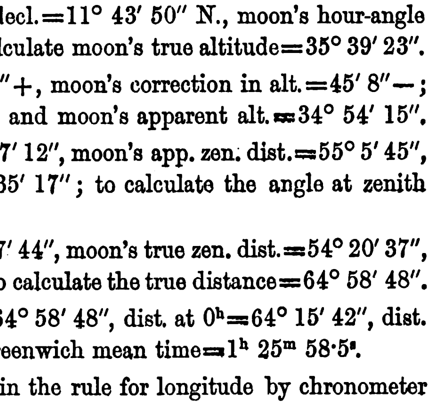
(5.) Sun's true zen. dist.$=32^\circ\,17'\,44''$, moon's true zen. dist.$=54^\circ\,20'\,37''$, and angle at zenith$=99^\circ$ $15'$ $15''$; to calculate the true distance$=64^\circ$ $58'$ $48''$.

(6.) True dist. at observation$=64^\circ$ $58'\,48''$, dist. at $0^h=64^\circ$ $15'\,42''$, dist. at $3^h=65^\circ$ $45'$ $56''$; to calculate Greenwich mean time$=1^h$ 25^m $58{\cdot}5^s$.

(7.) Ship mean time (found as in the rule for longitude by chronometer or otherwise)$=1^h$ 21^m $46{\cdot}5^s$, and Greenwich mean time$=1^h$ 25^m $58{\cdot}5^s$, to calculate the LONGITUDE$=4^m$ $12^s=1^\circ$ $3'$ W.

Construction of figure from preceding data.

(1.) Let P be the pole, Z the zenith, and S the true place of the sun; then, in the spherical triangle ZPS, are given PZ$=39^\circ$ $22'$ $30''$, PS$=67^\circ$ $17'$ $32''$, and angle SPZ$=1^h$ 23^m 24^s; to calculate the sun's true zenith dist. ZS$=32^\circ\,14'\,44''$: hence sun's true altitude SO$=57^\circ$ $42'$ $16''$. Let S$'$ be the apparent place of sun; then SS$'=32''$, and ∴ sun's apparent altitude S$'$O $=57^\circ$ $42'$ $48''$.



Again, let m be the true place of the moon; then, in the spherical triangle ZPm, are given PZ$=39^\circ\,22'\,30''$, P$m=78^\circ\,16'\,10''$, and angle ZP$m=3^h$ 6^m 55^s; to calculate the moon's true zenith distance Z$m=54^\circ$ $20'$ $37''$: hence moon's true altitude mO$'=35^\circ$ $39'$ $23''$. Let m' be the apparent place of the moon; then $mm'=45'$ $8''$, and ∴ moon's apparent altitude m'O$'=34^\circ$ $54'$ $15''$.

(2.) Join S$'m'$; then, in the spherical triangle ZS$'m'$, are given ZS$'=32^\circ$ $17'$ $12''$, Z$m'=55^\circ$ $5'$ $45''$, and apparent distance S$'m'=65^\circ$ $35'$ $17''$; to calculate the angle S$'$Z$m'=99^\circ$ $15'$ $15''$. Again, in the spherical triangle ZSm, are given ZS$=32^\circ$ $17'$ $44''$, Z$m=54^\circ$ $20'$ $37''$, and angle SZ$m=$

99° 15′ 15″; to calculate the true distance sm=64° 58′ 48″: hence the longitude is found as pointed out above.

PRACTICAL METHODS OF CLEARING THE LUNAR DISTANCE.

It may be necessary sometimes to be acquainted with other methods of clearing the lunar distance besides the one made use of in the preceding examples, since that may not be adapted to the tables in the student's hands. If, for instance, his tables do not contain the auxiliary angle A or a table of natural versines, the rule derived from the formula in p. 86 cannot be used by him. We will therefore now show how the fundamental formulæ in p. 85 may be altered, so as to obtain a rule adapted to other tables; we shall thus form, in fact, several distinct methods of clearing the distance, such as those found in Norie's, Riddle's, and Raper's works on navigation. The investigation of these several methods will not be without its use to the student, even if he should have no occasion to make a practical application of them.

SECOND METHOD OF CLEARING THE DISTANCE.

By means of versines, but not requiring the auxiliary angle A.

Since, as in former method, p. 85,

$$\frac{\cos. \mathrm{D}-\cos. z \,.\, \cos. z_1}{\sin. z \,.\, \sin. z_1}=\frac{\cos. d-\sin. a \,.\, \sin. a_1}{\cos. a \,.\, \cos. a_1},$$

$$\therefore \frac{\cos. \mathrm{D}-\cos. z \,.\, \cos. z_1}{\sin. z \,.\, \sin. z_1}-1=\frac{\cos. d-\sin. a \,.\, \sin. a_1}{\cos. a \,.\, \cos. a_1}-1;$$

$$\text{or } \frac{\cos. \mathrm{D}-(\cos. z \,.\, \cos. z_1+\sin. z \,.\, \sin. z_1)}{\sin. z \,.\, \sin. z_1}$$

$$=\frac{\cos. d-(\cos. a \,.\, \cos. a_1+\sin. a \,.\, \sin. a_1)}{\cos. a \,.\, \cos. a_1};$$

$$\therefore \frac{\cos. \mathrm{D}-\cos. (z\backsim z_1)}{\sin. z \,.\, \sin. z_1}=\frac{\cos. d-\cos. (a\backsim a_1)}{\cos. a \,.\, \cos. a_1};$$

$$\therefore \cos. \mathrm{D}=\cos. (z\backsim z_1)+\frac{\cos. d-\cos. (a\backsim a_1)}{\cos. a \,.\, \cos. a_1} \,.\, \sin. z \,.\, \sin. z_1$$

$$=\cos. (z\backsim z_1)-2 \sin. \tfrac{1}{2}(d+\overline{a\backsim a_1}) \,.\, \sin. \tfrac{1}{2}(d-\overline{a\backsim a_1}) \,.\, \frac{\sin. z \,.\, \sin. z_1}{\cos. a \,.\, \cos. a_1};$$

$$\therefore \text{vers.} \mathrm{D}=\text{vers.} (z\backsim z_1)+2 \sin. \tfrac{1}{2}(d+\overline{a\backsim a_1}) \,.\, \sin. \tfrac{1}{2}(d-\overline{a\backsim a_1}) \,.\, \frac{\sin. z \,.\, \sin. z_1}{\cos. a \,.\, \cos. a_1}$$

$$=\text{vers.} (z\backsim z_1)+\text{vers.} \theta. \text{ By assuming}$$

$$\text{vers.} \theta=2 \sin. \tfrac{1}{2}(d+\overline{a\backsim a_1}) \,.\, \sin. \tfrac{1}{2}(d-\overline{a\backsim a_1}) \,.\, \frac{\sin. z \,.\, \sin. z_1}{\cos. a \,.\, \cos. a_1}.$$

These formulæ determine the true distance D without using the auxiliary angle A, as in the first method.

EXAMPLE OF CLEARING THE DISTANCE BY SECOND METHOD.

175. Find the true distance D of the sun and moon, having given apparent distance $d=35° 47' 24''$, moon's app. alt. $a=57° 11' 25''$, sun's app. alt. $a_1=34° 21' 32''$, moon's true zenith distance $z=32° 19' 50''$, and sun's true zenith distance $z_1=55° 39' 46''$.

$$\text{vers. } D = \text{vers. } (z \backsim z_1) + \text{vers. } \theta,$$

$$\text{vers. } \theta = 2 \sin. \tfrac{1}{2}(d+\overline{a \backsim a_1}) . \sin. \tfrac{1}{2}(d-\overline{a \backsim a_1}) . \frac{\sin. z . \sin. z_1}{\cos. a . \cos. a_1}$$

$$\therefore \log. \text{vers. } \theta = 6\cdot301030 + \log. \sin. \tfrac{1}{2}(d+a \backsim a_1) +$$
$$\log. \sin. \tfrac{1}{2}(d-\overline{a \backsim a_1}) + \log. \sin. z + \log. \sin. z_1 - (\log. \cos. a + \log. \cos. a_1) - 20.$$

Calculation.

a....................	57°	11′	25″		6·301030	
a_1....................	34	21	32		9·689792	
$a \backsim a_1$...............	22	49	53		9·052480	
d....................	35	47	24		9·728194	9·733883
$d+\overline{a \backsim a_1}$...........	58	37	17		9·916839	9·916727
$d-\overline{a \backsim a_1}$...........	12	57	31		44·688335	19·650610
$\therefore \tfrac{1}{2}(d+\overline{a \backsim a_1})$...	29	18	38·5		19·650610	
$\tfrac{1}{2}(d-\overline{a \backsim a_1})$.......	6	28	45·5	log. vers. θ...	5·037725	
z....................	32	19	50	$\therefore$ vers. θ...	109076	
z_1....................	55	39	46	vers. $\overline{z \backsim z_1}$...	81776	
$z \backsim z_1$..............	23	19	56	$\therefore$ vers. D...	190852	
					$\therefore$ D$=35° 59' 14''$.	

This work may be somewhat shortened by using the table of haversines, as in the following method.

THIRD METHOD OF CLEARING THE DISTANCE.

Using haversines, but not requiring the auxiliary angle A.

By second method,

$$\text{vers. } \theta = 2 \sin. \tfrac{1}{2}(d+\overline{a \backsim a_1}) . \sin. \tfrac{1}{2}(d-\overline{a \backsim a}) . \frac{\sin. z . \sin. z_1}{\cos. a . \cos. a_1},$$

$$\therefore \tfrac{1}{2} \text{vers. } \theta \text{ or hav. } \theta = \sin. \tfrac{1}{2}(d+\overline{a \backsim a_1}) . \sin. \tfrac{1}{2}(d-\overline{a \backsim a_1}) . \sin. z . \sin. z_1 . \sec. a . \sec. a_1$$

$$= \sqrt{\text{hav. } (d+\overline{a \backsim a}) . \text{hav. } (d-\overline{a \backsim a})} . \sin. z . \sec. a . \sin. z_1 . \sec. a_1$$

From which θ may be found and thence D, as in second method.

EXAMPLE OF CLEARING THE DISTANCE BY THIRD METHOD.

176. Find the true distance D of the sun and moon, having given apparent distance $d=35° 47' 24''$, moon's app. alt. $a=57° 11' 25''$, sun's app. alt.

$a_1=34°\ 21'\ 32''$, moon's true zen. dist. $z=32°\ 19'\ 50''$, and sun's true zen. dist. $z_1=55°\ 39'\ 46''$.

$$\text{hav. } \theta=\sqrt{\text{hav.}(d+\overline{a\sim a_1})\ .\ \text{hav.}(d-\overline{a\sim a_1})\ .\ \text{sin. } z\ .\ \text{sec. } a\ .\ \text{sin. } z_1\ .\ \text{sec. } a_1},$$
$$\text{and vers. D}=\text{vers. } \overline{z\sim z_1}+\text{vers. } \theta.$$

a	57°	11′	25″	hav. $(d+\overline{a\sim a_1})$	9·379578
a_1	34	21	32	hav. $(d-\overline{a\sim a_1})$	8·104941
$a\sim a_1$	22	49	53		2)17·484519
d	35	47	24		8·742259
$d+\overline{a\sim a_1}$	58	37	17	sin. z	9·916838
$d-\overline{a\sim a_1}$	12	57	31	sec. a	0·083270
z	32	19	50	sin. z_1	9·728177
z_1	55	39	46	sec. a_1	0·266137
$z\sim z_1$	23	19	56	hav. θ	8·736681
				$\therefore\ \theta=$	27° 0′ 35″
				vers. θ	109069
				vers. $\overline{z\sim z_1}$	81775
				vers. D	190844
				$\therefore$ true dist. D=	35° 59′ 12″

FOURTH METHOD OF CLEARING THE DISTANCE.

The previous method simplified, by tabulating a part of the formula.

In the expression, versine $\theta=2\ \text{sin.}\ \frac{1}{2}(d+\overline{a\sim a_1})\ .\ \text{sin.}\ \frac{1}{2}(d-\overline{a-a_1})\ .\ \frac{\text{sin. } z\ .\ \text{sin. } z_1}{\text{cos. } a\ .\ \text{cos. } a_1}$, the quantity $\frac{\text{sin. } z_1}{\text{cos. } a_1}$ changes very slowly for all values of z_1 and a_1; since z_1 and a_1 are the true zen. dist. and app. alt. of the sun or star, and therefore $\frac{\text{sin. } z_1}{\text{cos. } a_1}=\frac{\text{cos. true alt.}}{\text{cos. app. alt.}}=1$ nearly. We may therefore compute beforehand the values of the expression $2\ .\ \frac{\text{sin. } z_1}{\text{cos. } a_1}$ for all altitudes of the sun or star, and form a small table of the results. This has accordingly been done (see Table B), and we can take the value of $\frac{2\ \text{sin. } z_1}{\text{cos. } a_1}$ out by inspection (entering the table with the app. alt. of the heavenly body); the labour of calculating $\frac{2\ \text{sin. } z_1}{\text{cos. } a_1}$ is thus avoided. The value of the true distance D may then be expressed as follows:

$$\text{vers. D}=\text{vers. } \overline{z\sim z_1}+\text{vers. } \theta, \text{ and}$$
$$\text{vers. } \theta=\text{B sin. } z\ .\ \text{sec. } a\ .\ \sqrt{\text{hav. }(d+\overline{a-a_1})\ .\ \text{hav. }(d-\overline{a-a_1})},$$
where log. B=quantity in following table.

Construction of Table B.

Since $\text{vers.}\,\theta = 2\,\text{sin.}\,\tfrac{1}{2}(d+\overline{a\backsim a_1})\,.\,\text{sin.}\,\tfrac{1}{2}(d-\overline{a\backsim a_1})\ \dfrac{\text{sin.}\,z\,.\,\text{sin.}\,z_1}{\text{cos.}\,a\,.\,\text{cos.}\,a_1}$

$\therefore\ \text{vers.}\,\theta = 2\sqrt{\text{hav.}\,(d+\overline{a\backsim a_1})\,.\,\text{hav.}\,(d-\overline{a-a_1})}\,:\,\text{sin.}\,z\,.\,\text{sec.}\,a\,.\,\text{sin.}\,z_1\,.\,\text{sec.}\,a_1,$

$\therefore\ \text{log. vers.}\,\theta - 6 = \cdot 301030 + \text{log. sin.}\,z_1 - 10 + \text{log. sec.}\,a_1 - 10 + \tfrac{1}{2}\{\text{log. hav.}\ (d+\overline{a\backsim a_1}) + \text{log. hav.}\,(d-\overline{a\backsim a_1})\} - 10 + \text{log. sin.}\,z - 10 + \text{log. sec.}\,a - 10,$

or $\text{log. vers.}\,\theta = 6\cdot 301030 + \text{log. sin.}\,z_1 + \text{log. sec.}\,a_1 - 20$
$+\tfrac{1}{2}\{\text{log. hav.}\,(d+\overline{a\backsim a_1}) + \text{log. hav.}\,(d-\overline{a\backsim a_1})\} + \text{log. sin.}\,z + \text{log. sec.}\,a - 30\cdot$

The quantities $6\cdot 301030 + \text{log. sin.}\,z_1 + \text{log. sec.}\,a_1 - 20$, may be computed for all altitudes of the sun or star, and tabulated. Let the sum be denoted by log. B: then $\text{log. B} = 6\cdot 301030 + \text{log. sin.}\,z_1 + \text{log. sec.}\,a_1 - 20$;
and the formula for finding θ will then become

$\text{log. vers.}\,\theta = \tfrac{1}{2}\{\text{log. hav.}\ (d+\overline{a\backsim a_1}) + \text{log. hav.}\ (d-\overline{a\backsim a_1})\} + \text{log. sin.}\,z + \text{log. sec.}\,a + \text{log. B} - 30.$

177. *Example.* Find the value of log. B, when the sun's apparent altitude $a_1 = 34°\ 21'\ 32''$, and sun's true zenith distance $z_1 = 55°\ 39'\ 46''$.

$\text{log. B} = 6\cdot 301030 + \text{log. sin.}\,z_1 + \text{log. sec.}\,a_1 - 20.$

	6·301030
log. sin. z_1...	9·916839
„ sec. a_1...	0·083273
∴ log. B.....	6·301142 for alt. 34°.

And in the same manner may all the other values be computed.

TABLE B.

FOR THE *SUN*. ARGUMENT.	SUN'S APP. ALT.	FOR A *STAR* OR *PLANET*. ARGUMENT.	SUN'S APP. ALT.
App. alt.	Log. B.	App. alt.	Log. B.
90	6·301134	90	6·301153
75	6·301135	30	6·301152
65	6·301136	20	6·301151
60	6·301137	15	6·301150
55	6·301138	12	6·301149
50	6·301139	10	6·301148
45	6·301140	9	6·301147
40	6·301141	8	6·301146
35	6·301142	7	6·301144
30	6·301143	6	6·301141
25	6·301144	5	6·301137
20	6·301145	4	6·301130
10	6·301144		
8	6·301143		
7	6·301142		
6	6·301139		
4	6·301130		

EXAMPLE OF CLEARING THE DISTANCE BY FOURTH METHOD.

178. Find the true distance D of the sun and moon, having given the apparent distance $d=35° 47' 24''$, sun's app. alt. $a_1=34° 21' 32''$, moon's app. alt. $a=57° 11' 25''$, sun's true zenith distance $z_1=55° 39' 46''$, and moon's true zenith distance $z=32° 19' 50''$.

Calculation.

$$\text{vers. } \theta = \text{B} \sin . z . \sec . a . \sqrt{\text{hav. } (d+\overline{a\sim a_1}) . \text{hav. } (d-\overline{a\sim a_1})}$$

$$\text{vers. D} = \text{vers. } \overline{z\sim z_1} + \text{vers. } \theta.$$

a_1	34°	21′	32″	½ log. hav. $(d+\overline{a-a_1})$	4·689789
a	57	11	25	½ log. hav. $(d-\overline{a-a_1})$	4·052470
$a-a_1$	22	49	53	„ sin. z	9·728177
d	35	47	24	„ sec. a	0·266137
$d+\overline{a-a_1}$	58	37	17	„ B	6·301138
$d-\overline{a-a}$	12	57	31	„ vers. θ	5·037711
z	32	19	50	∴ vers. $\theta=$	109072
z_1	55	39	46	vers. $(z\sim z_1)=$	81776
$z\sim z_1$	23	19	56	∴ vers. D=	190848
				and ∴ D=	35° 59′ 12″

FIFTH METHOD OF CLEARING THE DISTANCE.

By rejecting odd seconds.

The log. sines, &c., in most collections of Nautical Tables are given only to the nearest minute, or quarter of a minute; it will therefore be necessary in all the previous methods (excepting the first, p. 86) to proportion for the odd seconds of the altitudes and distance. Some part of this tedious operation may be avoided by adopting the following arrangement.

Reject the seconds in the apparent altitudes, but add them to the respective true zenith distances; reject also the seconds in the apparent distance.* Compute the true distance D as before. To the value of D thus found add the seconds rejected from the apparent distance d; the result will then be the correct true distance required.

EXAMPLE OF CLEARING THE DISTANCE BY FIFTH METHOD.

179. Given the app. dist. $d=35° 47' 24''$, sun's app. alt. $a_1=34° 21' 32''$ moon's app. alt. $a=57° 11' 25''$, sun's true zenith distance $z_1=55° 39' 46''$, moon's true zenith dist. $z=32° 19' 50''$; to find the true distance.

* If tables of sines, &c., computed to minutes only are used, this operation must be modified by sometimes *adding* a number of seconds to the apparent distance, so as to make $\frac{1}{2}(d+\overline{a\sim a_1})$ always to consist of degrees and minutes only; the seconds thus added must be subtracted from the computed distance to get the true distance.

$$\text{vers. } \theta = \text{B . sin. } z \text{ . sec. } a \text{ . } \sqrt{\text{hav. } (d+\overline{a \sim a_1}) \text{ . hav. . } (d-\overline{a \sim a_1})}$$
$$\text{vers. D} = \text{vers. } (z \sim z_1) + \text{vers. } \theta.$$

a...............57° 11′ (25″)	$\therefore z = 32°\ 19'\ 50'' + 25'' = 32°\ 20'\ 15''$	
a_1...............34 21 (32)	$z_1 = 55\ \ 39\ \ 46\ + 32\ = 55\ \ 40\ \ 18$	
$a - a_1$..........22 50	$z \sim z_1 = 23\ \ 20\ \ \ 3$	
d...............35 47 (24)	½ log. hav. $(d + \overline{a - a_1})$	...4·689761
$d + a - a_1$......58 37	½ log. hav. $(d - a - a_1)$	...4·052192
$d - (a - a_1)$ ...12 57	„ sin. z	...............9·728277
	„ sec. a	...............0·266039
	„ B	....................6·301138
	„ vers. θ	.............5·037407
	$\therefore$ vers. θ	=108995
	vers. $(z \sim z_1)$	= 81790
	vers. D	=190785
	or D	=35° 58′ 51″+24″
	$\therefore$ true distance	=35 59 15

By this method it appears that we only require to proportion for one quantity, namely sin. z, the moon's true zenith distance.

SIXTH METHOD OF CLEARING THE DISTANCE.

Requiring only the common tables of log. sines, &c.

In triangle ZMS, p. 84,

$$\text{cos. Z} = \frac{\text{cos. D} - \text{cos. } z \text{ . cos. } z_1}{\text{sin. } z \text{ . sin. } z_1},$$

$$\therefore \text{cos. D} = \text{cos. } z \text{ . cos. } z_1 + \text{sin. } z \text{ . sin. } z_1 \text{ . cos. Z}$$
$$= \text{cos. } z \text{ . cos. } z_1 + \text{sin. } z \text{ . sin. } z_1 \text{ . } (2 \text{ cos}^2 . \frac{\text{Z}}{2} - 1)$$
$$= \text{cos. } z \text{ . cos. } z_1 - \text{sin. } z \text{ . sin. } z_1 + 2 \text{ sin. } z \text{ . sin. } z_1 \text{ . cos}^2 . \frac{\text{Z}}{2}$$
$$= \text{cos. } (z+z_1) + 2 \text{ sin. } z \text{ . sin. } z_1 \text{ . cos}^2 . \frac{\text{Z}}{2}.$$

But $\text{cos. D} = 2 \text{ cos}^2 . \frac{\text{D}}{2} - 1$, and $\text{cos. } (z+z_1) = 2 \text{ cos}^2 . \frac{z+z_1}{2} - 1$.

Making these substitutions, we have

$$\text{cos}^2 . \frac{\text{D}}{2} = \text{cos}^2 . \frac{z+z_1}{2} + \text{sin. } z \text{ . sin. } z_1 \text{ . cos}^2 . \frac{\text{Z}}{2}$$
$$= \text{cos}^2 . \frac{z+z_1}{2} \left\{ 1 + \frac{\text{sin. } z \text{ . sin. } z_1 \text{ . cos}^2 . \frac{\text{Z}}{2}}{\text{cos}^2 . \frac{z+z_1}{2}} \right\}$$
$$= \text{cos}^2 . \frac{z+z_1}{2} \{ 1 + \tan^2 . \theta \} = \text{cos}^2 . \frac{z+z_1}{2} \text{ . sec}^2 . \theta \text{ . . (1)};$$

by assuming $\tan.^2\theta = \dfrac{\sin.\ z\ .\ \sin.\ z_1\ .\ \cos^2.\ \frac{Z}{2}}{\cos^2.\frac{z+z_1}{2}}$

Now to find the value of $\cos^2.\frac{Z}{2}$ in terms of the sides of the triangle ZM′S′, p. 84, we have (see *Trigonometry*, Part II. p. 58, formula P)

$$\cos^2.\frac{Z}{2} = \sec.\ a\ .\ \sec.\ a_1\ .\ \cos.\ \tfrac{1}{2}(a+a_1+d)\ .\ \cos.\ \tfrac{1}{2}(a+a_1-d).$$

Substituting this in the value of $\tan.^2\theta$,

$$\therefore \tan.\ \theta = \frac{\sqrt{\sin. z . \sec. a . \sin. z_1 . \sec. a_1 . \cos. \frac{1}{2}(a+a_1+d) \cos. \frac{1}{2}(a+a_1-d)}}{\cos.\ \frac{1}{2}(z+z_1)} \quad ..(2)$$

But $\cos.\ \frac{1}{2}(z+z_1) = \sin.\ \frac{1}{2}(A+A_1)$, if A and A_1 represent the true altitudes of the heavenly bodies.

Making these substitutions, we have

$$\tan.\ \theta = \frac{\sqrt{\cos. A . \sec. a . \cos. A_1 . \sec. a_1 . \cos. \frac{1}{2}(a+a_1+d) . \cos. \frac{1}{2}(a+a_1-d)}}{\sin.\ \frac{1}{2}(A+A_1)} \quad ...(3)$$

$= \dfrac{M}{\sin.\ \frac{1}{2}(A+A_1)}$, if we assume

$$M = \sqrt{\cos.\ A\ .\ \sec.\ a\ .\ \cos.\ A_1\ .\ \sec.\ a_1\ .\ \cos.\ \tfrac{1}{2}(a+a_1+d)\ .\ \cos.\ \tfrac{1}{2}(a+a_1-d)}$$

To simplify these formulæ:

$$\text{From (1), } \cos.\frac{D}{2} = \frac{\cos.\ \frac{1}{2}(z+z_1)}{\cos.\ \theta} = \frac{\sin.\ \frac{1}{2}(A+A_1)}{\cos.\ \theta};$$

and since

$$\tan.\ \theta = \frac{M}{\sin.\ \frac{1}{2}(A+A_1)} = \frac{\sin.\ \theta}{\cos.\ \theta},\ \therefore \frac{M}{\sin.\ \theta} = \frac{\sin.\ \frac{1}{2}(A+A_1)}{\cos.\ \theta}\ \therefore \cos.\ \frac{D}{2} = \frac{M}{\sin.\ \theta};$$

whence it appears that the true distance D may be found by means of the following formulæ:

$$M = \sqrt{\cos.\ A\ .\ \sec.\ a\ .\ \cos.\ A_1\ .\ \sec.\ a_1\ .\ \cos.\ (a+a_1+d)\ .\ \cos.\ \tfrac{1}{2}(a+a_1-d)}$$

$$\tan.\ \theta = \frac{M}{\sin.\ \frac{1}{2}(A+A_1)}, \qquad \cos.\ \frac{D}{2} = \frac{M}{\sin.\ \theta}.$$

EXAMPLE OF CLEARING THE DISTANCE BY SIXTH METHOD.

180. Given the sun's app. alt. $a_1 = 18° 22' 13''$, sun's true alt. $A_1 = 18° 19' 30''$, moon's app. alt. $a = 13° 21' 21''$, moon's true alt. $A = 14° 15' 24''$, and apparent distance $d = 61° 19' 49''$; to find the true distance D.

$$M = \sqrt{\cos.\ A_1\ .\ \sec.\ a_1\ .\ \cos.\ A\ .\ \sec.\ a\ .\ \cos.\ \tfrac{1}{2}(a+a_1+d)\ .\ \cos.\ \tfrac{1}{2}\ (a+a_1-d)}$$

$$\tan.\ \theta = \frac{M}{\sin.\ \frac{1}{2}(A+A_1)}, \qquad \cos.\ \frac{D}{2} = \frac{M}{\sin.\ \theta}.$$

a	13°	21′	21″	$\frac{1}{2}(a+a_1+d)$	46°	31′	41·5″
a_1	18	22	13	$\frac{1}{2}(a+a_1-d)$	14	48	7·5
$a+a_1$	31	43	34	A	14	15	24
d	61	19	44	A_1	18	19	30
$a+a_1+d$	93	3	23	$\text{A}+\text{A}_1$	32	34	54
$a+a_1-d$	29	36	15	$\therefore \frac{1}{2}(\text{A}+\text{A}_1)$	16	17	27

log. cos. A_1	9·977398		
„ sec. a_1	0·022716		
	0·000114		
„ cos. A	9·986414		
„ sec. a	10·011908		
„ cos. $\frac{1}{2}(a+a_1+d)$	9·837587		
„ cos. $\frac{1}{2}(a+a_1-d)$	9·985343		
	39·821366		
	19·910683	....	19·910683
„ sin. $\frac{1}{2}(\text{A}+\text{A}_1)$	9·447953	log. sin. θ	9·975639
„ tan. θ	10·462730	„ cos. $\frac{\text{D}}{2}$	9·935044
	$\therefore \theta=70°\ 59'\ 17''$	$\therefore \frac{\text{D}}{2}$	30° 33′ 42″
			2
		$\therefore$ true dist. =	61 7 24

The preceding method also involves the necessity of proportioning for seconds; the labour, however, may be diminished by neglecting the seconds in the apparent altitudes and distance, and by applying them to the true altitudes and calculated distance, as pointed out in the fifth method. Moreover, since the apparent altitude of the sun or star is nearly equal to the true altitude, therefore cos. A_1 . sec. a_1 (two of the factors in the formula for computing M) may be calculated beforehand and formed into a small table: this has accordingly been done (see Table C). Adopting these simplifications, the preceding example may be worked out as follows:

EXAMPLE OF CLEARING THE DISTANCE BY SIXTH METHOD, SUPPRESSING SECONDS AND USING AN AUXILIARY TABLE.

181. Given sun's app. alt. $a_1=18°\ 22'\ 13''$, sun's true alt. $a_1=18°\ 19'\ 30''$, moon's app. alt. $a=13°\ 21'\ 21''$, moon's true alt. $\text{A}=14°\ 15'\ 24''$, and apparent distance $d=61°\ 19'\ 49''$; to find the true distance D.

$$\text{M}=\sqrt{\cos.\ \text{A}\ .\ \sec.\ a\ .\ \cos.\ \tfrac{1}{2}(a+a_1+d)\ .\ \cos.\ \tfrac{1}{2}(a+a_1-d)\ .\ \text{C}}$$

$$\tan.\ \theta=\frac{\text{M}}{\sin.\ \frac{1}{2}(\text{A}+\text{A}_1)},\qquad \cos.\ \frac{\text{D}}{2}=\frac{\text{M}}{\sin.\ \theta}.$$

a	13°	21′	(21″)		$\text{A}-21''=$	14°	15′	3″
a_1............	18	22	(13)		$\text{A}_1-19\ =$	18	19	17
	31	43				32	34	20
d	61	19	(49)		$\frac{1}{2}\text{A}(\text{A}+\text{A}_1)$...	16	17	10
Sum.........	93	2		....	$\frac{1}{2}$ sum	46	31	
Diff..........	29	36		....	$\frac{1}{2}$ diff.	14	48	

log. cos. A...............	9.986426		
„ sec. a...............	10·011897		
„ cos. $\frac{1}{2}$ sum.........	9·837679		
„ cos. $\frac{1}{2}$ diff..........	9·985347	log. M............	19·910731
„ C	114	„ sin. θ	9·975657
	39·821463	„ cos. $\frac{1}{2}$ D ...	9·935074
„ M	19·910731	∴ $\frac{1}{2}\text{D}=$	30° 33′ 18″
„ sin. $\frac{1}{2}(\text{A}+\text{A}_1)$......	9·447831	∴ D=	61 6 36 + 49″
„ tan. θ...............	10·462900	or true dist.=	61 7 25

TABLE C.

FOR THE *SUN.* ARGUMENT. SUN'S APP. ALT.		FOR A *STAR* OR *PLANET.* ARGUMENT. STAR'S APP. ALT.	
App. alt.	Log. C.	App. alt.	Log. C.
90	0·000104	90	0·000123
75	0·000105	30	0·000122
65	0·000106	20	0·000121
60	0·000107	15	0·000120
55	0·000108	12	0·000119
50	0·000109	10	0·000118
45	0·000110	9	0·000117
40	0·000111	8	0·000116
35	0·000112	7	0·000114
30	0·000113	6	0·000111
25	0·000114	5	0·005107
20	0·000115	4	0·000100
10	0·000114	3	0·000090
8	0·000113		
7	0·000112		
6	0·000109		
4	0·000100		

Construction of Table C.

In the expression,

$$\text{M}=\sqrt{\cos.\ \text{A}_1\ \sec.\ a_1\ .\ \cos.\ \text{A}\ .\ \sec.\ a\ .\ \cos.\ \tfrac{1}{2}(a+a_1+\text{A})\ .\ \cos.\ \tfrac{1}{2}(a+a_1-d)},$$

A_1, the true altitude of the sun or star, is nearly equal to a_1, its apparent alt.; the quantity $\cos.\ \text{A}_1\ .\ \sec.\ a_1=\frac{\cos.\ \text{A}}{\cos.\ a_1}=1$ nearly for all altitudes: it therefore may be computed and formed into a table.

Let $\text{C}=\cos.\ \text{A}_1\ .\ \sec.\ a_1$, $\therefore$ log. C=log. cos. A_1 + log. sec. a_1-20.

182. Find by calculation the value of log. C, when the sun's true altitude $\text{A}_1=18°\ 19'\ 30''$, and app. alt. $a_1=18°\ 22'\ 13''$.

$$\begin{array}{ll} \text{log. cos. A}_1\ldots & 9\cdot977398 \\ \text{,, sec. } a_1\ldots & 10\cdot022716 \\ \hline \therefore \text{log. C}= & 0\cdot000114 \text{ for alt. } 18°; \end{array}$$

It is also evident that Table C may be formed from Table B, by subtracting log. 2=·301030.

SEVENTH METHOD OF CLEARING THE DISTANCE.

By the Common Rules of Spherical Trigonometry.

The true distance MS may be computed as follows (fig. p. 84):

1. In triangle M_1ZS_1 the three sides are given, namely, the two apparent distances ZM_1 and ZS_1 and the observed distance M_1S_1 to find the angle Z.

2. In triangle MZS are given the two sides MZ and SZ, the true zenith distances of the heavenly bodies, and the included angle Z just found, to compute the third side MS, *the true distance required.*

The practical inconvenience of this method arises from the necessity of taking out the log. sines, &c., to the nearest second, a work of considerable labour with the common tables of logarithm sines, &c., which seldom give the arcs nearer than 15″. To obviate this the true distance is now usually found in terms of the versines, the arcs in the table of versines being given to the nearest second.

Other rules have been proposed from time to time for clearing the distance. The methods investigated above have all been derived from the same fundamental formulæ, and from them may be deduced the rules given in most works on Navigation. There are, however, several approximate methods which might be examined with advantage by the student, not so much on account of their practical value, as for the ingenuity they exhibit in the attempt to shorten the labour of calculation; but these and other matters connected with lunar observations must be left for future discussion in the Third Part. (See Admiral Shadwell's Nautical Works.)

THE VARIATION OF THE COMPASS.

INVESTIGATION OF RULES FOR FINDING THE VARIATION OF THE COMPASS.

The true bearing of a heavenly body can be calculated by either of the three following methods.

First. By computing the true bearing of a heavenly body at its rising or setting, called an *amplitude.*

Second. By observing its altitude, and thence with its declination and the latitude of the observer determining the angle at the zenith or true bearing of the body: this angle, or the corresponding arc of the horizon which it subtends, is called the *azimuth* of the heavenly body.

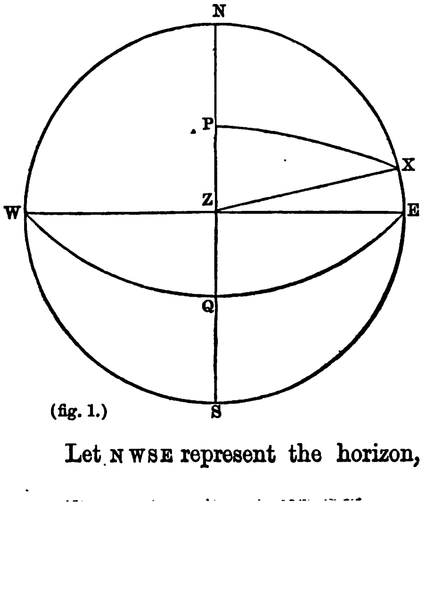
Third. By noting the time by means of a chronometer, and thence determining the hour-angle (note, p. 52); then, knowing the hour-angle and the declination and latitude, the angle at the zenith or azimuth may be computed as before.

If the compass bearing is observed at the time of the observation, the *variation of the compass*, that is, the difference between the true and compass bearing, may be determined.

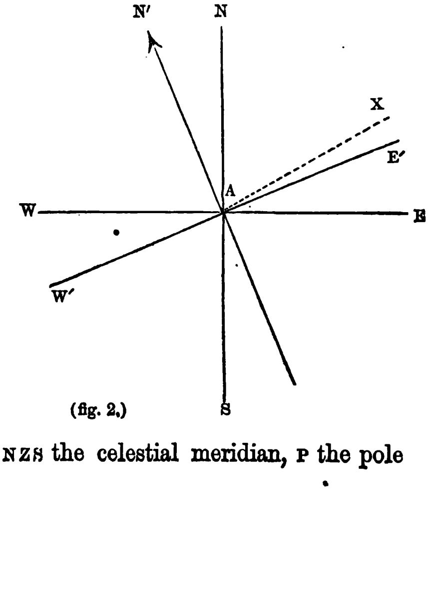
THE AMPLITUDE.

PROBLEM XXVIII.

Given the latitude of the ship, and the declination of a heavenly body; to find the amplitude of the heavenly body.



(fig. 1.)



(fig. 2.)

Let N W S E represent the horizon, N Z S the celestial meridian, P the pole

Z the zenith, and X a heavenly body at rising; then the arc EX, or angle EZX, is its amplitude.

In the quadrantal triangle PZX, are given PZ the colatitude, PX the pol. dist. or co-decl., and $ZX=90°$; to calculate the angle PZX, and thence its complement EZX, or the *amplitude*.

By Rule XIV. *Trig.* Part I., for quadrantal triangles,

$$\text{(fig. 1) } \cos. PX = \sin. PZ \,.\, \cos. PZX,$$

$$\text{or } \sin. \text{decl.} = \cos. \text{lat.} \,.\, \sin. \text{amplitude};$$

$$\therefore \sin. \text{ampl.} = \frac{\sin. \text{decl.}}{\cos. \text{lat.}} = \sin. \text{decl.} \,.\, \sec. \text{lat.}$$

EXAMPLE.

183. Given the lat. of ship$=47° 50'$ N., and the decl. of sun at rising$=17° 54' 44''$ N.; to find the true bearing or amplitude. If the compass bearing at the same time was E. $7° 10'$ N., find also the variation of the compass.

(1.) To find the true bearing (fig. 1).

Construction. Let X be the place of the sun at rising, and complete the figure; then, in the quadrantal triangle ZPX, are given the colat. $PZ=42° 10'$, the pol. dist. $PX=72° 5' 16''$, and the zen. dist. $ZX=90°$; hence the angle PZX, and therefore its complement EZX, may be found by Trigonometry, Rule XIV., or otherwise by the above formula.

$$\sin. \text{ampl.} = \sin. \text{decl.} \,.\, \sec. \text{lat.}$$

$$\begin{array}{ll} \log. \sin. \text{decl.} \ldots\ldots & 9{\cdot}487936 \\ \text{,, } \sec. \text{lat.} \ldots\ldots & \underline{0{\cdot}173090} \\ \therefore \text{,, } \sin. \text{ampl.} \ldots & 9{\cdot}661026 \end{array}$$

$$\therefore \text{true bearing} = \text{E. } 27° 16' 15'' \text{ N.} = EX.$$

(2.) To find the variation of the compass (fig. 2).

Let NAS represent the true meridian; draw WAE at right angles to NS, then E and W are the true east and west points. Make the angle $EAX=27° 16' 15''$=the true bearing of sun; then X will represent the place of the sun with respect to the true east point E. To represent the position of the sun with respect to the magnetic east point, take $XAE'=7° 10'$, and draw E'AW'; then E' and W' will represent the east and west points of the compass, and X will E. $7° 10'$ N. by compass. Now draw AN' at right angles to E'W'; then the line AN' will indicate the position of the magnetic meridian with respect to the true meridian AN, which is in this case to the west of AN, the true meridian, and therefore the variation of the compass is said to be westerly: hence

$$\begin{array}{lllll} \text{True bearing } \ldots\ldots\text{E.} & 27° & 16' & 15'' & \text{N.} \\ \text{Compass bearing} \ldots\text{E.} & \underline{7} & \underline{10} & \underline{0} & \text{N.} \end{array}$$

$\therefore$ variation$=20 \quad 6 \quad 15$, which by the figure is evidently westerly;

$\therefore$ var. of compass$=20 \quad 6 \quad 15$ W.

EXAMPLES FOR PRACTICE.

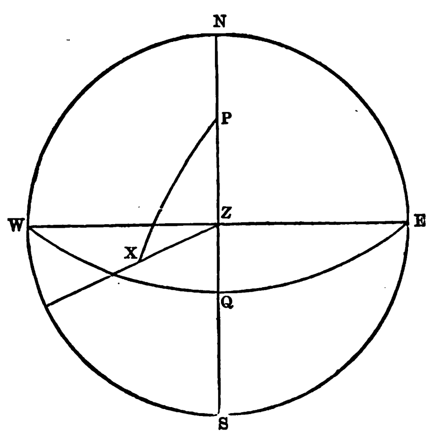
184. Given the lat. $=56° 40'$ N., and sun's decl. at setting $=10° 58' 8''$ N.; required the true bearing. If the compass bearing at the same time was observed to be W. $5° 50'$ S., required also the variation of the compass. Construct figures to show the true bearing and compass bearing.

Ans. Variation $=26° 5' 15''$ E.

185. Given the lat. $=49° 12'$ N., and the sun's decl. at setting $=16° 44' 16''$ S.; required the true bearing. If the compass bearing at the same time was observed to be W. $10° 42'$ S.; required also the variation of the compass. Construct figures to show the true bearing and the compass bearing. *Ans.* Variation $=15° 27'$ W.

186. Given the lat. $=50° 48'$ N., and the sun's decl. at rising $=16° 25' 9''$ N.; required the true bearing. If the compass bearing at the same time was observed to be E. $2° 10'$ S., required also the variation of the compass. Construct figures to show the true bearing and compass bearing.

Ans. Variation $=28° 44'$ W.



THE ALTITUDE AZIMUTH.

PROBLEM XXIX.

Given the altitude and declination of a heavenly body, and latitude of the place; to find the azimuth.

Let NSWE represent the horizon, NZS the celestial meridian, P the pole, and Z the zenith. Then, if X be the place of the heavenly body when its altitude was observed, we have given, in the spherical triangle XZP, the three sides, to find an angle; namely the zenith distance ZX, the polar distance PX, and the colatitude PZ, to find the angle PZX = the azimuth or true bearing of X.

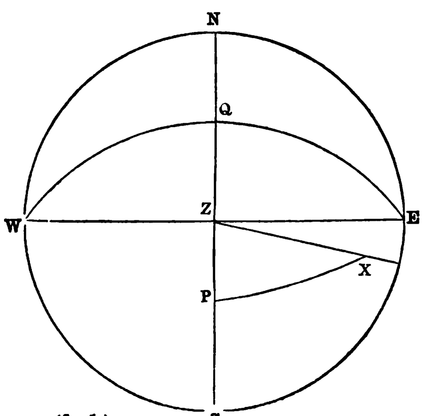
187. Given the lat. $=51° 10'$ S., the true alt. of sun $=5° 25' 37''$, and its decl. $=16° 41' 42''$ S.; required the true bearing, the sun being east of meridian. If the compass bearing was observed at the same time to be N. $87° 50'$ E., required also the variation of the compass.

(1.) To find the true bearing (fig. 1, p. 109).

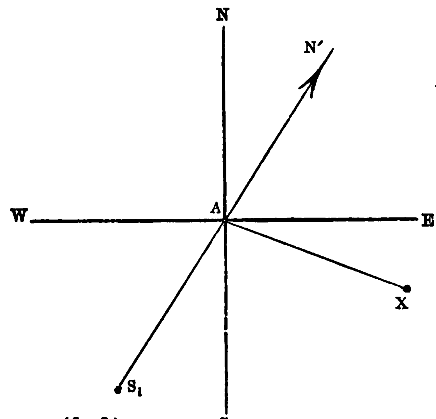
Let NWSE represent the horizon, Z the zenith, and P the south pole; and let X be the place of the sun. Then, in the spherical triangle ZPX, are given ZX the zenith dist. $=84° 34' 33''$, PX the pol. dist. $=73° 18' 18''$, and PZ the colat. $=38° 50'$; to find the angle PZX, the true bearing of the sun.

ZX.........	84°	34′	33″..................	0·001950			
PZ.........	38	50	0	0·202693			
	45	44	23	4·935422			
PX.........	73	18	18	4·377030			
	119	2	41	9·517095			
	27	33	55	∴ PZX=	69°	59′	30″
				or true bearing=S.	69	59	30 E.

(2.) To find the variation of the compass (fig. 2).



(fig. 1.)



(fig. 2.)

Let NAS represent the true meridian, and make the angle SAX=69° 59′ 30″, the true bearing; then X will represent the position of the sun with respect to the true meridian. But by the observation with the compass, the compass bearing=N. 87° 50′ E., or S. 92° 10′ E. Make, therefore, the angle XAS_1=92° 10′; then the angle S_1AX will represent the compass bearing, and the line S_1AN' will be the position of the magnetic meridian; the variation is manifestly easterly: thus

True bearing=S.	69°	59′	30″	E.
Compass bearing=S.	92	10	0	E.
∴ variation of compass=	22	10	30	E.

EXAMPLES FOR PRACTICE.

188. Given the lat.=50° 15′., the altitude of sun=7° 17′ 51″ (east of meridian), and its decl.=22° 21′ 48″ S.; required the true bearing. If the compass bearing was observed at the same time to be S. 61° 15′ E., required also the variation of the compass. Construct the figures to show the true bearing and variation. *Ans.* Variation=20° 11′ E.

189. Given the lat.=50° 30′ N., the altitude of sun=8° 28′ 14″ (west of meridian), and its decl.=23° 9′ 26″ N.; required the true bearing. If the compass bearing was observed at the same time to be N. 89° 40′ W.,

required also the variation of the compass. Construct the figures to show the true bearing and variation. *Ans.* Variation$=26° 3' 15''$ E.

190. Given the latitude$=52°$ N., the altitude of sun$=12° 35' 38''$ (east of meridian), and its decl.$=12° 52'$ N.; required the true bearing. If the compass bearing was observed at the same time to be E. $36° 30'$ S., find also the variation of the compass. Construct figures to show the true bearing and variation. *Ans.* Variation$=41° 21' 30''$ W.

THE TIME AZIMUTH.

PROBLEM XXX.

Given the hour-angle and declination of sun, and the latitude; to find the azimuth.

(1.) To find true bearing of sun.

Let NWSE (fig. 1) represent the horizon, NZS the celestial meridian, P the pole, Z the zenith, and X the place of the sun when the time by chronometer was noted; then, if the error of the chronometer on mean time at the place is known, the hour-angle P may be computed; we have then, in the triangle PZX, two sides and the included angle given to find an angle, namely, the co-declination PX, the colat. PZ, and the included angle P, to find the angle PZX, the true bearing or azimuth (*Trig.*, Part I., Rule XI.).

(2.) To find the variation of the compass.

If the compass bearing of the sun was also observed when the time was noted, the variation of the compass may then be determined as before.

191. Given the hour-angle of the sun$=1^h 12^m 34^s$ (west of meridian), the decl.$=13° 44' 21''$ N., and the latitude of the ship$=50° 48'$ N.; to find the true bearing. If the compass bearing at the time of observation for finding the hour-angle was S. $51° 55'$ W., required also the variation of the compass.

(1.) To find true bearing PZX (fig. 1, p. 111).

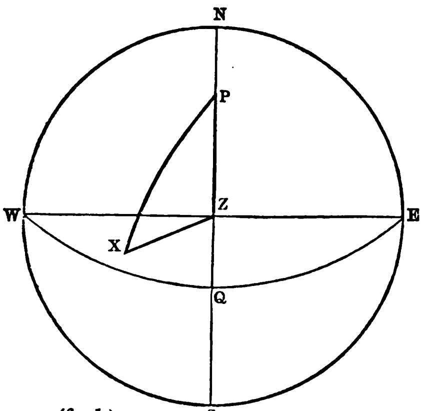
Let X be the place of the sun. Then, in the spherical triangle PZX, are given PX$=76° 15' 39''$, PZ$=39° 12'$, and the angle P$=1^h 12^m 34^s$; to calculate the angle PZX, the true bearing or azimuth.

Calculation (Rule XI. *Trig.* Part I.).

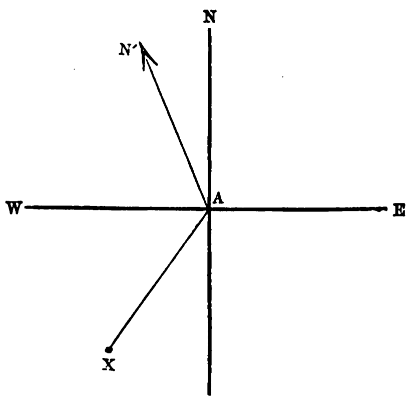
$39° 12' 0''$	1.		2.	
76 15 39	10·796826		10·796826	
115 27 39	9·502137		9·976882	
37 3 39	10·072869		10·272522	
57 43 49	10·371832		11·046230	
18 31 49	$66° 59' 15''$		$84° 51' 45'$	
P$=1^h 12^m 34^s$			66 59 15	
$\frac{1}{2}$P$=0$ 36 17	∴ true bearing$=$N.		151 51 0	W.

(2.) To find the variation of the compass.

Let NAS (fig. 2) represent the true meridian, and make the angle NAX= 151° 51′, the true bearing; then X will represent the place of the sun when



(fig. 1.)



(fig. 2.)

the compass bearing was taken. But the compass bearing was observed to be S. 51° 55′ W., or N. 128° 5′ W., the north point of the needle must therefore be 128° 5′ from X. Make XAN′=128° 5′; then N′A will represent the position of the needle, which is evidently to the west of NS, the true meridian; therefore the variation is westerly: thus

True bearing=N.	151°	51′	W.
Compass bearing=N.	128	5	W.
∴ variation of compass=	23	46	W.

EXAMPLES FOR PRACTICE.

192. Given the hour-angle of the sun=1^h 33^m 33^s (east of meridian), the decl.=23° 12′ 22″ S., and the latitude of ship=52° 10′ N.; to find the true bearing. If the compass bearing at the time of observation was N. 170° 20′ E., required also the variation of the compass. Construct figures to show the true bearing and variation of the compass.

Ans. Variation=12° 13′ W.

193. Given the hour-angle of the sun=2^h 4^m 56^s (west of meridian), the decl.=23° 12′ 55″ S., and the latitude of ship=48° 50′ N.; to find the true bearing. If the compass bearing at the time of observation was S. 51° 40′ W., required also the variation of the compass. Construct figures to show the true bearing and variation of the compass.

Ans. Variation=22° 25′ 15″ W.

194. Given the hour-angle of the sun=0^h 36^m 21^s (east of meridian), the decl.=23° 15′ 18″ S., and the latitude of ship=39° 40′ N.; to find the

true bearing. If the compass bearing at the time of observation was N. 167° 50′ E., required also the variation of the compass. Construct figures to show the true bearing and variation of the compass.

Ans. Variation=2° 50′ 15″ E.

TO FIND THE TRUE BEARING OF A TERRESTRIAL OBJECT.

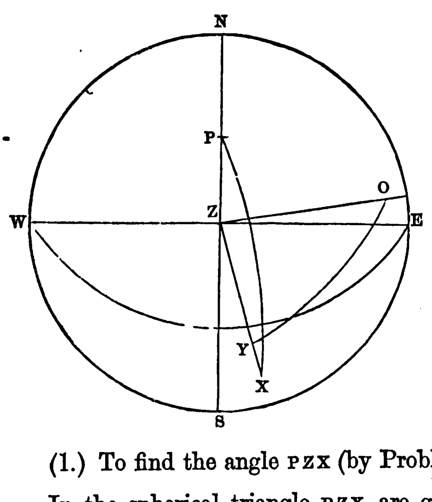
The bearing of a terrestrial object from a given station may be found by observing its angular distance (that is, the angular distance of its place projected on the celestial concave) from the sun or any other heavenly body whose true bearing from the meridian is given or can be calculated.

If the object is in the horizon of the observer, the apparent zenith distance of its place is evidently 90°; if elevated above the horizon, as the summit of a mountain, its altitude must be observed, and thus the zenith distance determined.

PROBLEM XXXI.

Given the angular distance of a terrestrial object from the sun, and the true bearing of the sun; to find the true bearing of the object.

Let O be the place of the terrestrial object (that is, its projection on the celestial concave), X the true place of the sun's center; and let NS represent the celestial meridian, P the pole, and Z the zenith of the station. Through O and X draw the circles of altitude OZ, XZ, and through X the circle of declination PX. Then the true bearing, PZO, of the object O is found by computing (1) the true bearing of the heavenly body X, namely the angle PZX, by Problems XXIX. or XXX.; and (2) the bearing of the object O from X, namely the angle OZX: the difference (in this case) will be the bearing of O, namely the angle PZO.



(1.) To find the angle PZX (by Problem XXIX. or altitude azimuth).

In the spherical triangle PZX, are given PZ the colatitude, PX the polar distance, and ZX the true zenith distance of the sun; to calculate the angle PZX, the true bearing of sun.

To find the angle PZX (by Problem XXX. or time azimuth).

In the spherical triangle PZX, are given PZ the colatitude, PX the polar

distance, and hour-angle ZPX (determined from ship mean time and the equation of time); to calculate the true zenith distance ZX and the azimuth PZX.

(2.) To find the angle OZY.

Let Y be the apparent place of the sun's center; then OY is the observed angular distance of the object O from the sun's center (=distance from sun's nearest limb + sun's semi.). In spherical triangle OZY, are given OZ=observed or apparent zenith distance of point O, ZY the apparent zenith distance of sun's center (found by correcting the true zenith distance for parallax and refraction), and OY the observed angular distance of object from sun; to calculate the angle OZY.

Then the true bearing or PZO=PZX−OZX.

EXAMPLE (BY ALTITUDE AZIMUTH).

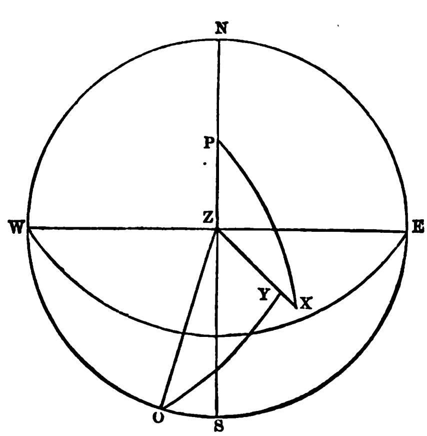
195. May 2, 1842, at $10^h\ 8^m$ A.M. mean time nearly, in lat. 50° 48′ N. and long. 1° W., observed the angular distance of the sun's nearest limb from an object O in the horizon to the right of it and W. of meridian to be 75° 17′ 20″. The apparent altitude of the sun's center at the same time was 48° 17′ 16″, and its true altitude was 48° 16′ 30″. Construct a figure, and find by calculation the true bearing of the object.

Elements from *Nautical Almanac* for Greenwich date, May 1, $22^h\ 12^m$:

(1.) Sun's decl. 15° 18′ 58″ N.; (2.) sun's semi. 15′ 53″.

Construction of figure.

Let NWSE represent the horizon, P the pole, Z the zenith, X the true place of the sun, and Y its apparent place. Let O be the projection of object on the horizon. Draw the circles of altitude ZO and ZX and circle of decl. PX, and join OY. Then the true bearing of O is the angle OZS to be computed.



(1.) To find the angle XZS, the supplement of PZX.

In the spherical triangle PZX, are given PZ = 39° 12′, ZX = 41° 43′ 30″, and PX=74° 41′ 2″; to calculate the angle PZX=138° 20′ 0″: hence XZS=41° 40′ 0″.

(2,) To find the angle OZY.

In the spherical triangle OZY, are given OZ=90°; ZY=41° 42′ 44″, and OY=75° 33′ 13″; to calculate the angle OZX=67° 59′ 0″.
Then OZX−XZS=S. 26° 19′ 0″ W., the bearing of O.

EXAMPLE (BY TIME AZIMUTH).

196. 20th Oct., at $10^h\ 42^m\ 29^s$ A.M. ship mean time, in lat 50° 48′ N. and long. by account 1° 6′ W., observed the angular distance of the sun's nearest limb from an object O to the left of it (see fig. p. 113) to be 78° 31′ 45″; the altitude of the object was 4° 28′. Construct a figure, and find by calculation the true bearing of the object.

Elements from *Nautical Almanac* for Greenwich date, Oct. 19, $22^h\ 47^m$.

(1.) Sun's decl. 10° 17′ 5″ S.; (2.) Sun's semi. 16′ 5″; (3.) Equation of time, $15^m\ 4{\cdot}5^s$ (additive to mean time); (4.) Correction in altitude, 1′ 44″.

Construction.

Let NWSE represent the horizon, P the pole, Z the zenith, X the true place of the sun, and Y its apparent place. Let O be the projected place of object at an altitude=40° 28′. Draw the circles of altitude ZO and ZX and circle of declination PX, and join OY. Then the true bearing of O is the angle NZO to be computed.

(1.) To find the true zenith distance ZX and azimuth PZX.

In the spherical triangle PZX, are given PZ=39° 12′, PX=100° 17′ 5″, and the included angle $ZPX=24^h-(22^h\ 42^m\ 29^s+15^m\ 4{\cdot}5^s)=1^h\ 2^m\ 26{\cdot}5^s$; to calculate the true zenith distance ZX=62° 34′ 31″, and true bearing of sun, namely PZX=162° 39′ 0″.

(2.) To find the angle OZY.

In the spherical triangle OZY, are given OZ=85° 32′, ZY=62° 34′ 31″ −1′ 44″=62° 32′ 47″, and OY=78° 47′ 50″; to compute the angle OZY= 79° 41′ 15″.
Hence the angle PZO=PZX−OZY=N. 82° 57′ 45″ E., the true bearing of terrestrial object required.

EXAMPLES FOR PRACTICE.

197. August 13, 1858, at $10^h\ 55^m\ 19^s$ A.M. ship mean time, in latitude 50° 48′ N. and longitude 1° 6′ W., observed the angular distance of the sun's farthest limb from an object O in the horizon to the right of sun and west of meridian to be 69° 4′ 45″. Construct a figure, and find by calculation the true bearing of the object.

Elements from the *Nautical Almanac* for Greenwich date, August 12, at $22^h\ 59^m\ 43^s$.

(1.) Sun's decl. 14° 43′ 0″ N.; (2.) Sun's semi. 15′ 50″; (3.) Equation of time, $4^m\ 39^s$ (subtractive from mean time); (4.) Correction in altitude, 0′ 41″. *Ans.* S. 27° 14′ 15″ W.

198. August 13, 1858, at $1^h\ 9^m\ 21^s$ P.M. ship mean time in lat. 50° 48′ N. and long. 1° 6′ W., observed the angular distance of the sun's nearest limb from an object o in the horizon to the left of sun and east of meridian to be 125° 31′ 32″. Construct a figure, and find by calculation the true bearing of the object.

Elements from *Nautical Almanac* for Greenwich date, August 13, at $1^h\ 13^m\ 45^s$:

(1.) Sun's decl. 14° 41′ 18″ N.; (2.) Sun's semi. 15′ 50″; (3.) Equation of time, $4^m\ 38^s$ (subtractive from mean time); (4.) Correction in altitude, 0′ 41″. *Ans.* N. 45° 22′ E.

The true bearing of a terrestrial object (as the spire of a church, seen from some given station, as a ship at anchor) being determined by the above problem, the variation of the compass on board is easily found by simply observing the compass bearing of the object; the difference between the bearings will evidently be the variation of the compass for that position of the ship. Moreover, if it is suspected that the iron on board has some important influence on the compass, this may in some measure be discovered by taking successive observations of the compass bearing of the terrestrial object while the ship is swinging, or whenever she happens to lie with her head in different positions with respect to the magnetic meridian. This method of ascertaining what is called the *deviation of the compass* may often be of use when more accurate methods of forming a table of deviations cannot be adopted.

To obtain the true bearing of the terrestrial object with the greatest accuracy, the heavenly body observed should be at a considerable distance from the meridian; and the angular distance of the terrestrial object from the heavenly body should also be large (not less than 80° or 90°). If these limitations are made, the effects of errors of observation will be considerably diminished; but these and other directions connected with this subject belong rather to marine surveying, where this problem is of great utility.

CHAPTER V.

CORRECTIONS.

Magnitude and figure of the earth.

In all the common rules and problems of Nautical Astronomy investigated in the preceding pages, the form of the earth has been considered to be that of a sphere. On this supposition the meridians would be great circles, and the *length* of a degree of latitude in every part of the earth would be equal. But observations and actual measurements of arcs of a meridian, made in different parts of the world, have made it apparent that the lengths of a degree of the meridian are not invariable, but that they *increase from the equator to the poles*, suggesting to us the figure of an oblate spheroid.

The following table contains the results of such measurements :

	Country.	Latitude of middle point of arc measured.	Length of 1° in feet.
1	Sweden	66° 20′ 10″	365782
2	Russia	58 17 37	365368
3	England	52 35 45	364971
4	France	46 52 2	364872
5	France	44 51 2	364535
6	Rome	42 59 0	364262
7	United States	39 12 0	363786
8	India	16 8 22	363044
9	India	12 32 21	363013
10	Peru	1 31 0	362808

The observations recorded in the above table prove that the curvature of the earth must diminish from the equator to the pole : this is sufficient to show that the earth is not a sphere, and that, in fact, it must approach in form to that of an oblate spheroid.

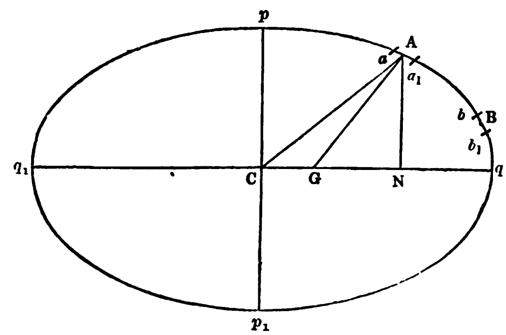
Assuming it to be such a figure, we may compute the lengths of the equatorial and polar diameters from the measured lengths of a degree of the meridian in two places differing considerably in latitude.

As this knowledge of the dimensions of the earth is necessary in several important problems in Nautical Astronomy, we will here investigate an

expression for calculating the major and minor diameters qq_1 and pp_1, and also the distance CA of any point A from the center of the earth.

PROBLEM XXXII.

To calculate the lengths of the equatorial and polar diameters of the earth.



Let pqp_1q_1 represent a section of the earth passing through the poles pp_1; A and B the middle points of two degrees aa_1, bb_1, the lengths of which are supposed to be known from the preceding table.

Draw AG, a normal at A; then the angle AGq is called the *latitude of A* (p. 9).

Let l and l_1 represent the latitudes of A and B;

a and b the semi-major and semi-minor axes;

n the normal AG at A;

D and D_1 the lengths of aa_1 and bb_1 the degrees in latitudes A and B, known from observation.

Then circle of curvature at $A=360\times D$ nearly$=2\pi r$;

$$\therefore \text{ radius of curvature } r=\frac{360\times D}{2\pi}=57{\cdot}29577\times D.$$

Similarly, radius of curvature r_1 at $B=57{\cdot}29577\times D_1$.

The difference of the equatorial and polar radii is small: call this difference c; then an approximate value of c may be found (by neglecting terms involving $\frac{c^2}{a^2}$, &c., which are small), and thence a and b. This may be done by expressing the normal n in terms of a and c and the latitude l; and then, by the differential calculus, n in terms of r. A similar expression being found for r_1, one of the unknown terms a and c may be eliminated between the two expressions, and the other determined as in the following investigation.

Let the coördinates CN and AN of the point A be denoted by x and y; then

$$\text{(by Conic Sections), } y^2=\frac{b^2}{a^2}(a^2-x^2), \text{ and } GN=\frac{b^2x}{a^2}.$$

In triangle AGN, $y=n \sin. l$, and $GN=n \cos. l$;

$$\therefore \frac{b^2}{a^2}(a^2-x^2)=n^2 \sin.^2 l, \text{ and } \frac{b^2x}{a^2}=n \cos. l; \therefore x^2=\frac{n^2a^4}{b^4}\cos.^2 l.$$

Substituting this value of x^2 in former equation,

$$\frac{b^2}{a^2}(a^2-\frac{n^2a^4}{b^4}\cos.^2 l)=n^2 \sin.^2 l, \therefore n^2=\frac{b^4}{a^2\,(1-\frac{a^2-b^2}{a^2}\sin.^2 l)}$$

$$\therefore n=\frac{b^2}{a}\,(1-\frac{a^2-b^2}{a^2}\sin.^2 l)^{-\frac{1}{2}}. \quad \text{But } c=a-b, \therefore b=a-c;$$

$$\therefore n=\frac{b^2}{a}\,(1-\frac{2ac-c^2}{a^2}\sin.^2 l)^{-\frac{1}{2}}$$

$$=\frac{b^2}{a}\,(1-\frac{2c}{a}\sin.^2 l)^{-\frac{1}{2}} \text{ nearly, since } \frac{c^2}{a^2} \text{ is small}$$

$$=\frac{b^2}{a}\,(1+\frac{c}{a}\sin.^2 l) \text{ neglecting terms involving higher powers of } \frac{c}{a}$$

$$\text{Again, } r=\frac{\{1+\left(\frac{dy}{dx}\right)^2\}^{\frac{3}{2}}}{-\frac{d^2y}{dx^2}}=\frac{\{b^2x^2+a^2(a^2-x^2)\}^{\frac{3}{2}}}{a^4b}$$

and $n^2=GN^2+AN^2$

$$=\frac{b^4x^2}{a^4}+\frac{b^2}{a^2}(a^2-x^2)=\frac{b^2}{a^4}\{b^2x^2+a^2(a^2-x^2)\}$$

$$\therefore n^3=\frac{b^3}{a^6}\{b^2x^2+a^2(a^2-x^2)\}^{\frac{3}{2}} \quad \therefore \frac{n^3a^6}{b^3}=\{b^2x^2+a^2(a^2-x^2)\}^{\frac{3}{2}}$$

Substituting this value of the numerator in r, we have

$$r=\frac{n^3a^2}{b^4} \quad \therefore r=\frac{a^2}{b^4}\cdot\frac{b^6}{a^3}(1+\frac{c}{a}\sin.^2 l)^3$$

$$=\frac{b^2}{a}(1+\frac{3c}{a}\sin.^2 l) \text{ nearly}=\frac{(a-c)^2}{a}\,(1+\frac{3c}{a}\sin.^2 l) \text{ nearly}$$

$$=\left(a-2c+\frac{c^2}{a}\right)\cdot(1+\frac{3c}{a}\sin.^2 l) \text{ nearly}$$

$$r=(a-2c)\,.(1+\frac{3c}{a}\sin.^2 l) \text{ nearly}=a+3c\sin.^2 l-2c \text{ nearly.}$$

Similarly $r_1=a+3c\sin.^2 l_1-2c \quad \therefore r-r_1=3c\,(\sin.^2 l-\sin.^2 l_1)$

$$\therefore c=\frac{r-r_1}{3\,(\sin.^2 l-\sin.^2 l_1)}=\frac{57\cdot29577\,(D-D_1)}{3\,(\sin.^2 l-\sin.^2 l_1)}$$

$$\text{or } c \text{ (in miles)}=\frac{57\cdot29577\,(D-D_1)}{3\times1760\times3}\operatorname{cosec.}(l+l_1)\operatorname{cosec.}(l-l_1).$$

Hence this practical rule to compute c, the difference between the semi-major and semi-minor axes of the earth.

Add together the constant log. $\bar{3}{\cdot}558367$ (the log. of $\frac{57{\cdot}29577}{3\times 1760\times 3}$), the log. of the difference of the lengths (in feet) of a degree in the two latitudes, and the log. cosecants of the sum and difference of the two latitudes: the natural number of the sum (rejecting 20 in the index) will be the value of c (the difference between the semi-major and semi-minor axes) in miles.

199. Find the value of c from the measurements (2 and 8) of a degree in Russia and India (see table, p. 116).

$$\begin{array}{rrrr} l= & 58^\circ & 17' & 37'' \\ l_1= & 16 & 8 & 22 \\ \hline l+l_1= & 74 & 25 & 59 \\ l-l_1= & 42 & 9 & 15 \end{array} \qquad \begin{array}{ll} \text{const. log.}\ldots\ldots\ldots\ldots & \bar{3}{\cdot}558367 \\ \text{log. } (\text{D}-\text{D}_1)\ \ldots\ldots\ldots & 3{\cdot}366236 \\ \text{,, cosec. } (l+l_1)\ldots & 0{\cdot}016230 \\ \text{,, cosec. } (l-l_1)\ldots & 0{\cdot}173195 \\ \hline \text{log. } c= & 1{\cdot}114028 \end{array}$$

$$\begin{array}{rr} \text{D}= & 365368 \\ \text{D}_1= & 363044 \\ \hline \text{D}-\text{D}_1= & 2324 \end{array} \qquad \therefore\ c=13{\cdot}00 \text{ miles.}$$

With the measurements in England and India (3 and 8), the value of

$$\begin{array}{lr} & c=12{\cdot}58 \\ \text{Measurements 1 and 10} & c=12{\cdot}83 \\ \text{,, } \quad 3 \text{ ,, } \quad 9 & c=12{\cdot}13 \\ \text{,, } \quad 2 \text{ ,, } \quad 9 & c=12{\cdot}59 \\ \hline \text{mean value of} & c=12{\cdot}64 \text{ nearly.} \end{array}$$

The value of c being found, we may compute a by the formula

$$r=a-2c+3c\sin.^2 l,\ \therefore\ a=r+2c-3c\sin.^2 l$$
$$=57{\cdot}29577\,\text{D}+(2-3\sin.^2 l)\cdot\frac{57{\cdot}29577\,(\text{D}-\text{D}_1)}{3\,(\sin.^2 l-\sin.^2 l_1)}$$
$$=\frac{57{\cdot}29577}{3}\cdot\frac{2(\text{D}-\text{D}_1)+3\,(\text{D}_1\sin.^2 l-\text{D}\sin.^2 l_1)}{\sin.^2 l-\sin.^2 l_1}$$
$$=\frac{57{\cdot}29577}{9\times 1760}\left(2\,\overline{\text{D}-\text{D}_1}+3\,\overline{\text{D}_1\sin.^2 l-\text{D}\sin.^2 l_1}\ \text{cosec.}\,\overline{l+l_1}\ \text{cosec.}\,\overline{l-l_1}\right)$$

Hence this practical rule to compute the semi-major axis of the earth.

To the log. of the length of a degree (D_1) in feet, in one latitude, add twice the log. sine of the other latitude. Again, to the log. of the length of a degree (D) in one latitude, add twice the log. sine of the other latitude (rejecting 20 from the index in each case). Take the difference between the natural numbers of the resulting logarithms, and multiply by 3. Add thereto twice the difference between the lengths of a degree in each latitude, and take out the logarithm of the result. To this logarithm add const. log. $\bar{3}{\cdot}558367$, and the log. cosecants of the sum and difference of the two lati-

tudes. The sum (rejecting the tens in index) is the log. of the semi-major axis of the earth; which find in the tables.

200. Find the value of a (the semi-major diameter of the earth) from the measurements (3 and 9) of a degree in England and India (see table, p. 116).

$$\begin{array}{rrrr} l & =52^\circ & 35' & 45'' \\ l_1 & =12 & 32 & 21 \\ \hline l+l_1 & =65 & 8 & 6 \\ l-l_1 & =40 & 3 & 24 \end{array} \qquad \begin{array}{rr} \text{D} & =364971 \\ \text{D}_1 & =363013 \\ \hline & 1958 \\ & 2 \\ \hline 2(\text{D}-\text{D}_1) & =3916 \end{array}$$

$$\begin{array}{lr} \text{log. D}_1 \ldots\ldots & 5\cdot5599222 \\ \text{,, sin. } l \ldots & 9\cdot9000231 \\ \text{,, sin. } l \ldots & 9\cdot9000231 \\ \hline & 5\cdot3599684 \\ & 229070 \\ & 17203 \\ \hline & 211867 \\ & 3 \\ \hline & 635601 \\ 2(\text{D}-\text{D}_1) \ldots & 3916 \\ \hline & 639517 \end{array} \qquad \begin{array}{lr} \text{log. D} \ldots\ldots\ldots & 5\cdot5622584 \\ \text{,, sin. } l_1 \ldots & 9\cdot3366737 \\ \text{,, sin. } l_1 \ldots & 9\cdot3366737 \\ \hline & 4\cdot2356058 \\ & 17203 \\ & \\ \text{const. log.} \ldots\ldots\ldots & \bar{3}\cdot5583670 \\ \text{log. cosec. } (l+l_1) \ldots & 0\cdot0422486 \\ \text{,, cosec. } (l-l_1) \ldots & 0\cdot1914211 \\ \text{,, } 639517 \ldots\ldots\ldots & 5\cdot8058521 \\ \hline \text{log. } a \ldots & \bar{3}\cdot5978888 \end{array}$$

$\therefore$ semi-major diameter $a=3961\cdot7$ miles.

201. Find the value of a, the semi-major diameter, from the measurements (2 and 8), p. 116. *Ans.* $a=3961\cdot8$ miles.

202. Find the value of a, the semi-major diameter, from the measurements (2 and 9), p. 116. *Ans.* $a=3962\cdot6$ miles.

From the last three results we find the mean value of semi-major diameter $a=3962$ miles, and $c=12\cdot64$ miles; $\therefore$ b the semi-minor diameter$=3962-12\cdot64=3949$ miles nearly, and $\frac{c}{a}$ (called the compression)$=\frac{12\cdot64}{3962}=\frac{1}{313\cdot4}$ nearly.

The distance CA of any point A on the surface of the earth from the center C will be useful hereafter in finding the correction of the moon's equatorial horizontal parallax. It may be computed by the following problem.

PROBLEM XXXIII.

To calculate the distance of a place on the surface of the earth from the center.

Let A (fig. p. 117) be the place, then its distance AO from the center may be investigated as follows :

$$\text{Since } CA^2=CN^2+AN^2 \text{ and subnor. } GN=\frac{b^2}{a^2}CN$$

$$\therefore CN=\frac{a^2}{b^2}\cdot GN=\frac{a^2}{b^2}n\cos. l=\frac{a^2}{b^2}\cdot\frac{b^2}{a}(1+\frac{c}{a}\sin.^2 l)\cos. l\ldots\ldots(\text{p. } 118)$$

$$=a\cos. l\,(1+\frac{c}{a}\sin.^2 l) \text{ and } AN=n\sin. l=\frac{b^2}{a}(1+\frac{c}{a}\sin.^2 l)\sin. l$$

$$=\frac{(a-c)^2}{a}(1+\frac{c^2}{a}\sin.^2 l)\sin. l$$

$$=\left(a-2c+\frac{c^2}{a}\right).(1+\frac{c}{a}\sin.^2 l)\sin. l$$

$$=(a-2c).(1+\frac{c}{a}\sin.^2 l)\sin. l \text{ nearly.}$$

Substituting these values of CN and AN,

$$CA^2=\{a^2\cos.^2 l+(a-2c)^2\sin.^2 l\}.(1+\frac{c}{a}\sin.^2 l)^2$$

$$=(a^2\cos.^2 l+a^2\sin.^2 l-4ac\sin.^2 l+4c^2\sin.^2 l).(1+\frac{2c}{a}\sin.^2 l) \text{ nearly}$$

$$=a^2+2ac\sin.^2 l-4ac\sin.^2 l \text{ nearly (neglecting other terms, which are small)}$$

$$=a^2+2ac\sin.^2 l$$

$$\therefore CA=a\,(1-\frac{2c}{a}\sin.^2 l)^{\frac{1}{2}}=a\,(1-\frac{c}{a}\sin.^2 l) \text{ nearly}$$

$$=a-c\sin.^2 l=3962-12{\cdot}64\sin.^2 l.$$

Hence this practical rule to find the distance of any place from the center of the earth :

To the constant log. 1·101747 (the log. of 12·64) add twice the log. sine of the latitude of the place : the natural number of the result (rejecting the tens in index), subtracted from the semi-major diameter, 3962, is the distance required.

203. Required the distance of a place in latitude 50° 48′ from the center of the earth.

const. log.	1·101747
log. sin. lat.	9·889271
„	9·889271
log. cor.	0·880289
∴ cor.	7·6 miles.

Hence distance from center=3962−7·6=3954·4 miles.

204. Required the distance of a place in latitude 30° from the center of the earth. *Ans.* 3958·84 miles.

From these investigations it is manifest that the earth, although not an exact sphere, is very nearly so; accordingly, in the common rules of Navigation, the earth is considered as a sphere, and a meridian a *great* circle (and not an ellipse); and the *latitude* of a place an arc of the meridian intercepted between the place and the equator. The correct definition of the latitude, namely taking into consideration the true figure of the earth, will be considered in the next problem.

PROBLEM XXXIV.

Given the true latitude of a place, to find the reduced or central latitude.

Let true lat. AG$q=l$, and reduced lat. AC$q=l_1$ (fig. p. 117), then $l-l_1$=the correction required.

$$\tan. l=\frac{\text{AN}}{\text{GN}}\ \tan. l_1=\frac{\text{AN}}{\text{CN}}$$

$$\therefore \frac{\tan. l}{\tan. l_1}=\frac{\text{CN}}{\text{GN}}=\frac{a^2}{b^2}\ (\text{since GN}=\frac{b^2}{a^2}\text{ CN, p. 117})$$

$$\therefore \tan. l_1=\frac{b^2}{a^2}\tan. l=\frac{(a-c)^2}{a^2}\tan. l=\left(1-\frac{2c}{a}\right)\tan. l \text{ nearly.}$$

$$\text{Now } \tan. (l-l_1)=\frac{\tan. l-\tan. l_1}{1+\tan. l\ .\ \tan. l_1}$$

$$=\frac{\tan. l\ .\ \left(1-1+\frac{2c}{a}\right)}{1+\tan.^2 l\left(1-\frac{2c}{a}\right)}$$

$$=\frac{2c}{a}\cdot\frac{\tan. l}{1+\tan.^2 l}\ (\text{since } 1-\frac{2c}{a}=1 \text{ nearly})$$

$$=\frac{c}{a}\sin. 2l=\frac{1}{313{\cdot}4}\sin. 2l$$

Since $l-l_1$ is only a few minutes, the circular measure $\left(\frac{\text{arc }(l-l_1)}{\text{rad.}}\right)$ may be substituted for $\tan. (l-l_1)$;

$$\therefore \text{arc }(l-l_1)\text{ in min.}=\frac{57^\circ{\cdot}29577\times 60}{313{\cdot}4}\ .\ \sin. 2l=11 \sin. 2l \text{ nearly.}$$

Hence this practical rule for finding the correction of the latitude for the spheroidal figure of the earth, and thence the reduced latitude:

To the constant log. 1·041393 (the log. of 11) add log. sine of twice the latitude (rejecting 10 in the index): the natural number of the result will

be the correction of latitude, which being subtracted from the given latitude, the remainder is the reduced or central latitude required.

205. Given the true latitude of a place A=50° 46′ N.; to find the reduced latitude.

$$\text{Reduced lat.}=l-\text{cor.}=l-11\ \sin.\ 2l.$$

true lat. l= 50° 46′	log. 11	1·041393
∴ $2l$=101 32	„ sin. $2l$	9·991141
	„ cor.	1·032534
	∴ cor. ......=10·8′=	10′ 48″
	true lat. l=	50 46 0 N.
	∴ reduced lat.=	50 35 12 N.

206. Given the true latitude=62° 10′ N.; calculate the reduction of the latitude for the spheroidal figure of the earth, and thence find the reduced latitude. *Ans.* Red. lat.=62° 0′ 55·2″ N.

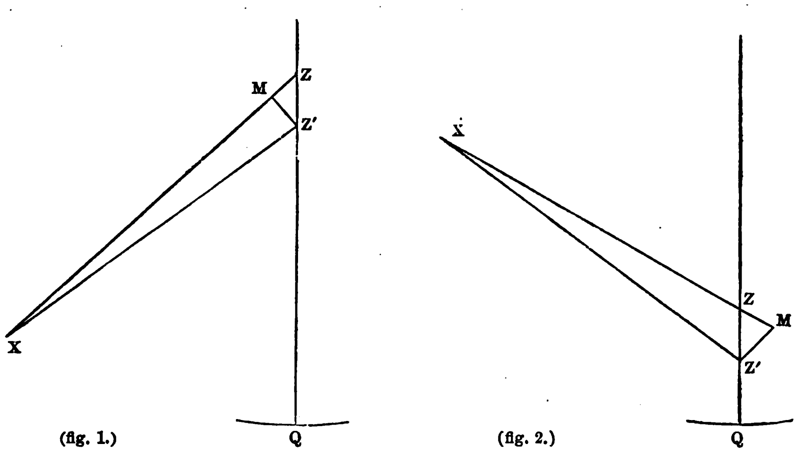
Construction of Table h *in Inman's Nautical Tables.*

By means of the above formula, table *h*, for finding the reduced latitude, may be computed.

PROBLEM XXXV.

Given the true zenith distance of a heavenly body, and its azimuth or bearing; to calculate the reduced zenith distance.

Let Z be the true zenith, Z′ the reduced zenith, and X a heavenly body whose azimuth is XZQ. Then ZX is the true zenith distance, and Z′X is the



(fig. 1.) (fig. 2.)

reduced zenith distance. With center X and distance XZ′ describe the arc Z′M: then the triangle ZZ′M, being very small, may be considered as a right-angled

plane triangle, and ZM is the difference between the true and reduced zenith distances, or $Z'X = ZX \pm ZM$; the sign to be used being easily determined by a diagram, remembering that Z' is always nearer the equator than Z.

To calculate the correction ZM we have, in the right-angled triangle ZMZ', $ZM = ZZ' \cos. MZZ' = 11 \sin. 2l \cos.$ azimuth; since ZZ'=reduction of latitude (Prob. XXXIV.), and the angle MZZ' is the azimuth or bearing of the body.

When the bearing is nothing, or the heavenly body is on the meridian, it is evident that $ZM = ZZ'$ the reduction of the latitude.

207. Calculate the reduced zenith distance of a heavenly body when the true zenith distance (corrected for refraction) is 35° 34′; the latitude of the place being 50° 46′ N., and the azimuth of the body S. 10° W.

$$l = 50° \ 46' \qquad ZM = ZZ' \cos. Z$$
$$\therefore 2l = 101 \ \ 32 \qquad = 11 \sin. 2l \cos. \text{az.}$$

Calculation.

log. 11 ....1·041393
„ sin. $2l$....................9·991141
„ cos. azimuth ....9·993351
„ ZM ....1·025885

$\therefore ZM = 10{\cdot}6' = \quad 10' \ 36''$
and $ZX = 35 \quad 34 \quad 0$

$\therefore$ reduced zenith distance$=35 \quad 23 \quad 24$

208. Calculate the reduced zenith distance of a heavenly body when the true zenith distance (corrected for refraction) is 35° 34′; the latitude of the place being 50° 48′ N., and the bearing or azimuth N. 10° W.

By means of the figure we see that ZM, the correction (found as in last example), must be added to the true zenith distance to get the reduced zenith distance.

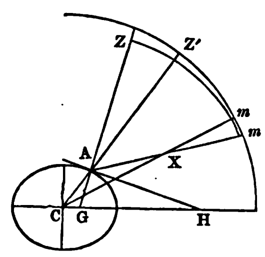
$$\therefore \text{reduced zenith distance} = 35° \ 34' \ \ 0'' + 10' \ 36''$$
$$= 35 \ \ 44 \ \ 36$$

209. Calculate the reduced zenith distance of a heavenly body when its true zenith distance (corrected for refraction) is 22° 30′; the latitude of the place being 40° 48′ S., and the bearing or azimuth S. 50° E. *Ans.* 22′ 37′.

Parallax.

The place of a heavenly body as seen, or supposed to be seen, from the center of the earth, is called its *true* or geocentric place; the place of a

heavenly body as seen from any point on the surface is called its apparent place. Thus, let A be any point on the surface of the earth, X a heavenly body, as the moon. Through X draw the straight lines A X m', C X m, from the surface and center to the celestial concave. Then $m m'$ being produced, will pass through the reduced zenith Z$'$. The arc $m'm$ or angle A X C is called the *diurnal* parallax; and if H be the same heavenly body in the horizon of the spectator at A, then the angle A H C is called the *horizontal parallax* of the heavenly body. The circle m'Z$'$ coincides very nearly with m'Z, a circle through the true zenith Z, since the figure of the earth differs very little from that of a sphere; and therefore m'Z$'$ is very nearly a vertical circle.



PROBLEM XXXVI.

Given the apparent reduced zenith distance and horizontal parallax; to calculate the diurnal parallax, and thence the true reduced zenith distance.

Let A be the spectator on the surface of the earth, Z$'$ his reduced zenith, X a heavenly body. Through X draw A X, C X, and produce the lines to m' and m. Then the arc Z$'m'$ is the distance of the body from the reduced zenith Z$'$, as seen from the surface of the earth, or it is the apparent reduced zenith distance (corrected for refraction); and Z$'m$ is the reduced zenith distance as seen from the center, or it is the true reduced zenith distance; and the arc mm' is the diurnal parallax.

To compute mm'.

Let $AC=r$ the radius of the earth, $CX=CH=d$ the distance of body,

$Z'Am'=Z'$ the apparent reduced zenith distance,

$AXC=p=mm'$ the diurnal parallax, $AHC=H$ the hor. par.

In triangle A X C we have $\dfrac{r}{d}=\dfrac{\sin. AXC}{\sin. CAX}=\dfrac{\sin. AXC}{\sin. Z'Am'}$

$$=\frac{\sin.\text{ diurnal parallax}}{\sin.\text{ app. red. zen. dist.}}=\frac{\sin. p}{\sin. Z'}\ldots\ldots\ldots\ldots\ldots(a)$$

Again, in A H C, considered as a right-angled triangle,

$$\frac{r}{d}=\sin. H=\sin.\text{ horizontal parallax}\ldots\ldots\ldots\ldots\ldots(\beta)$$

$\therefore$ equating (a) and (β) $\dfrac{\sin. p}{\sin. Z'}=\sin. H$; or $\sin. p=\sin. H \sin. Z'$.

But since the parallax is small even for the moon, the nearest of the heavenly bodies, we may substitute the circular measure of the angles $\left(\dfrac{\text{arc}}{\text{rad.}}\right)$ for their sines;

$$\therefore \frac{p}{\text{rad.}}=\frac{H}{\text{rad.}}\ .\ \sin. Z',\ \text{or}\ p=H\ .\ \sin. Z';$$

that is, diurnal par.$=$hor. par. $\times$ sin. apparent reduced zenith distance (corrected for refraction).

210. Given the apparent reduced zenith dist. of the moon$=77° \ 45' \ 36''$, and the horizontal parallax$=59' \ 33{\cdot}2''$. Calculate the diurnal parallax, and thence find the true reduced zenith distance.

$$p = \text{H sin. z}'$$

H$=59' \ 33{\cdot}2''$	log. H..........3·553057
60	„ sin. z'......9·990014
$3573{\cdot}2''$	„ p...........3·543071
	$\therefore p=3492'=$ 58′ 12″
	app. red. zen. dist.$=$77 45 36
	$\therefore$ true red. zen. dist$=$76 47 24

211. Calculate the diurnal parallax of the moon when the apparent reduced zenith distance is $45° \ 30'$, and the horizontal parallax $55' \ 42{\cdot}5''$.

Ans. $39' \ 44''$.

In the preceding problem the diurnal parallax p is determined when the *apparent* reduced zenith distance is given. If the true reduced zenith distance (as z$'m$ in the figure) is only given, to find the diurnal parallax, the above formula requires to be modified. This may be done as follows.

PROBLEM XXXVII.

Given the *true* reduced zenith distance and horizontal parallax; to calculate the diurnal parallax.

Let the true reduced zen. dist. z$'cm=$z (see last fig.)

$$\text{Then } z'=z+p\text{; and since sin. } p=\text{sin. H . sin. z}',$$

$$\therefore \text{sin. } p=\text{sin. H . sin. } (z+p)$$

$$=\text{sin. H } \{\text{sin. z . cos. } p+\text{cos. z . sin. } p\}$$

$$\text{or tan. } p=\text{sin. H } \{\text{sin. z}+\text{cos. z . tan. } p\}$$

$$\therefore \text{tan. } p\,(1-\text{sin. H . cos. z})=\text{sin. H . sin. z}$$

$$\therefore \text{tan. } p=\text{sin. H . sin. z}\frac{1}{1-\text{sin. H . cos. z}}$$

$$=\text{sin. H . sin. z } \{1+\text{sin. H . cos. z}+\ .\ .\ .\ .\ .\}$$

Substituting $\frac{\text{arc}}{\text{rad.}}$ for tan. p, sin. H, &c., we have

$$\frac{p}{\text{rad.}}=\frac{\text{H}}{\text{rad.}}\text{ . sin. z } \{1+\frac{\text{H}}{\text{rad.}}\text{ . cos. z}+\ .\ .\ .\ .\ .\ .\ .\ .\ \}$$

$$\therefore p=\text{H . sin. z}+\frac{\text{H}^2}{\text{rad.}}\text{ . sin. z . cos. z}+\ .\ .\ .\ .\ .\ .\ .\ .$$

$$=\text{H . sin. z}+\frac{\text{H}^2}{57{\cdot}29577\times 60\times 60}\cdot\frac{\text{sin. 2z}}{2}+\ .\ .\ .$$

$\therefore p=\text{H . sin. z}+\dfrac{\text{H}^2\text{ . sin. 2z}}{57{\cdot}29577\times 7200}$ nearly (by neglecting terms involving the higher powers of H).

By means of this formula the diurnal parallax may be computed when Z, the true reduced zenith distance, and the horizontal parallax are given.

212. Given the true reduced zenith distance of the moon=76° 47′ 24″, and the horizontal parallax=59′ 33·2″=3573·2″; to calculate the diurnal parallax.

$$p = \text{H} \cdot \sin. \text{Z} + \frac{\text{H}^2 \cdot \sin. 2\text{Z}}{57{\cdot}29577 \times 7200}$$

log. H	3·553057	3·553057	3·857333
„ sin. Z	9·988353	3·553057	1·758119
	3·541410	9·648322	5·615452
		6·754436	
	Nat. No. 3478″	5·615452	
	14	1·138984	

$\therefore p = 3492 = 58' \, 12''$, as before (see p. 126).

Parallax in altitude.

The reduced zenith Z′ is very near the true zenith Z, since the figure of the earth is very nearly a sphere (Prob. XXXII.); and the plane passing through Z′ and the heavenly body is nearly a vertical plane. In the common problems of Nautical Astronomy, therefore, it is usual to suppose the true zenith Z to coincide with the reduced zenith Z′, and the diurnal parallax to take place in a circle of altitude. The diurnal parallax then becomes the *parallax in altitude.*

The formula (p. 125), namely,

diurnal parallax=hor. par. × sin. app. red. zen. dist. (corr. for refraction),

may therefore be written thus,

parallax in altitude=hor. par. × sin. zenith distance (corrected for refraction),

or,

parallax in altitude=hor. par. × cos. alt. (corrected for refraction).

A table of the correction in altitude (which combines correction for parallax and refraction) for different altitudes has been computed from the last formula, and is used in navigation as sufficiently accurate for correcting the apparent altitude for the effects of parallax and refraction.

The method of computing this quantity may be seen in the following example:

213. Given the apparent altitude of the moon's center=42° 45′ 30″, refraction=1′ 3″, and the horizontal parallax=55′ 45″=3345″; to calculate the true altitude.

Par. in alt. = hor. par. × cos. alt. (corrected for refraction).

obs. alt.	42°	45′	30″	log. cos. alt.	9·865945
refraction		1	3−	„ hor. par.	3·524396
	42	44	27	„ par in alt.	3·390341
		40	56+	∴ par. in alt. = 2456″, or 40′ 56″	
∴ true alt.	43	25	23		

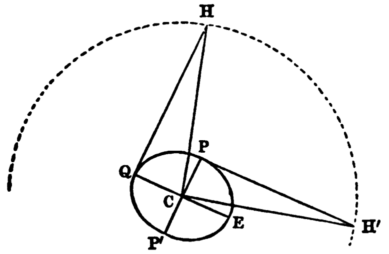
which result agrees with that found in the table: thus, by Inman's table,

		Obs alt.	42°	45′	30″
hor. par. ....55′.........	39′ 20″				
45″......	33				
Cor for refract. and par. ...39	53			39	53
		True alt. =	43	25	23

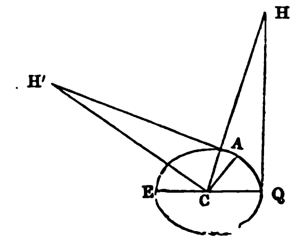
214. Given the apparent altitude of moon's center = 72° 42′ 15″, refraction = 18″, and horizontal parallax = 58′ 49″; required the true altitude (by calculation and by table). *Ans.* By calculation, 72° 59′ 26″. By table, 72° 59′ 26″.

The horizontal parallax $= \frac{\text{rad. of earth}}{\text{dist. of body}}$ (p. 125), therefore the horizontal parallax will vary with the radius of the earth; and as the earth is an oblate spheroid (p. 120), its radius diminishes from the equator to the pole: hence the equatorial horizontal parallax must be the greatest. This may be seen by a figure (fig. 1); for if P be the pole of the earth, and EQ the equator, H a heavenly body in the horizon of a spectator at Q, and H′ the same heavenly body in the horizon of P, then it is evident that the angle H is greater than H′, since CQ is greater than CP; that is, the equatorial horizontal parallax H is the greatest, and the horizontal parallax of a heavenly body H′, as seen from the pole, is the least.

The horizontal parallax of the moon, put down in the *Nautical Almanac*, is the equatorial horizontal parallax; to find the horizontal parallax for any other place, a correction must be applied to that taken from the *Nautical Almanac*, and this correction is evidently subtractive.



(fig. 1.)



(fig. 2.)

PROBLEM XXXVIII.

Given the horizontal parallax of the moon at the equator; to find the horizontal parallax at any other place on the surface of the earth.

Let A (fig. 2) be the given place, QH, AH′, tangents at Q and A. If equa-

torial horizontal parallax=H, horizontal parallax at A=H′, the equatorial semi-diameter CQ=a, and CA=r, CH=CH′=R, and lat of A=l.

Then, in triangle CHQ, $\frac{a}{\text{R}}=\sin.\ \text{H}.$

„ „ CH′A, $\frac{r}{\text{R}}=\sin.\ \text{H}'$ nearly, since the earth is nearly a sphere.

$$\therefore \sin \text{H}' : \sin.\ \text{H} :: \frac{r}{\text{R}} : \frac{a}{\text{R}} :: r : a, \quad \therefore \sin.\ \text{H}'=\frac{r}{a}\ .\ \sin.\ \text{H};$$

and since the angles H and H′ are small, the arcs may be substituted for the sines, $\therefore \text{H}'=\frac{r}{a}\ .\ \text{H}.$

Now the value of CA or r is equal to $a-c\ .\ \sin.^2 l$ (p. 121), whence $\frac{r}{a}=1-\frac{c}{a}\ .\ \sin.^2 l$ (where $\frac{c}{a}=\frac{1}{313\cdot4}$, p. 120);

$$\therefore \text{H}'=(1-\frac{c}{a}\ .\ \sin.^2 l)\ .\ \text{H}=\text{H}-\frac{c}{a}\ .\ \sin.^2 l\ .\ \text{H}.$$

The quantity $\frac{c}{a}\ .\ \sin.^2 l\ .\ \text{H}$ is the correction to be subtracted from H. This correction has been calculated for different latitudes, and forms part of table *h* in Inman's *Nautical Tables*.

215. Calculate the correction for the equatorial horizontal parallax of the moon; the horizontal parallax taken out of the *Nautical Almanac* being 59′ 53·7″, and the latitude of the spectator 50° 48′ N.

$$\text{Cor.}=\frac{c}{a}\ .\ \sin.^2 l\ .\ \text{H},$$

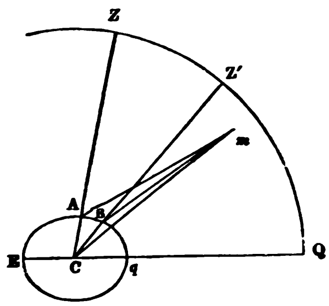
$\frac{c}{a}=\frac{1}{313\cdot4}$	$\therefore$ log. $\frac{c}{a}=\bar{3}\cdot503900$
59′ 53·7″	log. sin. l...9·889271
60	„ sin. l...9·889271
3593·7″=H	„ H.......3·555541
	„ cor......0·837983
	$\therefore$ cor.=6·89″

216. Find the correction for the equatorial parallax of the moon; the horizontal parallax from the *Nautical Almanac* being 54′ 32·5″, and the latitude of spectator 32° 42′ N. *Ans.* 3·048″.

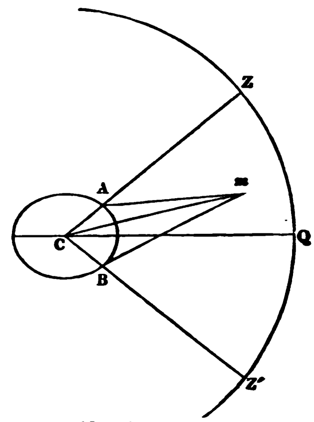
PROBLEM XXXIX.

Given the meridian zenith distances of a heavenly body observed at the same instant at two distant places on the same meridian; to calculate the parallax.

Let A and B be the two places on the same meridian, m a heavenly body on the meridian of A and B.



(fig. 1.)



(fig. 2.)

First. Suppose the two places to be on the same side of the equator, as in fig. 1.

Let ZQ the reduced lat of $A=l$,

Z'Q „ „ $B=l'$.

Z A m the reduced zen. dist. of m at $A=z$,

Z'B m „ „ $B=z'$.

Horizontal parallax at $A=H$, and at $B=H'$.

$AC=r$, $BC=r'$, dist. of body$=d$.

Then diurnal parallax at $A=H \cdot \sin. z = AmC$ (p. 125),

„ „ $B=H' \cdot \sin. z' = BmC$,

and $AmC = ZAm - ACm = z - ACm$,

$BmC = Z'Bm - BCm = z' - BCm$;

$$\therefore H \cdot \sin. z - H' \cdot \sin. z' = z - z' - (ACm - BCm)$$

$$= z - z' - ZCZ' = z - z' - (l - l').$$

$$\text{But hor. parallax} = \frac{\text{rad. of earth}}{\text{dist. of body}} \text{ (p. 125)},$$

$$\therefore H = \frac{r}{d}, \text{ and } H' = \frac{r'}{d} \quad \therefore H' = H \cdot \frac{r'}{d}$$

$$\therefore H \cdot \sin. z - H \cdot \frac{r'}{r} \cdot \sin. z' = z - z' - (l - l');$$

and if we assume $\frac{r'}{r}=1$, which it is very nearly,

$$\text{then } H = \frac{z - z' - (l - l')}{\sin. z - \sin. z'} = \frac{z - z' - (l - l')}{2 \cos. \frac{1}{2}(z + z') \sin. \frac{1}{2}(z - z')}$$

$$= \tfrac{1}{2}\{z - z' - (l - l')\} \cdot \sec. \tfrac{1}{2}(z + z') \text{ cosec. } \tfrac{1}{2}(z - z').$$

Next. Suppose the heavenly body to be on different sides of Z and Z'; and the places of observation on different sides of the equator, as in fig. 2.

Then, as before,

$$\text{H} . \sin. z = \text{A}m\text{C} = z - \text{AC}m$$
$$\text{H}' . \sin. z' = \text{B}m\text{C} = z' - \text{BC}m$$

$$\therefore \text{H} . \sin. z + \text{H}' . \sin. z' = z + z' - (\text{AC}m + \text{BC}m)$$
$$= z + z' - \text{ZCZ}' = z + z' - (l + l').$$

Let H=H, which it is nearly:

$$\therefore \text{H} = \frac{z+z'-(l+l')}{\sin. z + \sin. z'} = \frac{z+z'-(l+l')}{2 \sin. \frac{1}{2}(z+z') . \cos. \frac{1}{2}(z-z')}$$
$$\therefore \text{H} = \tfrac{1}{2}\{z+z'-(l+l')\} . \text{cosec.} \tfrac{1}{2}(z+z') . \sec. \tfrac{1}{2}(z-z').$$

By means of this formula, the horizontal parallax of a heavenly body may be calculated from the meridian zenith distances observed at two given places on the same meridian at the same instant.

When the heavenly body is not on the same meridian at the same time, the above formula must be modified to suit the case, or more exact methods used.

217. Calculate the horizontal parallax of the planet Mars, supposing the following observations to be taken at two places on the same meridian at the same instant.

In lat. A. ......59° 20′ 30″ N., the zenith distance was 68° 14′ 6″
„ B. ......33 55 5 S., „ „ 25 2 0

$$\text{H} = \tfrac{1}{2}(z+z'-\overline{l+l'}) . \text{cosec.} \tfrac{1}{2}(z+z') . \sec. \tfrac{1}{2}(z-z')$$

z	68°	14′	6″	l	59°	20′	30″	N.
z'	25	2	0	l'	33	55	5	S.
$z+z'$	93	16	6	$l+l'$	93	15	35	
$z-z'$	43	12	6	$z+z'$	93	16	6	
$\frac{1}{2}(z+z')$	46	38	3	$\therefore z+z'-(l+l')=$			31	
$\frac{1}{2}(z-z')$	21	36	3	$\frac{1}{2}(z+z'-\overline{l+l'})=$			15·5	

log. cosec. $\frac{1}{2}(z+z')$..................0·138481
„ sec. $\frac{1}{2}(z-z')$....................0·031621
„ „ 15·5.......................1·190332
„ H................................1·360434

$\therefore$ hor. par.=22·93″

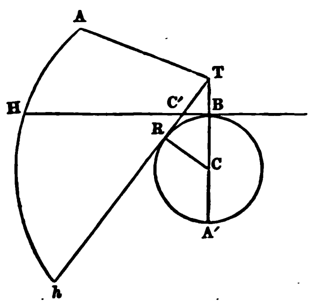
DIP.

The altitude of a heavenly body observed from a place above the surface of the earth, as from the deck of a ship, will evidently be greater than the altitude observed from the surface, since the spectator brings the image of the body down to his horizon, which is lower than the horizon seen from the earth's surface beneath him. The difference of the altitudes from this cause is called the dip.

PROBLEM XL.

Given the height of the eye above the sea, to calculate the dip.

Let a tangent at B, the point directly beneath the spectator at T, meet the celestial concave at H; from T draw the tangent Th, touching the earth at R. Then, if A be the place of a heavenly body, AH is the altitude observed from B, the surface of the earth, and Ah is the altitude from T. The arc Hh is the dip for the height TB of the spectator above the surface of the earth. Through T draw the diameter TA′, and join RC. Then, the triangles TRC, TC′B, being similar, the angles TCR and TC′B, or HC′h, are equal. Let rad. CR$=r$, and TB, the height of spectator in feet$=a$.



$$\text{Then tan. TCR}=\frac{\text{TR}}{\text{RC}}=\frac{\sqrt{\text{TC}^2-\text{RC}^2}}{\text{RC}}=\sqrt{\frac{(a+r)^2-r^2}{r}}=\sqrt{\frac{2a}{r}}\text{ nearly, (since}$$

a^2 is small compared with r), and tan. TCR=tan. HC′h=HC′h nearly, since the angle is small.

$$\text{But HC}'h=\frac{\text{arc}}{\text{rad.}}=\frac{\text{dip}}{57{\cdot}29577^\circ\times 60}\quad\therefore\frac{\text{dip}}{57{\cdot}29577^\circ\times 60}=\sqrt{\frac{2a}{r}}$$

$$\text{or dip}=\frac{57{\cdot}29577^\circ\times 60\times\sqrt{2}}{\sqrt{3960\times 1760\times 3}}\cdot\sqrt{a}=1{\cdot}063\times\sqrt{a}$$

This value of the dip, however, is affected by refraction; the amount of this cannot be very accurately determined on account of the variable nature of refraction at low altitudes. Experiments seem to show that refraction diminishes the amount of dip about $\frac{1}{15}$th of itself; according to Inman its value is about $\frac{1}{13}$th of itself, others make it about $\frac{1}{14}$th. If we take it at $\frac{3}{40}$ths of itself, the results will correspond very nearly with those found in most tables.

$$\text{Hence dip}=\frac{37}{40}\times 1{\cdot}063\times\sqrt{a}={\cdot}984\sqrt{a}.$$

218. Calculate the dip for the height of the eye above the sea = 20 feet.

$$\text{dip}={\cdot}984\sqrt{20}.$$

log. ·984........................$\bar{1}$·992995
,, $\sqrt{20}$........................0·650515

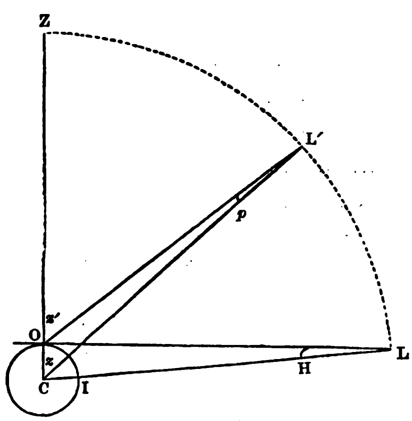
,, dip........................0·643510

$\therefore$ dip=4·4′=4′ 24″

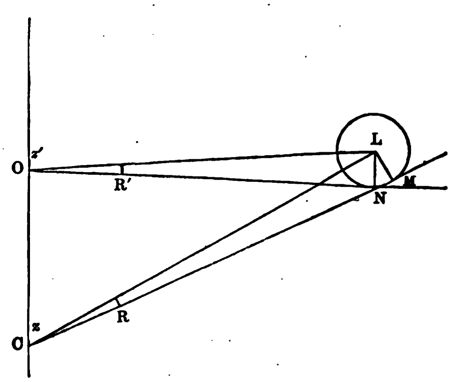
219. Calculate the dip for the height of the eye above the sea=110 feet, and for height=30 feet. *Ans.* 10′ 19·2″, 5′ 23·4″.

AUGMENTATION OF THE MOON'S HORIZONTAL SEMIDIAMETER.

When the moon is above the horizon, as at L′ (fig. 1), its distance OL′ from a spectator at O is less than its distance OL when in the horizon at L. For the distance CL of the earth's center from the moon is about 60 times the earth's radius, therefore $CL = 60 \times CI$. But as the horizontal parallax π is small, OL is nearly equal to CL, and therefore LI is less than LO by nearly the earth's radius. Hence if two observers were placed at O and I, one would see the moon when at L in his horizon, and the other in his zenith; but to the spectator at O the moon would be a little more, and to the spectator at I a little less than 60 times its radius, and the diameter would appear to the former about 30″ less than to the latter. It is evident that at any intermediate altitude, as at L′, the distance OL′ is less than OL, and therefore the moon's diameter at L′ would appear to be greater than the true or horizontal diameter at L; that is, the diameter at L′ would be *augmented*. To find this correction, or, as it is called, augmentation of the moon's horizontal semidiameter, we must proceed as in the following problem.



(fig. 1.)



(fig. 2.)

PROBLEM XLI.

Given the horizontal semidiameter of the moon, to calculate its augmentation for a given altitude (fig. 2).

Let O be the place of the spectator, C the earth's center, and L the moon's, ON and CM lines drawn to touch the moon at N and M. Join OL and CL, and draw the perpendiculars LN LM to the points of contact N and M. The angles LCM LON measure respectively the true and apparent semidiameters of the moon, when its zenith distance is z. Let $LCM = R$, $LON = R'$, then $R' - R$ is the correction to be calculated.

In the right-angled triangles LON and LCM we have

$$\sin. R' = \frac{LN}{LO},\ \sin. R = \frac{LM}{LC},\ \therefore\ \frac{\sin. R'}{\sin. R} = \frac{LC}{LO},\ \text{since } LN = LM.$$

Let z and z' be the true and apparent zenith distances of the moon. Then in triangle LOC $\frac{\sin. z'}{\sin. z} = \frac{LC}{LO} = \frac{\sin. R'}{\sin. R} = \frac{R'}{R}$ (since the angles R' and R are small).

$$\therefore\ \frac{R' - R}{R} = \frac{\sin. z' - \sin. z}{\sin. z} = \frac{2\cos. \frac{1}{2}(z' + z)\sin. \frac{1}{2}(z' - z)}{\sin. z}$$

Let OLC, the parallax in altitude, $= p$, then $p = z' - z$, and $\therefore\ z = z' - p$, substituting the value of z in the above and reducing, we have

$$R' - R = 2R \operatorname{cosec.} (z' - p) \cos. (z' - \tfrac{1}{2}p) \sin. \tfrac{1}{2}p,$$

from which formula $R' - R$, the augmentation, may be computed.

220. Calculate the augmentation of the moon's horizontal semidiameter, when the apparent altitude of the center is 60° 10′, the horizontal parallax being 56′ 1″, and horizontal semidiameter (as given in the *Nautical Almanac*) 15′ 16″.

$$\text{Aug.} = 2R\ .\ \operatorname{cosec.} (z' - p)\ .\ \sin. \tfrac{1}{2}p\ .\ \cos. (z' - \tfrac{1}{2}p)$$

and p = hor. par. × cos. app. alt. (p. 125).

56′ 1″	log. hor. par. ....3·526468
60	„ cos. alt. ....9·696774
3361″ = hor. par.	„ p..................3·223242
	∴ p = 1672″

log. 2 semi. ....3·262925	∴ p = 1672 =		27′	52″
„ sin. $\frac{1}{2}p$ ....7·607780	z' =	29	50	0
„ cos. $(z' - \frac{1}{2}p)$ ...9·939262	$z' - p$ =	29	22	8
„ cosec. $(z' - p)$ ...0·309422	$\frac{1}{2}p$ =		13	56
„ aug...............1·119389	z' =	29	50	0
∴ aug. = 13·16″	$z' - \frac{1}{2}p$ =	29	36	4

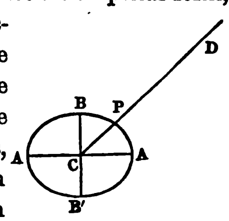
221. Calculate the augmentation of the moon's horizontal semidiameter, when the apparent altitude of the center is 32° 42′, the horizontal parallax being 54′ 42·5″, and horizontal semi. (in *Nautical Almanac*) 14′ 56″.

Ans. 7·86″.

PROBLEM XLII.

Given the altitude of the moon when near the horizon, and the inclination to the horizon of a line joining the moon's center and that of a distant object; to calculate the contraction of semidiameter on account of refraction.

When the moon is near the horizon its disc assumes an elliptical form, as ABB′, in consequence of the unequal effect of refraction at low altitudes. If, therefore, a contact is made between some distant object in the direction D and the moon's limb at the point P, the semidiameter PC, to be added to the distance to get the distance of the centers, must be less than CA, the semidiameter as found in the *Nautical Almanac*. If $CA=a$ and $CP=r$, then $a-r$ is the correction to be computed, or (as it is called) the contraction of the moon's semi. on account of refraction.



To calculate the contraction, let the angle $PCA=\theta$, then by the polar equation to the ellipse we have

$$r=\frac{b}{\sqrt{1-\epsilon^2\cos.^2\theta}}=b\,(1-\epsilon^2\cos.^2\theta)^{-\frac{1}{2}}=b\{1+\tfrac{1}{2}\epsilon^2\cos.^2\theta+\ .\ .\ .\ .\}$$
$$=b\{1+\tfrac{1}{2}\epsilon^2\cos.^2\theta\}$$

omitting all the other terms, since ϵ^2 ϵ^4 &c. are small.

$$=b\{1+\tfrac{1}{2}\epsilon^2-\tfrac{1}{2}\epsilon^2\sin.^2\theta\}\ .\ .\ .\ .\ (1)$$

Now, by conic sections, $\frac{b^2}{a^2}=1-\epsilon^2$, $\therefore\ a^2=\frac{b^2}{1-\epsilon^2}$.

$$\therefore\ a=\frac{b}{\sqrt{1-\epsilon^2}}=b(1-\epsilon^2)^{-\frac{1}{2}}$$
$$=b\{1+\tfrac{1}{2}\epsilon^2+\ .\ .\ .\ .\}=b\{1+\tfrac{1}{2}\epsilon^2\}\text{ nearly.}$$

Substituting this value of $b(1+\frac{1}{2}\epsilon^2)$ in (1) we have

$$r=a-\tfrac{1}{2}b\epsilon^2\sin.^2\theta,\ \therefore\ a-r=\tfrac{1}{2}b\epsilon^2\sin.^2\theta\ .\ .\ .\ .\ (2)$$

We may eliminate $\frac{1}{2}b\epsilon^2$ as follows:

Let $\theta=90°$, then $r=BC$ and $a-BC$=difference of refractions between the points B and C. This may be found in the Table of Refractions. Call this difference c;

$$\text{then } c=\tfrac{1}{2}b\epsilon^2\sin.^2 90=\tfrac{1}{2}b\epsilon^2.$$

Substituting this value of $\frac{1}{2}b\epsilon^2$ in (2) we have

$$a-r\text{ or the correction}=c\sin.^2\theta,$$

where c=difference of refraction for center C and vertex B, and θ=inclination of line joining the centers to the horizon.

222. Calculate the correction or contraction of moon's semidiameter when the altitude is 5° and the inclination of the line joining the centers is 60°, the moon's semidiameter being 16′ 0″.

$$\text{cor.}=c\sin.^2\theta$$

Refraction at	5° 0′...9′ 58″	log. c............1·397940
„	5 16...9 33	„ sin. 60° ...9·937531
	c= 25	„ sin. 60° ...9·937531
		„ cor......... 1·273002 $\therefore$ cor.=18·8″.

223. Calculate the correction or contraction of the moon's semidiameter when the altitude$=4^\circ\ 30'$ and the line joining the centers is inclined at an angle of 40°, the moon's semidiameter being $15'\ 30''$. *Ans.* $11{\cdot}5''$.

Correction of moon's meridian passage.

The time of the transit of a heavenly body can be found by means of Problem VI.; but in the case of the moon, the following approximate method of finding the time of her passage over a given meridian may be sometimes used with advantage.

The mean time of the moon's transit for every day at Greenwich is put down in the *Nautical Almanac.* At any place to the east of Greenwich, the time of the transit, owing to the moon's proper motion to the eastward, must take place sooner (independent of that due to the difference of longitude), and to a place to the westward of Greenwich, later than the time recorded in the *Nautical Almanac.* Thus, if we suppose the moon's daily motion to be 60 minutes: to a place 90° to the east of Greenwich the transit will take place 6 hours earlier than that at Greenwich (on account of the difference of longitude)$+\frac{90}{360}$ of 60^{m}, or 15 minutes, due to the moon's motion, supposed equable, to the eastward in the 6 hours before she reaches the meridian of Greenwich. To a place west of the first meridian, a retardation will take place for the same reason.

The moon's daily motion in RA varies between 40^{m} and 60^{m}, so that it would not be difficult to construct a small table of the correction of the transits given in the *Nautical Almanac* for any given longitude: this has accordingly been done, and may be found in Inman's *Nautical Tables,* p. 5.

The construction of the table may be explained as follows:

Let the moon's proper motion in 24 hours$=d$.

A meridian, as that of Greenwich, describes in that time one complete revolution, or 360°.

Let the longitude of some given place$=\text{D}^\circ$, and the proper motion of the moon for $\text{D}^\circ=x$.

Then $360^\circ : \text{D}^\circ :: d : x$ (supposing the moon's motion equable),

$$\text{or } x=\frac{\text{D}^\circ}{360^\circ}\times d.$$

By assuming different values for D° and d, the correction x may be easily calculated, and a table formed of the results.

Example. Given the daily motion of the moon$=45{\cdot}7^{\text{m}}$; required the correction of the meridian passage in the *Nautical Almanac* for a place in longitude 50° W.

$$\text{D}=50^\circ \qquad x=\frac{\text{D}^\circ}{360^\circ}\times d=\frac{50}{360}\times 45{\cdot}7=6{\cdot}3^{\text{m}}.$$
$$d=45{\cdot}7$$

And in a similar manner may all the other values in the table be calculated.

The moon's daily motion used should be that found by taking the difference between the two transits at Greenwich that happen before and after the one at the place: that is, if the place be in west longitude, the difference should be taken between the transit on the given day and the one following; if in east longitude, that on the given day and the one preceding. By observing this rule, the error arising from the unequal motion of the moon in RA is diminished.

An example or two of finding the time at Greenwich at the transit of the moon over a given meridian will show the use of the table.

224. April 27, required Greenwich mean time nearly at the transit of the moon over the meridian of a place in longitude 50° W.

By *Nautical Almanac*, mer. pass. on 27th...	11^h	$46{\cdot}3^m$
„ „ on 28th...	12	32·0
Moon's motion in 24 hours...		45·7
Correction (from table, or by calculation)...		6·3 +
∴ time of transit at place...	11	52·6
long. in time...	3	20·0 W.
∴ Greenwich mean time...	15	12·6

225. April 27th, required Greenwich mean time nearly at the transit of the moon over the meridian of a place in longitude 50° E.

	By *Nautical Almanac*, mer. pass. on 27th...	11^h	$46{\cdot}3^m$
	„ „ on 26th...	11	2·7
			43·6
Cor. $=\frac{50}{360}\times 43{\cdot}6$	Correction...		6·0 —
$=6{\cdot}06^m$	Transit at place...	11	40·3
or by table $=6{\cdot}0$	long. in time...	3	20·0 E.
	∴ Time at Greenwich...	8	20·3

Required the mean time at the place of the moon's meridian passage on July 19 (astronomical day), in longitude 60° W., and on July 27 (astronomical day), in longitude 175° E., having given the following quantities from the *Nautical Almanac*:

Gr. mer. pass. on July 19......	11^h	$24{\cdot}3^m$	July 27......	17^h	$30{\cdot}1^m$
„ „ 20......	12	19·2	„ 26......	16	49·5

Ans. Mer. pass. at place on July 19 at $11^h\ 33{\cdot}3^m$
„ „ July 27 at 17 11·1 = July 28 at $5^h\ 11{\cdot}1^m$ A.M.

CONSTRUCTION OF THE TABLES USED FOR CORRECTING THE QUANTITIES TAKEN OUT OF THE NAUTICAL ALMANAC FOR ANY GIVEN TIME FROM GREENWICH MEAN NOON.

The declination, right ascension, &c., of the sun and moon are inserted in the *Nautical Almanac* for every day at noon, or $0^h\ 0^m\ 0^s$ Greenwich mean time. To find the same quantities for any other Greenwich time, we may either multiply the hourly differences by the time elapsed since the preceding noon, or use the common rules of proportion, or, what is in some cases the simplest method, find the quantity to be added or subtracted by means of certain tables called Proportional Logarithms, the principal of which are the following:

(1.) The proportional logarithms (properly so called).
(2.) Greenwich date prop. logarithm for the sun.
(3.) Greenwich date prop. log. for the moon.
(4.) The logistic logarithms.

We will give the construction of the above tables, with a few examples to show their application and use.

(1.) *Construction of table of* PROPORTIONAL LOGARITHMS.

Definition. The logarithm of 180^m (or 3 hours), diminished by the logarithm of any other number of hours and minutes (less than 3 hours), is called the proportional logarithm of that number.

Thus prop. log. $2^h\ 42^m$=log. 180−log. 162=·045757.

(2.) *Construction of table of Greenwich date* PROPORTIONAL LOGARITHMS FOR THE SUN.

Definition. The logarithm of 1440^m (=24 hours), diminished by the logarithm of any other number of hours and minutes (less than 24 hours), is called the Greenwich date log. for sun for that number.

Thus Greenwich date log. sun for $17^h\ 42^m$=log. 1440−log. 1062= ·132238.

(3.) *Construction of table of Greenwich date* PROPORTIONAL LOGARITHMS FOR THE MOON.

Definition. The logarithm of 720^m (=12 hours), diminished by the logarithm of any other number of hours and minutes (less than 12 hours), is called the Greenwich date log. for moon for that number.

Thus Greenwich date log. moon for $9^h\ 48^m$=log. 720−log. 588= ·087955.

(4.) *Construction of table of* LOGISTIC LOGARITHMS.

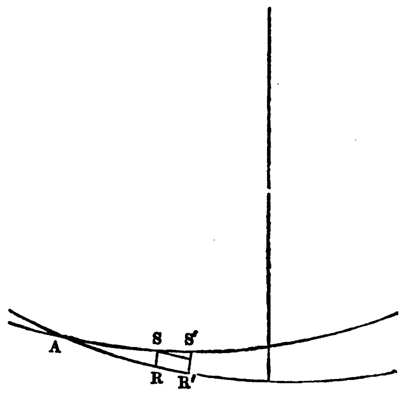
Definition. The logarithm of 3600^s (=1 hour), diminished by the logarithm of any other number of minutes and seconds (less than 1 hour), is called the logistic logarithm for that number.

Thus logist. log. $42^m\ 30^s$=log. 3600−log. 2550=·149762.

CORRECTION OF THE DECLINATION, RIGHT ASCENSION, &c., OF THE HEAVENLY BODIES FOR A GIVEN TIME FROM GREENWICH NOON.

The following examples will show the use of proportional logarithms for calculating the declination, right ascension, &c., for a given time from noon called "A GREENWICH DATE."

(1.) Given the declination of the sun at two consecutive noons at Greenwich; to find the declination at some intermediate time.



226. Find the sun's declination on April 27, 1846, at $10^h\ 42^m$ P.M. Greenwich date, $10^h\ 42^m$.

	Sun's decl. by *N. A.*	
On 27th	13° 48′ 3″ N. = SR	(see fig.)
„ 28th	14 7 4 N. = S′R′	
∴ change in 24 hours......	19 1 N.	

Let x=change in $10^h\ 42^m$,
then (supposing the motion in decl. is equable) we have

$$24^h : 10^h\ 42^m :: 19'\ 1'' : x,$$

$$\text{or } \log. 24 + \log. x = \log. 10^h\ 42^m + \log. 19'\ 1'',$$

$$\text{or } \log. x = -\log. 24^h + \log. 10^h\ 42^m + \log. 19'\ 1''.$$

Adding log. 3^h to both sides, and changing the signs,

$$\log. 3^h - \log. x = \log. 24^h - \log. 10^h\ 42^m + \log. 3^h - \log. 19'\ 1'',$$

$$\text{or prop. log. } x = \text{Gr. date log. sun } 10^h\ 42^m + \text{prop. log. } 19'\ 1''.$$

Greenwich date log. sun for $10^h\ 42^m$.........	35083
Prop. log. for 19′ 1″	97614
Prop. log. x......	1·32697
∴ x or change in $10^h\ 42^m$=	8′ 28·6″ N.
and declination at noon 13	48 3 N.
∴ decl. at Greenwich date......13	56 31·6 N.

It is evident the value of x may be found by applying the common rules of proportion or practice, or by multiplying the hourly difference, found in the *Nautical Almanac,* by the time elapsed since noon; and this latter method in some cases will be found the simplest.

(2.) Given the moon's semidiameter at noon and midnight; to find the semidiameter at some intermediate time.

227. Find the moon's semidiameter on April 27, 1846, at $10^h\ 42^m$ P.M. Greenwich date, $10^h\ 42^m$.

	Moon's semi.
27th noon......................	15′ 27·3″
27th mid.	15 21·8
Change in 12 hours	5·5

Let x=change in $10^h\ 42^m$, then $12^h : 10^h\ 42^m :: 5{\cdot}5 : x$,

$\therefore \log. x = -\log. 12^h + \log. 10^h\ 42^m + \log. 5{\cdot}5''$,

$\therefore \log. 3^h - \log. x = \log. 12^h - \log. 10^h\ 42^m + \log. 3^h - \log. 5{\cdot}5''$,

or prop. log. x=Greenwich date log. moon for $10^h\ 42^m$+prop. log. 5·5″.

Greenwich date log. moon for $10^h\ 42^m$	·04980
prop. log. 5·5″	3·29306
prop. log. x	3·34286
$\therefore x$ or change required=	0′ 4·9″—
and semi. at noon=	15 27·3
$\therefore$ semi. at Greenwich date=	15 22·4

(3.) Given the moon's declination for two consecutive hours; to find the declination for some intermediate time.

228. Find the moon's decl. for April 27, 1846, at $17^h\ 14^m\ 50^s$.
Greenwich date, $14^m\ 50^s$.

	Moon's decl. on 27th.
At 17^h............................	18° 59′ 30″ N.
„ 18	19 1 58 N.
	2 28

Let x=change for $14^m\ 50^s$, then $1^h : 14^m\ 50^s :: 2'\ 28'' : x$,

or $\log. x + \log. 1^h = \log. 14^m\ 50^s + \log. 2'\ 28''$,

$\therefore \log. x = -\log. 1^h + \log. 14^m\ 50^s + \log. 2'\ 28''$,

$\therefore \log. 3^\circ - \log. x = \log. 1^h - \log. 14^m\ 50^s + \log. 3^\circ - \log. 2'\ 28''$,

or prop. log. x=logistic log. $14^m\ 50^s$+prop. log. 2′ 28″.

logistic log. $14^m\ 50^s$	·60691
prop. log. 2′ 28″	1·86316
prop. log. x	2·47007
$\therefore x$=	0° 0′ 36·8″ N.
declination at 17^h	18 59 30 N.
declination at $17^h\ 14^m\ 50^s$............	19 0 6·8 N.

(4.) The lunar distances of certain stars used in finding the longitude being given in the *Nautical Almanac* for every third hour from Greenwich mean noon, to find the distance at any intermediate time by proportional logarithms.

229. Find the distance of Fomalhaut from the moon on Dec. 1, 1846, at $4^h\ 30^m$. Greenwich date, $1^h\ 30^m$.

	Star's distance at
3^h........................	$80°\ 45'\ 54''$
6^h........................	$82\ \ 17\ \ 51$
change in 3 hours	$1\ \ 31\ \ 57$

Let x=change in $1^h\ 30^m$, then $3^h : 1^h\ 30^m :: 1°\ 31'\ 57'' : x$,

$\therefore$ log. x=log. 3^h+log. $1^h\ 30^m$+log. $1°\ 31'\ 57''$.

Changing the signs, and adding log. 3° to each side, we have

log. 3°—log. x=log. 3^h—log. $1^h\ 30^m$+log. 3°—log. $1°\ 31'\ 57''$,

prop. log. x=prop. log. $1^h\ 30^m$+prop. log. $1°\ 31'\ 57''$.

prop. log. $1^h\ 30^m$ ....·30103

prop. log. $1°\ 31'\ 57''$ ....·29172

prop. log. x ....·59275

$\therefore x=\ 0°\ 45'\ 58{\cdot}5''$=change in $1^h\ 30^m$

distance at 3^h=80 45 54

$\therefore$ dist. at $4^h\ 30^m$=81 31 52·5

The converse of this, namely, to find the time corresponding to some given intermediate distance, may also be found by means of proportional logarithms. This is always required in the rule for finding the longitude by lunar observations (p. 87).

230. Given the lunar distances at 3^h and 6^h to be $80°\ 45'\ 54''$ and $82°\ 17'\ 51''$ respectively; to find the *time* when the lunar distance will be $81°\ 31'\ 52{\cdot}5''$.

Let x=the time elapsed since 3 o'clock.

lun. dist. at x..................	81°	31'	52·5"
,, at 3^h	80	45	54
,, at 6^h	82	17	51
increase since 3^h..............	0	45	58·5
increase in 3^h..................	1	31	57

Then $3^h : x :: 1°\ 31'\ 57'' : 45'\ 58{\cdot}5''$,

$\therefore$ log. x+log. $1°\ 31'\ 57$=log. 3^h+log. $45'\ 58{\cdot}5''$

log. 3^h—log. x=log. 3°—log. $45'\ 58{\cdot}5''$—(log. 3°—log. $1°\ 31'\ 57''$)

or prop. log. x=prop. log. $45'\ 58{\cdot}5''$—prop. log. $1°\ 31'\ 57''$.

prop. log. $45'\ 58{\cdot}5''$............·59275

prop. log. $1°\ 31'\ 57''$ ....·29172

prop. log. x=·30103

$\therefore x$ or time since 3 o'clock=$1^h\ 30^m$

$\therefore$ time corresponding to distance $81°\ 31'\ 52{\cdot}5''$ is $3^h+1^h\ 30^m$, or $4^h\ 30^m$.

We have supposed in the above examples the motions of the sun and moon in the interval between the given Greenwich times to be *uniform.* This is seldom the case; and therefore, for very accurate observations, a correction must be used called the equation of second differences, a rule for computing which will be investigated in *Navigation*, Part III.

Other corrections might have been noticed, such as the correction for refraction under different conditions of the air, the correction called the annual variation of the fixed stars, arising from precession, &c.; but the analytical investigation of these, and others of a similar nature, are not sufficiently elementary for the present volume.

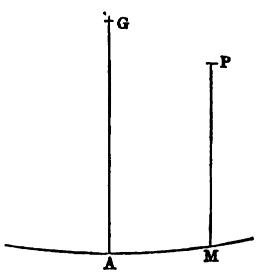
END OF NAUTICAL ASTRONOMY, PART II.

NAVIGATION.

CHAPTER VI.

INVESTIGATION OF RULES IN NAVIGATION OR PLANE SAILING.

THE position of a place on the surface of the earth is found by referring it to two lines drawn on the surface at right angles to one another; these lines are the *terrestrial equator*, and a line continued round the earth passing through its poles and through some well-known place, as *Greenwich*, called the *first meridian*. Thus, if AM represent a portion of the equator, and GA a portion of the first meridian, and P any place on the surface of the earth, and PM an arc of the meridian passing through P; then the position of the point P is said to be determined when the magnitudes of the arcs AM and PM are known. The arc AM is called the *longitude* of P.



If the earth were an exact sphere, the length of a degree of the meridian in every part of it would be the same; but observations and actual measurements of arcs in different parts of the world have proved to us that the figure of the earth is that of an oblate spheroid, whose equatorial and polar diameters are about 7924 and 7898 miles respectively (see p. 120).

From the above dimensions it is manifest that the earth, although not an exact sphere, is very nearly so; accordingly, in the common rules of Navigation which we are about to investigate, the earth will be considered as a sphere, and on this supposition a meridian is a great circle, and the arc of a great circle PM intercepted between the point P and the equator is the latitude of P.

The following are the principal terms in Navigation: the definitions of these terms, like those in Nautical Astronomy, must be thoroughly understood and committed to memory.

Course.
Distance.
Departure.
True difference of latitude.
Meridional difference of latitude.
Difference of longitude.
Middle latitude.

Definitions of terms in Navigation.

The course is the angle which the ship's track makes with all the meridians between the place left and the place arrived at.

The distance is the spiral line made by the ship's track in describing the course between the place left and the place arrived at.

The departure is the sum of all the arcs of parallels of latitude drawn between the place left and the place arrived at, through points indefinitely near to one another taken on the distance, and intercepted between the meridians passing through those points.

The true difference of latitude is the arc of a meridian intercepted between the parallels of latitude drawn through the place left and the place arrived at.

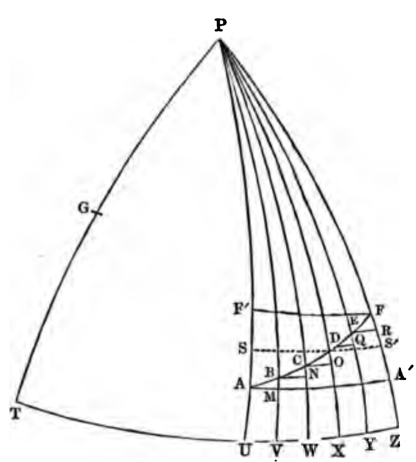
The meridional difference of latitude is the value in minutes of a great circle of the line on a Mercator's chart, into which the true difference of latitude has been expanded.

The difference of longitude is the arc of the terrestrial equator intercepted between the meridians passing through the place left and the place arrived at.

The middle latitude is the mean of the latitudes (supposed of the same name) of the place left and the place arrived at.

These definitions will be clearly understood by means of the following diagrams.

Let P represent the pole of the earth, TZ an arc of the equator, PT the meridian passing through a known place G, as Greenwich, A and F two other places on the earth (considered as a sphere), PU, PZ, their meridians.



Through the points A and F suppose a curve line AF to be drawn, cutting all the intermediate meridians PV, PW, PX, &c., *at the same angle;* that is, making the angle PAB=PBC=PCD=&c. Then this common angle PAF is called the *course.* The arc AF* is the *distance.* Draw the parallels of latitude AA′, FF′; then, since the latitude of A is the arc AU, and the longitude of A the arc TU, and the latitude of F is the arc FZ, and the longitude of F is the arc TZ; therefore the difference, or, as it is usually called, the *true difference of latitude,* between A and F is the arc AF′ or A′F, and *the difference of longitude* between A and F is the arc of the equator UZ. Again, suppose the intermediate meridians PV, PW, &c., to be

* AF is sometimes called the *rhumb line,* sometimes the *loxodromic curve,* sometimes the *equiangular spiral.*

drawn through points B, C, D, &c., taken on the line AF indefinitely near to each other; and through the points A, B, C, D, &c., the arcs of parallels of latitude AM, BN, CO, &c. On this supposition (namely that the points A, B, C, &c., are indefinitely near to each other) the elementary triangles ABM, BCN, CDO, &c., may be considered without any error to be right-angled *plane triangles.* *The departure* between A and F=AM+BN+CO+ . . . ER, the points A, B, C, &c., being supposed to be indefinitely near to each other.

If a parallel of latitude SS′ be drawn through the middle of AF′, then the arc of the meridian SU is called the mean or *middle latitude* between A and F. It is manifest that the arc SS′ will be nearly equal to AM+BN+ . . . DQ+ER, the departure, A and F being supposed to be on the same side of the equator. For short distances SS′ is substituted without any practical error for the departure, and one of the principal rules in Navigation deduced from it.

There are two kinds of charts; the Plane chart, and Mercator's chart.

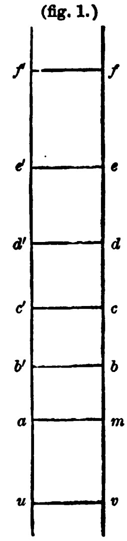
The Plane chart.

The plane chart is a representation of the earth's surface, considering it as a plane. When a small portion of the surface is concerned, this mode of representing it will lead to no practical error; hence coasting charts are usually constructed in this manner, in which the different headlands, lighthouses, &c., are laid down according to their bearings.

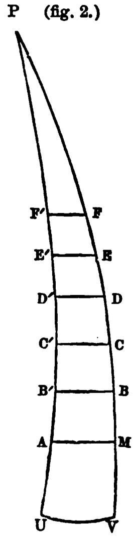
Mercator's chart.

The chart used at sea for marking down the ship's track, and for other purposes, is called Mercator's chart. It exhibits also the surface of the earth *on a plane;* but the meridians are drawn perpendicular to the equator, and therefore the arcs AM, B′B, &c., of parallels of latitude intercepted between any two meridians are increased to am, $b'b$, &c., and become equal to one another and to line uv, and therefore to the intercepted arc UV of the equator. If we wish to make the figures (supposed to be very small) $ambb'$, $b'bcc'$, &c., on the chart similar to AMBB′, B′BCC′, &c., of the globe, it is evident we must increase the sides bm, bc, &c., in the same proportion as am, $b'b$, &c. (that represent AM, BB′, &c.), have been increased. Let us therefore suppose the straight lines am, $b'b$, $c'c$, &c., have been drawn at such a distance from each other that the above similarity of figure is preserved (and

(fig. 1.)



(fig. 2.)



this can only be done by supposing the surfaces $ambb'$, $b'bcc'$, &c., *indefinitely* small, so that the surfaces AMBB′, B′BCC′, &c., may be considered as plane surfaces). Then a representation of the earth's surface, or any part of it, so constructed, is called a *Mercator's chart.*

The straight line mf, into which MF, the true difference of latitude between M and F, has been expanded, is called the *meridional difference of latitude* between M and F, and the values of bv, cv, &c., in minutes, are called the *meridional parts* of B, C, &c. : hence the *meridional difference of latitude* between two places is the difference of the meridional parts for the two places.

An approximate method of calculating the meridional parts for any latitude.

In order to construct a Mercator's chart, we must know the lengths of the lines corresponding to the latitude of every point on the globe, at small distances from each other. These lengths, or meridional parts, computed for every minute of latitude from 0° to 90°, form the table of meridional parts. It may be obtained with sufficient exactness in the following manner:

Suppose the meridians PV, PU (p. 145), and parallels AM, BB′, &c., are drawn sufficiently near to each other that the quadrilateral surfaces of the earth, AMBB′, B′BCC′, &c., thus formed, may be considered without any practical error to be *plane surfaces;* this may be done if the arcs MB, BC, &c., be not taken greater than 1 minute: then these quadrilateral surfaces being expanded into $ambb'$, $b'bcc'$, similar to them on the chart, we can prove that

$$\begin{aligned} bm &= \text{BM} \,.\, \text{sec. lat. M}, \\ bc &= \text{BC} \,.\, \text{sec. lat. B}, \\ cd &= \text{CD} \,.\, \text{sec. lat. C}, \\ de &= \text{DE} \,.\, \text{sec. lat. D}, \\ \&\text{c.} &= \&\text{c.} \end{aligned}$$

For (by *Trig.* Part II. Art. 69), $\frac{\text{AM}}{\text{UV}} = \text{cos. MV} = \text{cos. lat. M}$,

$$\therefore \text{sec. lat. M} = \frac{\text{UV}}{\text{AM}} = \frac{uv}{\text{AM}} = \frac{am}{\text{AM}}.$$

But since the quadrilateral surfaces on the globe and chart are similar,

$$\therefore \frac{bm}{\text{BM}} = \frac{am}{\text{AM}} = \text{sec. lat. M},$$

$$\begin{aligned} \therefore\ bm &= \text{BM} \,.\, \text{sec. lat. M}. \\ \text{similarly } bc &= \text{BC} \,.\, \text{sec. lat. B}, \\ cd &= \text{CD} \,.\, \text{sec. lat. C}, \\ de &= \text{DE} \,.\, \text{sec. lat. D}, \\ \&\text{c.} &= \&\text{c.} \end{aligned}$$

Let lat. of M$=l$, and BM$=$BC$=$CD$=$DE$=1$ minute; then lat. B$=l+1'$, lat. C$=l+2'$, lat. D$=l+3'$; $\therefore$ adding, we have

$$\text{sec. } l+\text{sec. } (l+1)+\text{sec. } (l+2)+\text{sec. } (l+3)=bm+bc+cd+de=me$$
$=$meridional difference of lat. between M and E.

If the point M be on the equator, then $l=0$, and the above expression gives the value of the meridional parts for 4 minutes: thus

$$bm+bc+cd+de=\text{sec. } 0'+\text{sec. } 1'+\text{sec. } 2'+\text{sec. } 3',$$
or the merid. parts for $4'=\text{sec. } 0^\circ+\text{sec. } 1'+\text{sec. } 2'+\text{sec. } 3'$.

A nearer approximation to the value of the meridional parts for 4 minutes would of course be obtained by taking the parts MB, BC, CD, &c., still smaller, as 1 second; for then the meridional parts for 4 minutes would be found from the expression,

MP for $4'=\text{sec. } 0''+\text{sec. } 1''+\text{sec. } 2''+\text{sec. } 3''+\ldots+\text{sec. } 3'\ 59''$.

Hence, generally, we have for any lat. l

Mer. parts for $l^\circ=\text{sec. } 0''+\text{sec. } 1''+\text{sec. } 2''+\ldots+\text{sec. } (l^\circ-1'')$.

And from a similar expression to this was the first table of meridional parts computed.

231. *Example.* If we suppose the meridional parts for 70° to have been found to be$=5965{\cdot}92$ (by the above or any other method), let it be required to calculate the meridional parts for 70° 10′.

MP for $70^\circ 10'=\text{sec. } 0'+\text{sec. } 1'+\text{sec. } 2'+\text{sec. } 3'+\ldots+\text{sec. } 69^\circ 59'(=5965{\cdot}92)$
$+\text{sec. } 70^\circ+\text{sec. } 70^\circ\ 1'+\ldots+\text{sec. } 70^\circ\ 9'$
$=5965{\cdot}92+\text{sec. } 70^\circ+\text{sec. } 70^\circ\ 1'+\ldots\text{sec. } 70^\circ\ 9'$.

log. sec.	70° 0′	......0·465948......	∴ nat. sec.=	2·9238
„	70 1	......0·466296......	„ =	2·9262
„	70 2	......0·466643	„ =	2·9285
„	70 3	......0·466991	„ =	2·9308
„	70 4	......0·467339	„ =	2·9332
„	70 5	......0·467688	„ =	2·9355
„	70 6	......0·468037	„ =	2·9379
„	70 7	......0·468386	„ =	2·9403
„	70 8	......0·468735	„ =	2·9426
„	70 9	......0·469085	„ =	2·9450
				29·3438
			mer. parts for 70°=	5965·92
			∴ mer. parts for 70° 10′=	5995·2638

232. Given the meridional parts for 70° 10′$=5995{\cdot}2638$; calculate the meridional parts for 70° 15′. *Ans.* Mer. parts$=6010{\cdot}03$.

PROOF OF THE RULES IN NAVIGATION.

From the definitions and principles given in pp. 144, 145, are deduced the following formulæ or equations, and these expressed in words constitute the common rules of Navigation for finding the *place* of a ship, that is, its latitude and longitude.

FUNDAMENTAL FORMULÆ IN NAVIGATION.

Departure = distance × sin. course . . . . . . (1)

True diff. lat. = distance × cos. course . . . . (2)

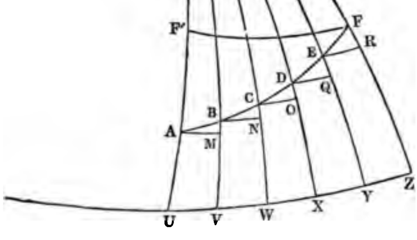
Diff. long. = meridional diff. lat. × tan. course . . (3)

In parallel sailing,
Distance = diff. long. × cos. lat. . . . . (4)

In middle latitude sailing,
Departure (nearly) = diff. long. × cos. mid. lat. . (5)

PROOF OF THE ABOVE FORMULÆ.

(1.) *Proof that* DEPARTURE = DIST. × SIN. COURSE.



Suppose the ship to sail from A to F, cutting all the intermediate meridians at the same angle PAF, PBF, &c., then this common angle is the course (p. 144), the arc AF is the distance (p. 144); and if in the figure the triangles ABM, BCN, &c., be taken so small that they may be considered as *right-angled plane triangles*, then AM + BN + . . . + ER is the departure (p. 145): it is also manifest that the angles BAM, CBN, DCO, &c., are equal to one another, each being the complement of the course.

∴ in triangle BAM, AM = AB . cos. BAM = AB . sin. course
„ BCN, BN = BC . cos. CBN = BC . sin. course
„ CDO, CO = CD . cos. DCO = CD . sin. course
&c. = &c.

adding AM+BN+CO+..=(AB+BC+CD+..) sin. course.
But AM+BN+CO+ . . . +ER=departure,
and AB+BC+CD+ . . . +EF=distance,
.·. departure=dist. × sin. course.

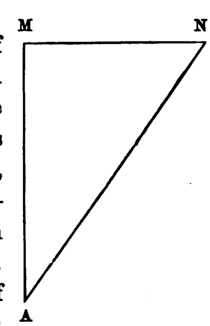
(2.) *Proof that* TRUE DIFF. LAT.=DIST. × COS. COURSE.

The same construction being made as in the last figure, we have in the elementary triangles ABM, BCN, CDO, &c.

BM=AB . sin. BAM=AB . cos. course,
CN=BC . sin. CBN=BC . cos. course,
DO=DC . sin. DCO=DC . cos. course,
&c.=&c. .·. adding
BM+CN+DO+ . . =(AB+BC+DC+ . .) cos. course.
But BM+CN+DO+ . . . +FR=AF′=true diff. lat.
and AB+BC+DC+ . . . +EF=AF=distance,
.·. true diff. lat=dist. × cos. course.

(3.) *Proof that* DIFF. LONG.=MERIDIONAL DIFF. LAT. × TAN. COURSE.

We have already seen that in the construction of a Mercator's chart the meridians are drawn parallel to each other and perpendicular to the equator, and therefore the parts of parallels of latitude AM, BN, CO, &c. (see last figure), are increased, and become equal to the corresponding parts UV, VW, WX, &c., of the equator; and that in order to preserve the similarity of the parts on the chart that correspond respectively to the triangles ABM, BCN, &c., the sides, BM, CN, DO, &c., must be increased in the same proportion as AM, BN, CO, &c., have been increased: when, therefore, AM+BN+CO, &c., the departure between A and F, has been increased, and become equal in length to UZ, the difference of longitude, the arcs BM+CN+DO, &c.=AF′, the true diff. lat. between A and F, will be expanded, and become equal to a straight line called *the meridional difference of latitude between* A and F. Moreover the lines into which each of the parts AB, BC, CD, &c., is expanded, will be inclined to the parallels at the same angle as the course, since the triangular surfaces on the chart into which the triangles ABM, BCN, &c., are expanded are made similar to ABM, BCN, &c., in every respect; and as AB, BC, &c., cut the meridians at the same angle, lines corresponding to them on the chart must be in one and the same straight line. Hence the meridional difference of latitude, difference of longitude, and the line into which the distance AF is expanded, form the sides of a right-angled plane triangle, as AMN; where if A represents the course, corresponding to the angle PAF on the globe, AM



will be the meridional difference of latitude, and MN the difference of longitude between A and F.

Since, in the right-angled triangle AMN, $\frac{MN}{AM}$=tan. A,

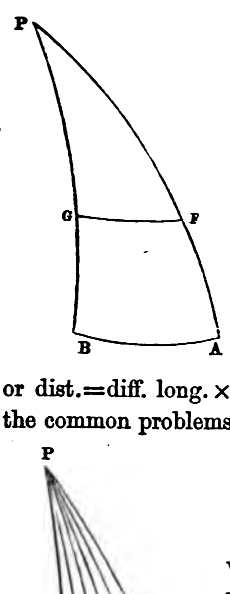
$$\text{therefore } \frac{\text{diff. long.}}{\text{mer. diff. lat.}} = \text{tan. course,}$$

$$\text{or diff. long.} = \text{mer. diff. lat.} \times \text{tan. course.}$$

(4.) PARALLEL SAILING.

Proof that DIST.=DIFF. LONG. × COS. LAT.

When the course is 90°, that is, when the ship is sailing on a parallel of latitude, tan. course=∾, and for this value of the angle formula (3) gives no assistance in finding the difference of longitude: but since the course in parallel sailing is due east or due west, the distance is in fact the *arc of a parallel of latitude* intercepted between the meridians passing through the two places; we may therefore find the relation between the distance, difference of longitude and latitude, by means of the well-known property that $\frac{GF}{AB}$=cos. FA (*Trig.* Part II. p. 76), in which FG is the distance, AB the diff. long., and AF the latitude of the ship.

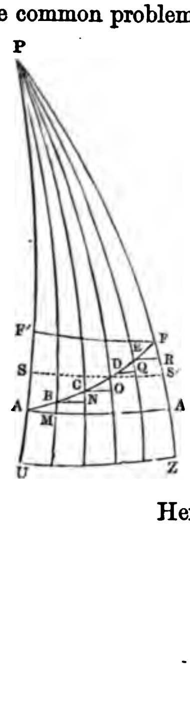


$$\text{Hence we have } \frac{\text{dist.}}{\text{diff. long.}} = \text{cos. lat.,}$$

or dist.=diff. long. × cos. lat. : a formula that will enable us to solve all the common problems in parallel sailing.

(5.) MIDDLE LATITUDE SAILING.

Proof that DEPARTURE=DIFF. LONG. × COS. MID. LAT.



Another formula, giving results sufficiently correct when the distance run is not great (such as in an ordinary day's sailing), is obtained by considering the parallel SS′, drawn through the middle of AF′ (the difference of latitude between A and F), as equal to AM+BN+CO+ . . ., the departure; for then we have (*Trig.* Part II. Art. 69) $\frac{SS'}{UZ}$=cos. SU,

in which SS′=departure nearly,

UZ=difference of long. between A and F,

and SU=the latitude of the middle point between A and F, and is therefore called the *middle latitude.*

$$\text{Hence } \frac{\text{dep.}}{\text{diff. long.}} = \text{cos. mid. lat.}$$

or departure=diff. long. × cos. mid. lat.

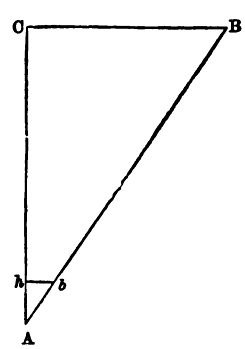
RULES IN NAVIGATION DERIVED FROM PLANE TRIGONOMETRY.

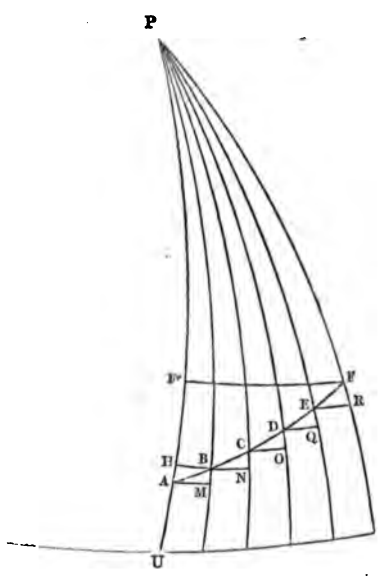
The principal rules in Navigation may be deduced from the formulæ just proved; but as they can be shown to depend also on the solution of a right-angled triangle (the proof of which will be given in the following problem), we will adopt the latter method of developing the rules, as being more practically useful.

PROBLEM XLIII.

The DISTANCE, TRUE DIFFERENCE OF LATITUDE, DEPARTURE, and COURSE, may be correctly represented by the three sides and one of the angles of a *right-angled plane triangle;* also the MERIDIONAL DIFFERENCE OF LATITUDE, and DIFFERENCE OF LONGITUDE, may be represented by two sides of a triangle which is similar to the same right-angled plane triangle.

Suppose the arc AF, the distance between A and F, to be divided into n *equal* parts AB, BC, CD, &c.; then AF$=n$AB, and the true difference of





latitude AF′ ($=$BM + CN + . . .)$=n$BM, since the elementary triangles ABM, BCN, &c., are on this supposition equal to each other in every respect, and also to the triangle ABH, formed by drawing BH parallel to AM. Let now a small plane triangle Abh be supposed to be constructed equal and similar to the triangle ABH (and this can only be done by supposing ABH to be in its evanescent condition, that is, indefinitely small); produce Ab to B, and Ah to C, so that AB$=$AF, the distance, and AC$=$AF′, the true difference

of latitude: join BC, then BC shall be at right angles to AC, and equal to AM + BN + CO + . . . the departure.

For since in triangle ABC, $AB = nAb$, and $AC = nAh$,

$\therefore$ AB and AC are cut proportionally in b and h,

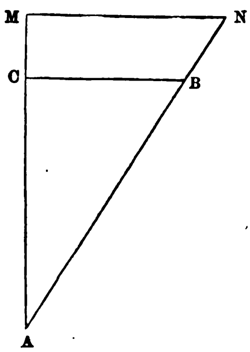
$\therefore$ BC is parallel to bh, and the angle at C is therefore a right angle.

Again, in the same triangle, ABC,

$$BC = AB \cdot \sin. A = \text{dist. sin. course.}$$

But by formula (p. 148), departure = dist. sin. course,

$$\therefore BC = \text{departure.}$$



Again, let the side AC be produced to M, so that AM may be equal to the meridional difference of latitude between A and F, and let the right-angled triangle AMN be completed; then the side MN will equal the difference of longitude UZ (fig. p. 148) between A and F.

For in triangle AMN,

$$MN = AM \tan. A = \text{mer. diff. lat.} \times \tan. \text{course.}$$

But by formula (p. 149),

$$\text{diff. long.} = \text{mer. diff. lat.} \times \tan. \text{course,}$$

$$\therefore MN = \text{difference of long.}$$

We thus see that questions in Navigation or plane sailing may be much simplified by representing the distance, true difference of latitude, departure, meridional difference of latitude, difference of longitude, and course, by the several parts of two similar right-angled plane triangles connected together in the form given in the above figure. In fact, all the principal rules (except in the case when the course is due east or west, and then we must use formula (4), p. 150) can thus be made to depend on the COMMON RULE IN PLANE TRIGONOMETRY FOR SOLVING RIGHT-ANGLED PLANE TRIANGLES.

A few examples will illustrate the method of working questions in Navigation by the common rules of Plane Trigonometry.

To find the course and distance (using meridional parts).

EXAMPLES.

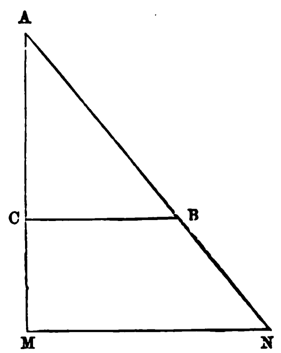
233. Required the course and distance from A to B, having given

Lat. A.........56° 45′ N.	Long. A.........39° 5′ W.
„ B.........49 10 N.	„ B.........29 17 W.

and the merid. diff. lat. (from the table of meridional parts) = 758′. The

course is evidently towards the south and east, the true diff. lat. is 455′, and diff. long. 588′. Construct, therefore, the figure as follows:

Let A represent the place sailed from, and from A draw AC due south=455′, the true diff. lat.; produce AC to M, and make AM=758′, the mer. diff. alt. From M draw MN towards the east, at right angles to AM, and make it equal to 588′, the diff. long.; join AN, and through C draw CB parallel to MN; then AB will represent the distance, and the angle A the course required.



(1.) In right-angled triangle AMN, given AM the mer. diff. lat., and MN the diff. long.; to find angle A = 37° 48′ : ∴ the course = S. 37° 48′ E. (see *Trigonometry*, Part I. Rule V.).

(2.) In triangle ABC, given AC the true diff. lat., and angle A; to find AB=576′, the distance (see *Trig.* Rule V. for right-angled triangles).

234. Given lat. A......70° 10′ S. Long. A......54° 40′ W.
„ B......74 40 S. „ B......57 10 W.

Construct a figure, and find the course and distance from A to B.

Ans. Course=S. 9° 30′ W., dist.=274 miles.

To find latitude and longitude in (using meridional parts).

235. Sailed from a place A, in latitude 56° 45′ N. and long. 39° 5′ W., S. 37° 48′ E., 576 miles to another place B; required the latitude and longitude of B.

Construction.

Let A (last fig.) represent the place sailed from. Draw AM, a part of the meridian, and at A in the straight line AM make the angle A=S. 37° 48′ E.; take AB=576′, the distance, and through B draw BC at right angles to AM; then AC will represent the true diff. lat. between A and B. Produce AC to M, and make AM=meridional diff. lat., and complete the triangle AMN; then MN is the diff. long.

To find true diff. lat. and diff. long. (*Trigonometry*, Rule V.).

(1.) In triangle ABC, are given the angle A the course, and AB the distance; to calculate AC the true diff. lat.=455′=7° 35′ S. Hence the lat. of B=49° 10′ N., and the mer. diff. lat. (found from table)=758′=AM.

(2.) In triangle AMN, are given the course A, and AM the mer. diff. lat.; to find MN the diff. long.=588′=9° 48′ E. Hence the long. of B= 29° 17′ W.

236. Sailed from A in latitude 20° 30′ N. and longitude 1° 40′ W., N. 45° 33′ 30″ E., 400 miles to another place B. Construct a figure, and find the lat. and long. of B. *Ans.* Lat. B=25° 10′ N., long. B=3° 30′ E.

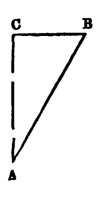
To find the course and distance (using middle latitude).

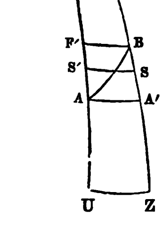
If in the right-angled triangle ABC (p. 153) there is given the true diff. lat. AC, and we also know or can find the *departure* BC, then the other parts of the triangle, namely the course A and distance AB, may also be found directly, and without the aid of the triangle AMN. Now it has been shown (p. 150) that the arc SS′ of the parallel of latitude, drawn through the middle latitude between two places A and F, may be considered equal to the departure for small distances; and on this assumption we have (formula, p. 148) dep. = diff. long. × cos. mid. lat.* When, therefore, the latitudes and longitudes of the two places are given, we can compute the departure by the above formula; and then, with the true diff. lat. already known, the course and distance may be found by the common rule in Trigonometry for right-angled plane triangles.

237. Find the course and distance from A to B (using middle latitude), having given

Lat. A......14° 40′ N.		Long. A......56° 40′ E.
„ B......18 20 N.		„ B......60 10 E.

Construction.





Through A and B, the two places, draw the meridians PU and PZ; draw the parallel BF′, and through S′, the middle of AF′, draw the parallel of latitude SS′; then SS′ = departure nearly, and $\frac{SS'}{UZ}$ = cos. S′U, or $\frac{\text{dep.}}{\text{diff. long.}}$ = cos. middle latitude (p. 150);

∴ dep. = diff. long. cos. mid. lat.

Again, in the right-angled triangle ABC, let AC represent the true diff. lat., and CB the departure (found from the above formula); then AB = distance, and angle A the course required.

* Mid. lat. = ½{lat. from + lat. in};
for (fig. p. 150) SU = AU + AS,
and SU = F′U − F′S;
∴ 2SU = AU + F′U (since AS = F′S),
or SU = ½{AU + F′U};
that is, mid. lat. = ½{lat. from + lat. in}.

Calculation.

In triangle CAB, $\tan. A = \frac{BC}{AC}$,

$$\text{or tan. course} = \frac{\text{dep.}}{\text{true diff. lat.}} = \frac{\text{diff. long. cos. mid. lat.}}{\text{true diff. lat.}}$$

diff. long. $=210'$; mid. lat. $=16° \ 30'$; true diff. lat. $=220'$

$\therefore$ course $=$ N. $42° \ 28'$ E.

In triangle CAB, $\frac{AB}{AC} = \sec. A$; $\therefore AB = AC \cdot \sec A$,

or dist. $=$ true diff. lat. sec. course; $\therefore$ dist. $=298$ miles.

*** In the following examples the student is supposed to know the points of the compass.

CORRECTIONS IN NAVIGATION.

Three corrections are sometimes necessary to be applied to the course steered by compass, to reduce it to the true course; and the converse. These are called:

(1.) The correction for variation of the compass.

(2.) The correction for deviation of the compass.

(3.) The correction for leeway.

(1.) *The Correction for Variation of the Compass.*

The magnetic needle seldom points to the true north. Its deflection to the east or west of the true north is called the *variation of the compass;* it is different in different places, and it is also subject to a slow change in the same place. The variation of the compass is ascertained at sea by observing the magnetic bearing of the sun when in the horizon, or at a given altitude above it. From this observation the true bearing is found by rules given in nautical astronomy. The difference between the true bearing and the observed bearing is the variation of the compass.

The method of correcting the course for variation will be more readily understood by means of a few examples.

Suppose the variation of the compass is found to be two points to the east, that is, the needle is directed two points to the right of the north point of the heavens; then the N.N.W. point of the compass card will evidently point to the true north, and every other point on the card will be shifted round two points. If, therefore, a ship is sailing *by compass* N.N.W., or, as it is expressed, the compass course is N.N.W., her true course will be north; that is, *two points to the right of the compass course.* In a similar manner it may be shown that, when the variation is two points westerly, the true course will be *two points to the left of compass course.* Hence this rule (the method *by construction* is given at p. 160):

To find the true course, the compass course being given.

Easterly variation allow to the right.

Westerly „ „ left.

From the preceding considerations it will be easy to deduce the converse rule, namely:

To find the compass course, the true course being given.

Easterly variation allow to the left.

Westerly „ „ right.

EXAMPLES.

1. Find the true course, having given the compass course N.W.$\frac{1}{2}$W. and variation 3$\frac{1}{4}$ W.

	pts.	qrs.
Compass course......	4	2 left of N.
variation.............	3	1 left.*
true course...........	7	3 left of N.=W.$\frac{1}{4}$N.

2. Find the compass course, having given the true course W.$\frac{1}{4}$N. and variation 3$\frac{1}{4}$ W.

	pts.	qrs.
True course..........	7	3 left of N.
variation..............	3	1 right.
compass course......	4	2 left of N.=N.W.$\frac{1}{2}$W.

Find the true course in each of the following examples:

	Compass course.	Var.	Answers.
3.	N.N.E.	2$\frac{1}{4}$W.	N.$\frac{1}{4}$W.
4.	N.W.	1$\frac{3}{4}$E.	N.N.W.$\frac{1}{4}$W.
5.	S.W.$\frac{3}{4}$W.	1$\frac{1}{2}$E.	W.S.W.$\frac{1}{4}$W.
6.	S.	2W.	S.S.E.
7.	W.	2$\frac{1}{2}$E.	N.W.b.W.$\frac{1}{2}$W.

Find the compass course in each of the following examples:

	True course.	Var.	Answers.
8.	N.N.E.$\frac{1}{2}$E.	$\frac{1}{4}$W.	N.N.E.$\frac{3}{4}$E.
9.	N.	1$\frac{1}{2}$E.	N.b.W.$\frac{1}{2}$W.
10.	S.S.W.	2W.	S.W.
11.	S.W.	0	S.W.
12.	N.b.W.$\frac{1}{4}$W.	1$\frac{1}{4}$W.	N.

(2.) *The Correction for Deviation of the Compass.*

This correction of the compass arises from the effect of the iron on board ship on the magnetic needle, in deflecting it to the right or left of the magnetic meridian. The increased quantity of iron used in ships has caused this correction to be attended to now more than formerly, as its effects and

* When names are alike (that is, both left or both right), *add:* when unlike, *subtract*, marking remainder with the name of the greater.

magnitude have become more perceptible. The amount of the deviation arising from this local cause varies as the mass of iron changes its position with respect to the compass. When a fore and aft line coincides with the direction of the magnetic meridian, the iron in the ship may be supposed to be nearly equally distributed on both sides of the needle, and its effect in deflecting the needle may be inappreciable. In other positions of the ship with respect to the magnetic meridian, the iron may produce a sensible deflection of the needle; and this deflection or deviation will in general be the greatest when the ship's head points to the east or west.

Various methods are used to determine this correction. The one usually adopted is to place a compass on shore, where it may be beyond the influence of the iron of the ship, or any other local disturbing force, and to take the bearing of the ship's compass, or some object in the same direction therewith; at the same time, the observer on board takes the bearing of the shore compass; then if 180° be added to the bearing at the shore compass, so as to bring it round to the opposite point, the difference between the result and the bearing at ship's compass will be the amount of the deviation of the compass for that position of the ship.

The ship is then swung round one or two points, and a similar observation made; and thus the local deviation found for a second position of the ship. This being repeated for every point or two points of the compass, the deviation is thus known for all positions of the ship. A table, similar to the one below, is then formed, and the courses corrected for this deviation by the following rules; which resemble those already given for correcting for variation.

Deviation of Compass of H.M.S. ——, for given positions of the ship's head.

Direction of ship's head.	Deviation of compass.		nearly	Direction of ship's head.	Deviation of compass.		nearly
N.	E.	2° 45′	or $\frac{1}{4}$ pt.	S.	W.	3° 0′	or $\frac{1}{4}$ pt.
N.b.E.	E.	4 57	or $\frac{1}{2}$,,	S.b.W.	W.	4 20	or $\frac{1}{2}$,,
N.N.E.	E.	7 30	or $\frac{3}{4}$,,	S.S.W.	W.	5 0	or $\frac{1}{2}$,,
N.E.b.N.	E.	9 0	or $\frac{3}{4}$,,	S.W.b.S.	W.	6 7	or $\frac{1}{2}$,,
N.E.	E.	10 0	or $\frac{3}{4}$,,	S.W.	W.	7 0	or $\frac{1}{2}$,,
N.E.b.E.	E.	10 55	or 1 ,,	S.W.b.W.	W.	7 27	or $\frac{3}{4}$,,
E.N.E.	E.	10 40	or 1 ,,	W.S.W.	W.	7 50	or $\frac{3}{4}$,,
E.b.N.	E.	9 55	or $\frac{3}{4}$,,	W.b.S.	W.	8 20	or $\frac{3}{4}$,,
E.	E.	8 50	or $\frac{3}{4}$,,	W.	W.	8 50	or $\frac{3}{4}$,,
E.b.S.	E.	7 15	or $\frac{3}{4}$,,	W.b.N.	W.	8 10	or $\frac{3}{4}$,,
E.S.E.	E.	5 35	or $\frac{1}{2}$,,	W.N.W.	W.	6 50	or $\frac{1}{2}$,,
S.E.b.E.	E.	3 40	or $\frac{1}{4}$,,	N.W.b.W.	W.	5 40	or $\frac{1}{2}$,,
S.E.	E.	1 50	or $\frac{1}{4}$,,	N.W.	W.	4 50	or $\frac{1}{2}$,,
S.E.b.S.	E.	0 20	or 0 ,,	N.W.b.N.	W.	3 20	or $\frac{1}{4}$,,
S.S.E.	W.	0 56	or 0 ,,	N.N.W.	W.	1 40	or 0 ,,
S.b.E.	W.	2 20	or $\frac{1}{4}$,,	N.b.W.	E.	1 10	or 0 ,,

To find the true course, having given the compass course and the deviation.

Easterly deviation allow to the right.
Westerly „ „ left.

EXAMPLES.

13. Correct the compass course W.b.S. for deviation ¾W. (known from table, p. 157).

	pts.	qrs.	
Compass course	7	0	right of S.
deviation..............	0	3	left.
true course............	6	1	right of S.

or W.S.W.¼W.

14. Correct the compass course N.W.½W. for deviation ½W. (from deviation table, p. 157), and also for variation of compass 3¼W.

			pts.	qrs.
Compass course			4	2 l. N.
deviation	0	2 l.		
variation.............	3	1 l.		
			3	3 l.
true course			8	1 l. N.
			16	
or true course.............			7	3 r. S.=W.¼S.

Find the true course in each of the following examples, by correcting for deviation from table, p. 157, and for variation :

	Compass course.	Var.	Answers.
15.	N.N.E.	2¼W.	N.½E.
16.	N.W.	1¾E.	N.N.W.¾W.
17.	S.W.¾W.	1½E.	S.W.b.W.¾W.
18.	S.	2W.	S.S.E.¼E.
19.	W.	2½E.	W.N.W.¼W.
20.	W.¾N.	1½W.	W.S.W.½W.

To find the compass course, having given the true course and deviation.

Easterly deviation allow to the left.
Westerly „ „ right.

NOTE.—The true course should first be corrected for variation (if any) by Rule, p. 156 (which is similar to the above), so as to get a compass course nearly, and then this course for deviation, from table, p. 157.

EXAMPLES.

21. Required the compass course, the true course being W.S.W.¼W., variation 0, and deviation ¾W. (see table).

	pts.	qrs.	
True course	6	1 r.	S.
deviation	0	3 r.	
compass course............	7	0 r.	S., or W.b.S.

22. Required the compass course, the true course being S.W., variation of compass 2¼E., and deviation as in table, p. 157.

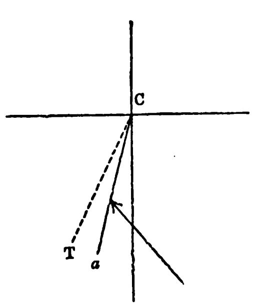
	pts.	qrs.	
True course	4	0 r.	S.
variation....................	2	1 l.	
compass course nearly...	1	3 r.	S., or S.b.W.¾W.
deviation	0	2 r.	
compass course............	2	1 r.	S.=S.S.W.¼W.

Required the compass course in each of the following examples (for deviation, see table, p. 157):

	True course.	Var.	Answers.
23.	N.½E.	2¼W.	N.N.E.
24.	N.N.W.¾W.	1¾E.	N.W.
25.	S.W.b.W.¾W.	1½E.	S.W.¾W.
26.	S.S.E.¼E.	2W.	S.
27.	W.N.W.¼W.	2½E.	W.
28.	W.S.W.½W.	1½W.	W.¾N.

(3.) *The Correction for Leeway.*

This correction is the angle which the ship's track makes with the direction of a fore and aft line: it arises from the action of the wind on the sails, &c., not only impelling the ship forwards, but pressing against it sideways, so as to cause the actual course made to be to *leeward* of the apparent course, as shown by the fore and aft line. The amount of leeway differs in different ships, depending on their construction, on the sails set, the velocity forwards, and other circumstances. Experience and observation, therefore, usually determine the amount of leeway to be allowed.



The method of correcting for leeway will be best seen by the following example:

Suppose the apparent course is S.S.W.½W., and leeway two points, the wind being S.E., required the correct course.

Draw two lines at right angles to each other towards the cardinal points

of compass, and a line, as c*a*, to represent (roughly) the course of the ship, and another to represent the direction of the wind (as the arrow in fig.); then it will be seen that the corrected course, as CT, will be to the *right* of the apparent course; *the observer being always supposed to be at the center* C, *and looking towards the cardinal point from whence the course is measured;* hence

	pts.	qrs.
Apparent course......	2	2 r. S.
leeway.................	2	0 r.
corrected course	4	2 r. S.=S.W.$\frac{1}{2}$W.

EXAMPLES.

Correct the following courses for leeway, so as to find the true courses:

	Apparent course.	Wind.	Leeway.	Answers.
29.	N.N.E.	W.N.W.	$1\frac{1}{2}$	N.E.$\frac{1}{2}$N.
30.	N.W.	N.N.E.	2	W.N.W.
31.	E.S.E.	S.	$2\frac{1}{2}$	E.$\frac{1}{2}$N.
32.	E.	N.b.E.	$\frac{3}{4}$	E.$\frac{3}{4}$S.

Correct the following compass courses for deviation, variation, and leeway, so as to find the true courses. The deviation is found in table, p. 157, and the variation of compass is supposed to be in each example $2\frac{1}{2}$ W.

	Course.	Wind.	Leeway.	Answers.
33.	N.W.$\frac{1}{4}$W.	W.S.W.	$2\frac{1}{2}$	N.W.$\frac{3}{4}$W.
34.	S.E.$\frac{1}{2}$E.	E.N.E.	$2\frac{1}{4}$	S.E.$\frac{1}{2}$E.
35.	W.$\frac{1}{4}$S.	S.S.W.	2	W.S.W.$\frac{1}{2}$W.
36.	N.$\frac{3}{4}$W.	W.b.N.	$1\frac{1}{2}$	N.b.W.$\frac{1}{2}$W.

These examples may be worked out in the following manner:

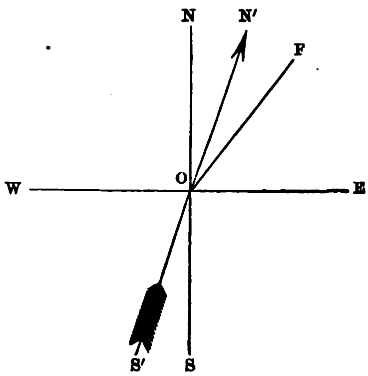
			pts.	qrs.
Ex. 33.	Compass course..........		4	1 l. N.
	deviation 0	2 l.		
	variation..........2	2 l.		
			3	0 l.
			7	1 l. N.
	leeway.....................		2	2 r.
	true course................		4	3 l. N.=N.W.$\frac{3}{4}$W.

In the preceding examples the courses, both true and compass, are corrected for variation and deviation by a formal rule. The student, however, should also know how to make these corrections by means of a construction, as in the following examples:

238. Given the true course=N. 42° 28′ E., and the variation of the compass=$1\frac{1}{2}$ points easterly; construct a figure to show the compass course.

Construction.

Let NS represent the true meridian; and since the variation of the compass is $1\frac{1}{2}$ points E., draw N′S′ $1\frac{1}{2}$ points, or 16° 52′, to the east of the true meridian; then N′S′ will represent the direction of the magnetic meridian, and the angle NON′ the variation of the compass. At the point O, in the straight line NO, make the angle NOF = 42° 28′; then NOF will represent the true course, N. 42° 28′ E., and N′OF will therefore be the compass course; and it is evident by the figure that



N′OF=NOF—NON′,

or compass course=true course—variation
=42° 28′−16° 52′=25° 36′;

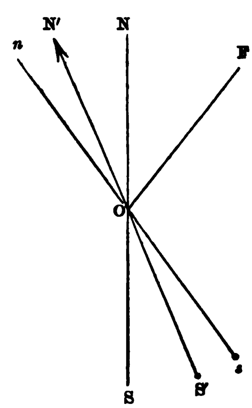
and since this angle is to the right of the magnetic meridian,

∴ the compass course=N. 25° 36′ E.

239. Given the true course=N. 25° 36′ E., the variation=2 points W., and deviation on account of local attraction=7° 20′ E.; to find the corrected compass course (by construction).

Construction.

Let NS represent the true meridian; and since the variation of the compass is 2 points westerly, draw *ns* 2 points, or 22° 30′, to the west of the true meridian; then *ns* will represent the direction of the magnetic meridian, and the angle NO*n* the variation of the compass. But the needle is deflected 7° 20′ to the east of the magnetic meridian; draw, therefore, N′S′ 7° 20′ to the right of *ns*; then N′O*n*=deviation of compass, and N′S′ will represent the position of the needle. At the point O, in the straight line NO, make the angle NOF = 25° 36′; then NOF will represent the true course, N. 25° 36′ E., and N′OF the corrected compass course required.



M

By the figure, N′OF = NOF + (NOn − nON′)
= 25° 36′ + (22° 30′ − 7° 20′)
= 25 36 + 15 10
= 40 46

∴ the corrected compass course = N. 40° 46′ E.

By practical rule (p. 158),

True course..................25° 36′ r. N.
Variation, 22° 30′ r.
Deviation, 7 20 l. ......15 10 r.
Compass course ....40 46 r. N. = N. 40° 46′ E.

EXAMPLE FOR PRACTICE.

240. Given lat. A......30° 15′ N. Long. A......19° 20′ E.
„ B......26 40 N. „ B......20 30 E.

variation of compass $2\frac{1}{4}$ points W.; deviation, 4° 20′ W. Construct figures, and find by calculation the true and compass course, and the distance (by middle lat. method). *Ans.* True course, S. 15° 58′ E., dist. 223·6′.
Compass course, S. 13° 40′ W.

To find latitude and longitude in (middle latitude method).

241. Sailed N. 42° 31′ E., 298·5 miles from a place A, in lat. 14° 40′ N. and long. 56° 40′ E., to another place B; required the latitude and longitude of B.

Construction.

Let AC (see fig. p. 154) represent a part of the meridian, and A the place sailed from. At the point A, in the straight line AC, make the angle CAB = 42° 31′ = the course, and let AB = 298·5 = distance, and from B drop a perpendicular BC on AC: then, in the right-angled triangle CAB, we have given the angle A and side AB; to find AC = true diff. lat., and CB = departure.

Again, let PU, PZ, be the meridians passing through the two places A and B; draw the parallel BF′, and bisect AF′ in S′, and through S′ draw SS′, an arc of a parallel; then SS′ = dep. nearly, and UZ = diff. of long. between A and B.

Calculation.

(1.) In right-angled triangle ABC,

AC = AB cos. A, ∴ true diff. lat. = dist. cos. course . . . (1)

Given dist. = 298·5	true diff. lat. ... 3° 40′ N.
course = 42° 31′	lat. from.........14 40 N.
to find true diff. lat. = 220′ = 3° 40′	∴ lat. of B ....18 20 N.
	∴ mid. lat. 16 30

(2.) Fig. p. 154, $\frac{\text{ss}'}{\text{uz}}$=cos. s′u, or $\frac{\text{dep.}}{\text{diff. long.}}$=cos. mid. lat.

∴ dep.=diff. long. cos. mid. lat. . . . . (2)

In triangle ABC, $\frac{\text{CB}}{\text{AB}}$=sin. A, or $\frac{\text{dep.}}{\text{dist.}}$=sin. course,

∴ dep.=dist. sin. course (3) ∴ equating (2) and (3),
diff. long. cos. mid. lat.=dist. sin. course,
∴ diff. long.=dist. sin. course sec. mid. lat.

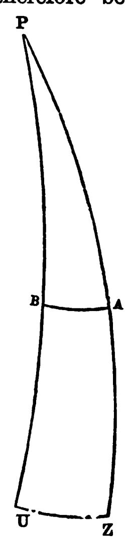
Given, dist.=298·5	∴ diff. long.=210·4= 3° 30′ E.
course=42° 31′	long. from............56 40 E.
mid. lat.=16 30	∴ long. of B............60 10 E.

EXAMPLE FOR PRACTICE.

242. Sailed from a place A, in latitude 42° 15′ N. and longitude 18° 30′ E., N. 46° 23′ E., 195·7 miles. Construct figures, and find by calculation the latitude and longitude in. *Ans.* Lat. in=44° 30′ N., long. in=21° 45′ E.

PARALLEL SAILING.

In parallel sailing the course is evidently due east or due west, and the distance is the arc of the parallel of latitude intercepted between the two places concerned. Questions in this kind of sailing must therefore be solved by means of the property in Spherical Trigonometry mentioned in p. 150, or by formula (4), p. 148, deduced from it.



A few examples will show the use of formula (4).

243. Required the course and distance from a place A, lat. 80° N. and long. 3° 50′ E., to another place B, lat. 80° N. and long. 6° 10′ W. The course is evidently due west.

To find the distance.

Let PZ, PU, represent the meridians of the two given places A and B, UZ the arc of the equator intercepted between them= diff. long. between A and B=10° 0′=600′, and AB the arc of the parallel of latitude through A and B; then AB=distance.

In the fig. $\frac{\text{AB}}{\text{UZ}}$=cos. AZ, or AB=UZ cos. AZ,

that is, dist.=diff. long. cos. lat. A; ∴ dist.=104·2 miles.

EXAMPLE FOR PRACTICE.

244. Required the course and distance from a place B, lat. 41° 30′ N. long. 30° 45′ W., to another place A, lat. 41° 30′ N. and long. 27° 45′ W.

Ans. Course due east, dist.=134·8 miles.

245. Sailed from a place A, due west 492·5 miles, to a place B; required the latitude and longitude of B. Lat. A=52° 10′ N., long. A=0° 29′ E.

To find diff. long.

In last. fig. $\frac{AB}{UZ}$=cos. AU, ∴ AB=UZ cos. AU,

or dist.=diff. long. cos. lat. A, ∴ diff. long.=dist. sec. lat. A.

Given dist.=492·5; lat. A=52° 10′ N.

∴ diff. long.=803′=13° 23′ W.
long. from .... 0 29 E.

∴ long. B=12 54 W.

and the lat. of B is evidently 52° 10′ N.

EXAMPLES FOR PRACTICE.

246. Sailed from A, due east 226·5 miles, to a place B; required the latitude and longitude of B. Lat. A=19° 20′ N., long. A=17° 30′ E.

Ans. Lat. B=19° 20′ N., long. B=21° 30′ E.

247. Two places in the same latitude north, whose difference of longitude is 900 miles, are distant from each other 600 miles; required the latitude they are in.

In last figure are given AB=600, UZ=900; to find AU the latitude.

$$\text{cos. AU}=\frac{AB}{UZ}=\frac{600}{900}=\frac{2}{3} \quad \therefore \text{latitude}=48^\circ\ 11'\ \text{N.}$$

248. Two places in the same latitude north, whose difference of longitude=150 miles, are distant from each other 130·8; required the latitude they are in. *Ans.* Lat.=29° 18′ N.

249. How many miles are there in a parallel of latitude in latitude 80°?

In last fig. are given UZ=1°=60′, and BU=80°; to find AB=length of a degree in lat 80°.

$$\frac{AB}{UZ}=\text{cos. BU, or AB}=\text{UZ . cos. BU}=60 \text{ cos. } 80^\circ \therefore \text{AB}=10{\cdot}4;$$

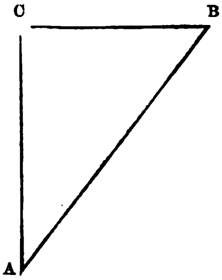
$$\therefore \text{number of miles in parallel}=360\times 10{\cdot}4=3744.$$

250. How many miles are there in a parallel of latitude in latitude 50° 48′ N.? *Ans.* 13652 miles.

Construction of the traverse table.

This table contains the true difference of latitude and departure corresponding to every course from 0° to 90°; and for every distance, from 1 nautical mile to about 300.

It is constructed as follows. A course and distance being assumed, the true difference of latitude and departure may be computed for that course and distance: thus, in the triangle ABC, right-angled at C, if the angle CAB represents a given course, and AB a given distance, the side AC will be the true difference of latitude, and CB the departure corresponding to that course and distance.



Given course=25° and distance=26 miles; compute corresponding true diff. lat. and departure.

In the triangle CAB, let A=25° and AB=26,

then true diff. lat.=AC=AB . cos. A=26 . cos. 25°, ∴ true diff. lat.=23·56′,

and dep.=BC=AB sin. A=26 sin. 25°; ∴ dep.=10·99.

When the difference of latitude and departure are computed in this manner *up* to 45°, the diff. lat. and dep. for courses *above* 45° may be found by interchanging the titles to the columns. Thus, let it be required to find the diff. lat. and dep. for course 65°, and distance 26° miles:

diff. lat. for 65°=26 cos. 65°=26 sin. 25°=*dep.* for 25°

dep. for 65°=26 sin. 65°=26 cos. 25°=*diff. lat.* for 25°.

Thus it appears that the diff. lat. and dep. for any course are the dep. and diff. lat. respectively for the *complement* of that course; this will easily explain the reason the quantities are tabulated in the following manner.

Form of traverse table.

DISTANCE TWENTY-SIX MILES.			
Course.	Diff. lat.	Dep.	Course.
1°	... ...	... ...	89°
2	... ...	... ...	88
25	23·56	10·99	65
45	... ...	... ...	45
Course.	Dep.	Diff. lat.	Course.

Application of the traverse table.

This table may be applied to a vast variety of problems that depend for their solution on the relation to the several parts of a right-angled triangle. Thus, suppose we have given the middle latitude and departure, and it is required to find the difference of longitude, we may take this quantity out of the traverse table by inspection as follows:

since diff. lat. = dist. cos. course (p. 148),
and dep. = diff. long. cos. mid. lat. (p. 148);

if, therefore, we enter the traverse table with the mid. lat. as a course, and the dep. as a diff. lat., the corresponding distance will be equal to the diff. long. required: thus,

Given the mid. lat. $=50^\circ\ 20'$ N., and the dep. $=14{\cdot}5'$; find by traverse table the diff. long. *Ans.* Diff. long. $=22{\cdot}7'$.

APPROXIMATE GREAT CIRCLE SAILING.

PROBLEM XLIV.

251. The shortest distance between two places on the surface of the earth is the arc of a great circle passing through them. If the latitudes and longitudes of the two places are known, this arc may be readily calculated by the common rule in Spherical Trigonometry for finding the third side of a spherical triangle, when the other two sides and the included angle are given (see *Trig.* Rule IX.). The practical inconvenience of sailing on a great circle arises from the necessity of continually altering the course. It is for this reason that the rules given in the preceding pages for sailing from one place to another, in which the course is constant, are usually adopted, although the distance run on such a course is not the shortest between the two places.

When the distance between the two places is considerable, as between New York and Liverpool, the following method of approximating to the shortest distance may be adopted with advantage:

(1.) Compute the shortest distance between the two places by *Trig.* Rule IX.

(2.) Take two or more convenient points on this arc, and find the latitude and longitude of those points (see following Examples).

(3.) Find the course and distance from the place of departure to the nearest point marked on the arc: from thence find the course and distance to the next point, and so on, by the common rule for finding the course and distance from one place to another (p. 152). The sum of the distances described on these several courses will not differ much from the shortest distance.

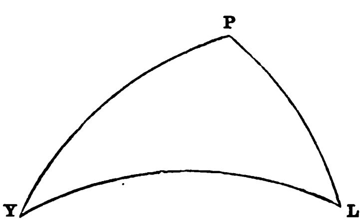
The points should not be farther apart than 1000 miles. The nearer they are taken to each other, the less will be the difference between the sum of the distances run and the shortest distance.

By proceeding in this manner as far as it is practicable, the advantage of sailing close to the arc of the great circle, and thus of shortening the distance, may be obtained without any difficulty.

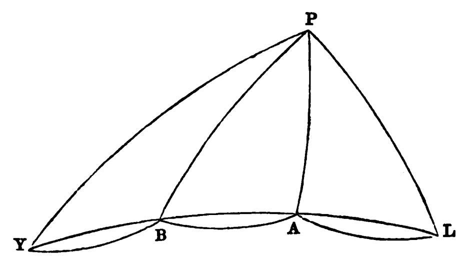
The following example, worked out at length, will explain more fully the method of proceeding :

252. Let it be required to find the shortest distance between New York (lat. 40° 42′ N., long. 73° 59′ W.) and Liverpool (lat. 53° 25′ N., long. 2° 59′ W.), also the latitudes and longitudes of certain points taken on the arc of shortest distance, also the course and distance from New York to the nearest point marked on the arc, the course and distance from thence to the second point, &c., and from the last point to Liverpool. Find also the course and distance from New York to Liverpool, in order to determine the distance saved by sailing in this manner near the arc of the great circle.

(1.) Let PY, PL (fig. 1), be the meridians of New York and Liverpool, and LY the arc of the great circle passing through the two places: then, in the spherical triangle PYL, are given PY the colatitude of Y=49° 18′, PL the colatitude of L=36° 35′, and the difference of longitude YPL=71° 0′; to calculate the arc YL, the shortest distance between Y and L (=47° 52′, or 2872 minutes, or nautical miles: see calculation, p. 168).



(fig. 1.)



(fig. 2.)

Divide the arc LY (fig. 2) into three parts at B and A, so that YB=1000 miles, BA=1000 miles, and therefore the remainder AL=872 miles.

To find the latitudes and longitudes of the points A and B, proceed as follows:

(2.) In the triangle PYL are given the three sides PL=36° 35′, PY= 49° 18′, and YL=47° 52′; to compute the angle PYL (=49° 27′: see calculation).

(3.) In triangle PYB are given PY=49° 18′, YB=1000′=16° 40′, and the angle PYB=49° 27′; to compute the arc PB (=40°), the colatitude of B; ∴ the latitude of B=50° N.

(4.) In same triangle PYB are given the three sides PY=49° 18′, PB= 40°, and YB=16° 40′; to compute the angle YPB (=19° 49′), the difference of longitude between New York and the point B; and thence the longitude of B is 54° 10′ W.

(5.) Again, in the triangle YPA are given PY=49° 18′, YA=2000′=

33° 20′, and the included angle PYL=49° 27′; to compute the arc PA, the colatitude of A (=35° 21′); ∴ the latitude of A=54° 39′ N.

(6.) In same triangle PYA are given the three side PY=49° 18′, YA=33° 20′, and PA=35° 21′; to compute the angle YPA (=46° 12′), the difference of longitude between New York and the point A; and thence the longitude of A is 27° 47′ W.

The latitude and longitude of the points B and A being thus found on the line of shortest distance, the course and distance from New York to B, thence from B to A, and lastly, from A to Liverpool, may be calculated by the common rule for finding the course and distance given in p. 152.

Calculation of the above arcs and angles.

(1.) To find arc YL (*Trigonometry*, Rule IX.).

PY......49° 18′	6·301030	304736	
PL......36 35	9·879746	24529	
12 43	9·775240	329265	
YPL....71 0	9·527908		∴ YL=47° 52′=2872 nautical miles.
	5·483924		

(2.) To find the angle PYL
(*Trig.* Rule VIII.).

PY......49° 18′	0·120254	
YL......47 52	0·129782	
1 26	4·512826	
PL......36 35	4·479940	
38 1	9·242802	
35 9		∴ PYL=49° 27′

(3.) To find arc PB
(*Trig.* Rule IX.).

PY......49° 18′	6·301030
YB......16 40	9·879746
32 38	9·457584
PYB....49 27	9·242900
	4·881260
	76090
	157861
∴ PB=40° 0′......	233951
and lat. of B=50 0 N.	

(4.) To find the angle YPB
(*Trig.* Rule VIII.).

PY......49° 18′	0·120254	
PB......40 0	0·191933	
9 18	4·351540	
YB......16 40	3·807819	
25 58	8·471546	
15 22		∴ YPB=19° 49′=diff. long.
long. of Y=73 59		
∴ long. of B=54 10 W.		

(5.) To find arc PA
(*Trig.* Rule IX.).

PY......49° 18′	6·301030
YA......33 20	9·879746
15 48	9·739976
PYL....49 27	9·242900
	5·163652
	145800
	38578
∴ PA=35° 21′...	184378
and lat. of A=54 39 N.	

(6.) To find the angle YPA (*Trigonometry*, Rule VIII.).

PY............49° 18′	0·120254	
PA............35 21	0·237644	
13 57	4·603161	
YA............33 20	4·226203	
47 17	9·187262	
19 23	∴ YPA=46° 12′=diff. long.	
	long. of Y=73 59	
	∴ long. of A=27 47 W.	

By Rule (p. 152), the course and distance from

New York to point B=N. 56° 11′ E.,	dist. 1002	miles.
From B to A=N. 73 53 E.,	„ 1006	„
From A to Liverpool=S. 85 10 E.,	„ 877	„
	2885	„
From New York to Liverpool=N. 75 10 E.,	„ 2982	„
∴ distance saved=	97	„

We thus see that the distance from New York to Liverpool, by Rule (p. 152), is 2982 miles; but the shortest distance between the two places is 2872 miles, and the sum of the distances described by the ship sailing on the three loops as above is 2885 miles; so that the distance saved by altering the course only twice is about 97 miles.

By taking a greater number of points than two on the arc YL, so as to bring the ship oftener to the line of shortest distance, the sum of the distances actually sailed will approximate nearer to the shortest distance. It will be seen, however, that even on the assumption of only altering the course once in a thousand miles, the sum of the distances run exceeds by about 12 miles only the shortest distance, and that the absolute saving between the two places, New York and Liverpool, is on this supposition nearly 100 miles.

EXAMPLE FOR PRACTICE.

253. Let it be required to find the shortest distance between Rio de Janeiro, lat. 22° 53′ S. and long. 43° 12′ W., and Java Head, lat. 6° 48′ S., and long. 105° 11′ E.; also the latitudes and longitudes of two points, A and B, taken on the arc of shortest distance so as to divide the arc into three equal parts; also the course and distance from Rio to A, from thence to B, and from B to Java Head. Find also the course and distance from Rio to the Cape, and also from the Cape to Java Head, in order to determine the difference of distances by the two methods.

Ans. The shortest distance between Rio and Java Head=137° 8′ 15″ or 8228·25 miles.

Latitude and longitude of point A, 44° 6′ S., 6° 32′ E.
„ „ B, 35 40 S., 66 28 E.

The required distances and courses are :

From Rio to A,	S. 62°	38′ E.	=2772	miles.
From A to B,	N. 79	35 E.	=2799	„
From B to Java Head,	N. 50	56 E.	=2751	„
Distance by great circle sailing			=8322	„
From Rio to Cape,	S. 77	50 E.	=3305	„
From Cape to Java Head,	N. 70	57 E.	=5090	„
Distance by common method			=8395	„

The above calculation shows us that the route by the great circle from Rio to Java Head is only about 70 miles shorter than that by the Cape. If we subdivide the arc into more than three parts, we shall be enabled to sail closer to the great circle, and thus approximate still nearer to the shortest distance: the greatest saving will not exceed 160 miles. This route, however, will not take us farther south than lat. 45° 10′,* which is generally sufficiently far to meet the westerly winds, and thereby secure the double advantage of the shortest distance with fair winds.

The above method of calculating the length of the arc of a great circle passing through two places, and of determining a certain number of convenient points thereon to which the ship may be directed so as to keep her as near to the great circle as possible, is direct and general. By dropping a perpendicular from P upon the arc passing through the two places, or the arc produced, or by means of special tables, the labour of the computation may be somewhat diminished, but with the disadvantage of sometimes introducing a distinction of cases, and thereby rendering the problem complicated. As the attempt at great circle sailing can only be of rare occurrence, and as we see that a simple application of the common rules in Spherical Trigonometry is sufficient to obtain all the essential parts of the problem, it does not seem advisable to increase the tabular part of our books of navigation by introducing special rules and tables for the purpose.

* The highest latitude reached by sailing on the arc of a great circle may be found by calculating the value of the perpendicular from the pole upon the arc. See Problems 145 and 146 in the volume of Astronomical Problems by the author.

The value of the meridional parts for any latitude may be correctly found by the following problem :

PROBLEM XLV.

To calculate the meridional parts for any latitude.

Let l=VD, lat. of any point D (fig. 2, p. 145),

M=corresponding angular measure of vd the meridional parts for l,

dl, dM, the contemporary increments of l and M, as the angular measures of DE, de, in fig.

$$\text{Now } \frac{de}{\text{DE}}=\frac{dd'}{\text{DD}'}=\frac{\text{UV}}{\text{DD}'}=\sec. l,$$

$$\text{or } \frac{d\text{M}}{dl}=\sec. l;$$

$$\therefore \text{M}=\int \sec. l dl=\int\frac{\cos. l dl}{\cos.^2 l}=\int\frac{\cos. l dl}{1-\sin.^2 l}$$

$$=\int\frac{\frac{1}{2}\cos. l dl}{1+\sin. l}-\int\frac{-\frac{1}{2}\cos. l dl}{1-\sin. l}$$

$$=\tfrac{1}{2}\log._\epsilon(1+\sin. l)-\tfrac{1}{2}\log._\epsilon(1-\sin. l)+\text{cor. } (=0)$$

$$=\log._\epsilon\sqrt{\frac{1+\sin. l}{1-\sin. l}}$$

Let $l_1=90-l$, the colatitude of l;

$$\therefore \text{M}=\log._\epsilon\sqrt{\frac{1+\cos. l_1}{1-\cos. l_1}}=\log._\epsilon\sqrt{\frac{2\cos.^2\frac{1}{2}l_1}{2\sin.^2\frac{1}{2}l_1}}=\log._\epsilon \cot. \tfrac{1}{2}l_1$$

Reducing this expression to common logarithms (by *Trig.* Part II. p. 98),

$$\text{M}=2\cdot3025851 \log._{10} \cot. \tfrac{1}{2}l_1$$

$$\text{Now M}=\frac{\text{arc}}{\text{rad.}}=\frac{vd \text{ (in min.)}}{\text{rad. (in min.)}}$$

$$=\frac{\text{mer. part for lat. } l}{57\cdot29577^\circ\times60}$$

$\therefore$ meridional parts for lat. l

$$=57\cdot29577\times60\times2\cdot3025851\times\log._{10}\cot. \tfrac{1}{2}l_1$$

and since log. $57\cdot29577\times2\cdot3025851=3\cdot8984895$

$\therefore$ log. mer. parts$=3\cdot8984895+\log\{\log._{10}\cot. \frac{1}{2}\text{colat.}-10\}$.

EXAMPLE.

Find the meridional parts for lat. 10° and lat. 60°.

	For lat. 10°.		For lat. 60°.
log. cot. ½ colat. 10	0·076187		0·571948
log. ·076187	$\bar{2}$·8818810	log. ·571948	$\bar{1}$·7573561
const. log.	3·8984895		3·8984895
log. mer. parts	2·7803705	log. mer. parts	3·6558456

$\therefore$ mer. parts for 10°=603·07 $\qquad$ $\therefore$ mer. parts for 60°=4527·37

CHAPTER VII.

SHORE OBSERVATIONS FOR DETERMINING TIME AND LONGITUDE.

Transit observations.

THE transit telescope is a meridional instrument for observing, with the assistance of a clock or chronometer, the time when a heavenly body passes the meridian. In the focus of the object-glass is placed a vertical wire, and on each side two or more wires parallel to and at equal distances from it; and the observation consists in noting the second and fractional part of a second when a heavenly body passes these wires: the middle wire is placed in the plane of the meridian. When the *error* of a chronometer or clock is to be determined by a transit of a heavenly body, the instrument should be placed accurately in the plane of the meridian, and to do this some nicety must be observed in making the usual adjustments; but when it is only required to obtain a *useful rate* for the chronometer, and the observation is confined to one particular star, any ordinary telescope furnished with a vertical wire may be made use of, and the position of the instrument may be only approximately close to the plane of the meridian.

The principal use to a seaman of transit observations is to enable him to obtain the error and rate of his chronometer; and this he may very often have an opportunity of doing, either by comparing it with the clock in a fixed observatory, or by deducing the error himself from observations made with his own portable transit. The few problems following on transit observations have reference to this latter object.

TO FIND THE ERROR OF A CHRONOMETER ON MEAN TIME AT A GIVEN PLACE BY COMPARING IT WITH A SIDEREAL CLOCK WHOSE ERROR IS KNOWN.

The clock of an observatory used for noting the transits of a heavenly body is generally regulated to show sidereal time, that is, to go 24 hours during one complete revolution of the earth, or of a fixed star about the earth; it is therefore called a sidereal clock (p. 15). This clock enables us to find sidereal time at any instant; and since we can, by Problem V. p. 27, determine the mean time corresponding to any sidereal time, we have thus the means of finding the error of a chronometer, supposed to be regulated to mean time by simply comparing it with the sidereal clock.

The practical rule deduced from Problem V. is as follows:

Compare the chronometer with the sidereal clock at some coincident beat; correct right ascension of mean sun at mean noon at Greenwich (as found in the *Nautical Almanac*) for the difference of longitude (p. 30); subtract right ascension of mean sun so corrected from sidereal time (adding 24 hours if necessary): the result will be mean time nearly at the place. Approximate to correct mean time, as pointed out in Problem V. The difference between mean time thus found and the time shown by the chronometer is the error of the chronometer on mean time at the place.

To find the error of chronometer on mean time at *Greenwich*, proceed as follows:

To the mean time at the place found as above add long. in time if west, and subtract if east; the result will be Greenwich mean time when the comparison was made, the difference between which and the time shown by the chronometer is the error of the chronometer on mean time at *Greenwich.*

254. Oct. 27, 1857, at $11^h\ 30^m$ A.M. mean time nearly, in longitude $1°\ 6'\ 3''$ W., a chronometer showed $11^h\ 32^m\ 11^s$ when a sidereal clock showed $13^h\ 52^m\ 18^s$: the error of the sidereal clock was $0^m\ 19{\cdot}28^s$ slow; required the error of the chronometer on mean time at Greenwich.

Element from *Nautical Almanac:*

Oct. 26, RA mean sun at mean noon.........	14^h	19^m	$13{\cdot}49^s$
Correction for long. $4^m\ 24^s$ W.*..............			·70
∴ RA mean sun at mean noon at place=	14	19	14·19
Sidereal clock showed	13	52	18·00
Error			19·28
∴ sidereal time=	13	52	37·28 + 24^h
RA mean sun at noon...........................	14	·19	14·19
Mean time at place nearly	23	33	23·09
Cor. for 23^h ......$3^m\ 46{\cdot}70^s$			
„ 33^m ...... 5·42			
„ 23^s ...... ·06		3	52·18
1st approx. mean time=	23	29	30·91
Cor. for 23^h ......$3^m\ 46{\cdot}70^s$			
„ 29^m...... 4·76			
„ 31^s...... ·08....................		3	51·54
2d approx. mean time=	23	29	31·55

* This correction may be found thus: $24^h : 4^m\ 24^s :: 3^m\ 56^s$ (change of RA of mean sun in 24 hours) : $x = {\cdot}7^s$; or the correction may be found in the common way by using a table, or by proportional logarithms.

Cor. for 23^h......3^m $46{\cdot}70^s$				
„ 29^m...... $4{\cdot}76$				
„ $31{\cdot}6^s$ ... $\cdot 09$....................			3^m	$51{\cdot}55^s$
∴ Correct mean time at place=	23	29	31·54	
Longitude in time		4	24·20	
Greenwich mean time, Oct. 26	23	33	55·74	
Chronometer showed	23	32	11·00	
∴ error of chronometer=		1	44·74 slow.	

NOTE. If the comparison is made a little *after* noon, the several approximations above made would be unnecessary, as may be seen in the second and third examples following.

EXAMPLES FOR PRACTICE.

255. Aug. 24, 1858, at 11^h 45^m A.M. nearly, in long. 1° 6′ 3″ W., a chronometer showed 11^h 41^m 40^s when a sidereal clock showed 9^h 43^m 4^s: the error of the sidereal clock was 7^m $46{\cdot}92^s$ slow; required the error of the chronometer on Greenwich mean time.

Element from *Nautical Almanac:* RA of mean sun at mean noon at Greenwich, Aug. 23, 10^h 5^m $57{\cdot}07^s$.

Ans. Error of chronometer, slow 3^m $43{\cdot}92^s$.

256. Aug. 24, 1858, at 0^h 16^m 19^s P.M. mean time nearly, in long. 1° 6′ 3″ W., a chronometer showed 0^h 17^m 0^s when a sidereal clock showed 10^h 18^m 30^s: the error of sidereal clock was 7^m $46{\cdot}92^s$ slow; required the error of the chronometer on Greenwich mean time.

Element from *Nautical Almanac:* RA of mean sun at mean noon at Greenwich, Aug. 24, 10^h 9^m $53{\cdot}62^s$.

Ans. Error of chronometer, slow 3^m $44{\cdot}12^s$.

257. Oct. 27, 1857, at 0^h 30^m P.M. mean time nearly, in long. 90° W., a chronometer showed 6^h 21^m 42^s when a sidereal clock showed 14^h 46^m 22^s: the error of the sidereal clock was 0^m $21{\cdot}28^s$ slow; required the error of the chronometer on Greenwich mean time.

Element from *Nautical Almanac:* RA of mean sun at mean noon at Greenwich, Oct. 27, 14^h 23^m $10{\cdot}04^s$.

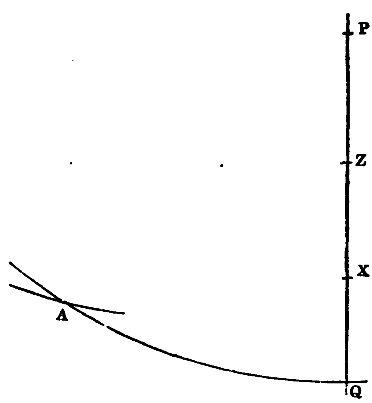
Ans. Error of chronometer, slow 0^m $48{\cdot}4^s$.

TO FIND ERROR OF SIDEREAL CLOCK BY STAR'S TRANSIT.

PROBLEM XLVI.

Given the time shown by a sidereal clock at the transit of a star; to determine the error of the clock.

Let PQ represent the celestial meridian, AQ the equator, A the first point of Aries, and X a heavenly body on the meridian. Then AQ represents sidereal time at the transit of the star; and this is evidently the same as the star's right ascension at that instant, a quantity found in the *Nautical Almanac*. Hence the difference between the star's right ascension and the time shown by the sidereal clock will be the error of the sidereal clock required.



EXAMPLES FOR PRACTICE.

258. Oct. 3, 1846, observed the transit of α Leonis over the three wires of the telescope at the following times, as noted by a sidereal clock; to determine the error of the sidereal clock, the right ascension of α Leonis by the *Nautical Almanac* being $10^h\ 0^m\ 12{\cdot}00^s$.

1st wire	9^h	58^m	$28{\cdot}0^s$
2d, or meridian wire	9	59	13·0
3d	9	59	58·2
			39·2
Time by clock	9	59	13·06
Sidereal time......................	10	0	12·00
∴ Error of sidereal clock=		0	58·94 slow.

259. Aug. 19, 1857, observed the transit of μ Sagittarii over the five wires of a transit telescope at the following times as noted by a sidereal clock; to determine the error of clock, the right ascension of μ Sagittarii by the *Nautical Almanac* being $18^h\ 5^m\ 15{\cdot}8^s$.

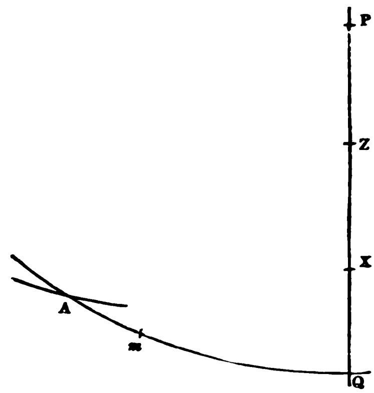
1st wire	18^h	5^m	12^s
2d „		5	31
3d „		5	50
4th „		6	9
5th „		6	28·5

Ans. Error of sidereal clock, fast $34{\cdot}3^s$.

TO FIND ERROR OF A MEAN SOLAR CLOCK OR CHRONOMETER BY STAR'S TRANSIT.

PROBLEM XLVII.

Given the time shown by a mean solar clock, or chronometer, at the transit of a star; to determine the error of the timekeeper.



Let PQ represent the celestial meridian, AQ the celestial equator, A the first point of Aries, m the mean sun, and X the place of the heavenly body on the meridian. Then the right ascension of the meridian, or sidereal time QA, is known, since it is the same as the right ascension of the star when on the meridian. The problem reduces itself, therefore, to finding mean solar time, having given sidereal time; and this can be done by means of Problem V. p. 27. The difference between mean time thus found and the time shown by the chronometer is the error of the chronometer on mean time at the place.

EXAMPLES FOR PRACTICE.

260. Oct. 3, 1846, in long. 1° 6′ W., observed the transit of Antares over the wires of the telescope at the following times, as noted by a chronometer regulated to mean time; to determine its error on mean time at the place.

Elements from *Nautical Almanac:* RA of mean sun at mean noon at Greenwich, $12^h\ 47^m\ 13{\cdot}80^s$; RA of Antares, $16^h\ 20^m\ 1{\cdot}00^s$.

Times of transit.

1st wire	3^h	25^m	$47{\cdot}5^s$
2d „	3	26	35·8
3d „	3	27	23·8
3)		19	47·1
∴ chronometer showed...	3	26	35·7

RA of Antares or sidereal time	16^h	20^m	$1{\cdot}00^s$
RA mean sun at mean noon at place...	12	47	14·50*
Mean time nearly=	3	32	46·50

* $0{\cdot}7^s$ is added to the right ascension of mean sun in *Nautical Almanac*, to adapt it to the meridian of the place; it is, in fact, equivalent to correcting the right ascension for a Greenwich date=long. of place. See Example (254).

Cor. for 3^h............$29 \cdot 57^s$			
„ 32^m............ $5 \cdot 26$			
„ $46 \cdot 5^s$ .... $0 \cdot 13$............			$34 \cdot 96^s$
Mean time, 1st approximation=3		32	11·54
Cor. for 3^h............$29 \cdot 57^s$			
„ 32^m............ $5 \cdot 26$			
„ $11 \cdot 54^s$......... $\cdot 03$............			$34 \cdot 86^s$
Correct mean time 3		32	11·64
Chronometer showed 3		26	35·70
∴ error of chronometer (slow)=		5	35·94

261. Aug. 27, 1857, in long. 1° 6′ 3″ W., observed the transit of μ Sagittarii over the wires at the following times, as noted by a mean solar clock; find the error of clock on *Greenwich* mean time.

Elements from *Nautical Almanac:* RA of mean sun at mean noon at Greenwich, $10^h\ 22^m\ 40 \cdot 24^s$; RA of star, $18^h\ 5^m\ 15 \cdot 73^s$.

1st wire ....7^h	42^m	12^s
2d „	42	32
3d „	42	51

Ans. Error, slow $3^m\ 11 \cdot 56^s$ on Greenwich mean time.

TO FIND ERROR OF SIDEREAL CLOCK BY SUN'S TRANSIT.

PROBLEM XLVIII.

Given the time shown by a sidereal clock at the transit of the sun; to determine the error of the clock.

Let PQ (fig. p. 176) represent the celestial meridian, AQ the celestial equator, A the first point of Aries, and X the place of the sun on the meridian: then AQ represents sidereal time at that instant; and this is known, since it is the same as the right ascension of the true sun X at apparent noon at the place of observation, and this can be deduced from the right ascension of the true sun, as given in the *Nautical Almanac* for apparent noon *at Greenwich.** The difference between the right ascension of true sun at apparent noon at the place thus found and the time shown by the sidereal clock will be the error of sidereal clock required.

262. Oct. 3, 1846, in long. 1° 6′ W., observed the transit of the sun's limbs over the wires at the following times, as noted by a sidereal clock; to determine the error of the clock.

* The right ascension of the sun for apparent noon at any other place whose longitude is given can be found as pointed out in *Navigation*, Part I.

Times by sidereal clock.

Western limb,	1st wire	....	12^h	31^m	$8{\cdot}0^s$
"	2d	"		32	2·5
"	3d	"		32	47·5
Eastern limb,	1st	"		33	28·5
"	2d	"		34	13·0
"	3d	"		34	58·0
			6)	18	37·5
Sidereal time by clock=			12	33	6·25

RA of sun at app. noon at Greenwich, by *Nautical Almanac*...............	12^h	36^m	$19{\cdot}06^s$
Cor. for long. 1° 6′ W.			·70+
.·. RA of sun at apparent noon at place=	12	36	19·76
Clock showed................................	12	33	6·25
.·. error of sidereal clock=		3	13·51 slow.

263. Aug. 24, 1858, at noon, in long. 1° 6′ 3″ W., observed the transit of the sun's limbs over the wires at the following times, as noted by a sidereal clock; to determine the error of sidereal clock.

Element from *Nautical Almanac:* RA of sun at app. noon at Greenwich, 10^h 12^m $6{\cdot}75^s$.

Times of transit by sidereal clock.

Western limb,	1st wire	....	10^h	2^m	$38{\cdot}0^s$
"	2d	"		2	57·0
"	3d	"		3	15·0
"	4th	"		3	33·5
"	5th	"		3	52·0
Eastern limb,	1st	"		4	49·0
"	2d	"		5	8·0
"	3d	"		5	25·5
"	4th	"		5	44·0
"	5th	"		6	2·0

Ans. Error on Greenwich mean time, slow 7^m $47{\cdot}05^s$.

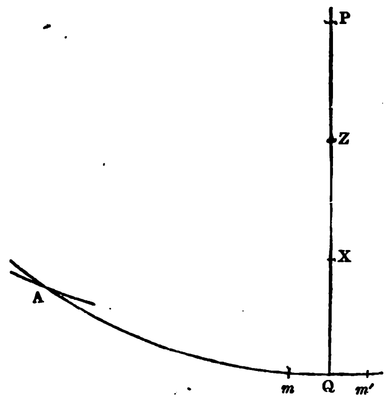
TO FIND ERROR OF MEAN SOLAR CLOCK, OR CHRONOMETER, BY SUN'S TRANSIT.

PROBLEM XLIX.

Given the time shown by a chronometer, or mean solar clock, at the transit of the sun; to determine the error of the timekeeper.

Let PQ represent the celestial meridian, AQ the celestial equator, X the place of the sun on the meridian, and m or m' the mean sun. Then apparent time at

the instant the sun is on the meridian$=0^h\ 0^m\ 0^s$, or $24^h\ 0^m\ 0^s$, and mean time$=0^h+Qm$, or $24^h-Qm'$. Now mQ or $m'Q$ represents the equation of time as given in the *Nautical Almanac*, corrected for the place of observation; by applying which to the apparent time, 0^h or 24^h (adding it to 0^h, or subtracting it from 24^h, according to the sign in the *Nautical Almanac*), it is evident that mean time is obtained for the instant the sun is on the meridian of the observer. The difference between mean time thus found and the time shown by the chronometer, is the error of the chronometer on mean time at the place.



264. May 18, 1846, in longitude 90° W., observed the transit of the sun's limbs over the wires at the following times, as noted by a chronometer regulated to mean time; to determine the error of the chronometer on mean time at the place.

May 18, at	0^h	0^m
Long. in time	6	0 W.
Greenwich, May 18	6	0

Equation of time from Nautical Almanac.

At 18	3^m	$51{\cdot}96^s$	subtractive from apparent time.
„ 19	3	49·73	
		2·23	
Cor. for 6^h		·56	
	3	51·40	=equation of time at transit.

Apparent time of transit	24^h	0^m	$0{\cdot}00^s$
Equation of time		3	51·40
Mean time at place	23	56	8·60

Times of transit.

Western limb,	1st wire	11^h	54^m	$13{\cdot}2^s$
„	2d „		55	0·0
„	3d „		55	46·6
Eastern limb,	1st „		56	27·8
„	2d „		57	14·2
„	3d „		58	1·0
			36	42·8
Mean time by clock		11	56	$7{\cdot}13+12^h$
„ at place		23	56	8·60
∴ error of chronometer=				1·47 slow.

265. August 24th, 1858, at noon, in long. 1° 6′ 3″ W., observed the transit of the sun's limbs over the wires at the following times, as noted by a chronometer; to determine the error of chronometer on Greenwich mean time.

Elements from *Nautical Almanac:* equation of time at app. noon on 24th, $2^m\ 12{\cdot}77^s$; on 25th, $1^m\ 56{\cdot}65^s$, additive to apparent time.

Times of transit by chronometer.

Western limb,	1st wire	......	0^h	1^m	$11{\cdot}0^s$
„	2d „	......		1	29·7
„	3d „	......		1	47·0
„	4th „	......		2	8·0 uncertain, from cloud.
„	5th „	......		2	24·5
Eastern limb,	1st „	......		3	21·4
„	2d „	......		3	40·7
„	3d „	......		3	58·4
„	4th „	......		4	16·0
„	5th „	......		4	34·7

Adding together first and last contact, second and last but one, &c., and taking the mean (rejecting the defective observation) for transit of center, we have

1st and last contact........................	5^m	$45{\cdot}7^s$
2d and last but one........................	5	45·7
3d „ „ two........................	5	45·4
4th „ „ three........................	5	48·7 lost.
5th „ „ four........................	5	45·9
	4)	2·7
	5	45·67
∴ transit of center by chron. =	2	52·84

Aug. 24................	0^h	0^m
Long. in time..........		4
Greenwich, Aug. 24...	0	4

Equation of time.

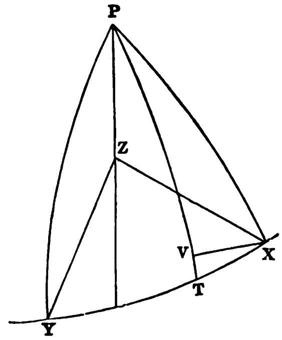
Aug. 24.........	2^m	$12{\cdot}77^s$
„	1	56·65
		16·11
Cor.		·04
	2	12·73

App. time of transit..................................	0^h	0^m	0^s
Equation of time......................................		2	12·73 +
Mean time at place....................................	0	2	12·73
Longitude in time		4	24·20
Mean time at Greenwich............................		6	36·93
Chronometer showed		2	52·84
∴ error of chron. on Greenwich mean time=		3	44·09 slow.

Investigation of the correction called the EQUATION OF EQUAL ALTITUDES.

PROBLEM L.

Given the elapsed time, and change of declination in the interval; to investigate an expression for the equation of equal altitudes.



Let P be the pole, Z the zenith, and X and Y the places of the sun at the times of the observations; therefore the zenith distances ZX and ZY are equal.

First. Suppose the polar distance to be increasing; then the polar distance PY will be greater than PX, and therefore the hour-angle ZPX is greater than the hour-angle ZPY, or greater than half the interval XPY. Make ZPT=ZPY, and call the angle XPT$=2x$, and the elapsed time, namely the angle XPY$=\epsilon$.

Then $\epsilon=$YPT+XPT$=2$ZPT$+2x$,

$\therefore \frac{1}{2}\epsilon=ZPT+x$: add x to both sides,

$\therefore \frac{1}{2}\epsilon+x=ZPT+2x=$ZPX=hour-angle from noon, or the time that must elapse (by chronometer) before the sun is on the meridian PZ.

Second. Suppose the polar distance to be decreasing, or PY less than PX, then in a similar manner it may be shown that the time to noon or the angle ZPX$=\frac{1}{2}\epsilon-x$.

The value of x in seconds of time is called the EQUATION OF EQUAL ALTITUDES.

Investigation of equation of equal altitudes.

About Z as a center, describe the arc XTY, and join TZ. Then PT=PY (for YZ=TZ, and ZPT was made equal to ZPY, and PZ is common to the two triangles). Again, about P as a center describe the arc of a parallel of decl. XV: then TV is the difference between PX and PT or PY; that is, it is the change of declination in the interval of elapsed time ϵ.

Let TV, the change of decl. in elapsed time$=d'$,

PX, the polar dist. at 1st observation$=p$,

and latitude of place$=l$.

The triangle XVT, being very small, may be considered as a right-angled plane triangle, V being the right angle.

$\therefore$ XV=TV . cot. VXT$=d'$. cot. VXT$=d'$. cot. PXZ,

and {*Trig.* Part II. p. 75} XV=XPV . sin. PX$=2x$. sin. p,

$\therefore 2x$. sin. $p=d'$. cot PXZ....................(1)

To eliminate cot. PXZ from this expression. Since

cot. PXZ . sin. ZPX=cot. PZ . sin. PX—cos. ZPX . cos. PX,*

* Cot. A . sin. B=cot. a . sin. c—cos. B . cos. c (*Trig.* Part II. p. 58).

$$\text{or cot. } PXZ \text{ . sin. } (\tfrac{1}{2}\epsilon \text{ nearly}) = \text{tan. } l \text{ . sin. } p - \text{cos. } (\tfrac{1}{2}\epsilon \text{ nearly}) \text{ . cos. } p;$$

$$\therefore \text{cot. } PXZ = \text{tan. } l \text{ . sin. } p \text{ . cosec. } \tfrac{1}{2}\epsilon - \text{cot. } \tfrac{1}{2}\epsilon \text{ . cos. } p \text{ nearly.}$$

Substituting this value of cot. PXZ in (1), we have

$$2x \text{ sin. } p = d' \text{ tan. } l \text{ . sin. } p \text{ . cosec. } \tfrac{1}{2}\epsilon - d' \text{ cot. } \tfrac{1}{2}\epsilon \text{ . cos. } p,$$

$$\text{or } 2x = d' \text{ tan. } l \text{ . cosec. } \tfrac{1}{2}\epsilon - d' \text{ cot. } \tfrac{1}{2}\epsilon \text{ . cot. } p.$$

From this formula x may be computed, and the practical rule in *Navigation*, Part I., deduced as follows:

$$\text{Since } 2x = d' \text{ . tan. } l \text{ . cosec. } \tfrac{1}{2}\epsilon - d' \text{ . cot. } \tfrac{1}{2}\epsilon \text{ . cot. } p \text{ ....(in arc)}$$

$$\therefore = \frac{d'}{15} \text{ tan. } l \text{ . cosec. } \tfrac{1}{2}\epsilon - \frac{d'}{15} \text{ cot. } \tfrac{1}{2}\epsilon \text{ . cot. } p \text{..........(in time)}$$

$$\therefore x = \tfrac{1}{30}d' \text{ . tan. } l \text{ . cosec. } \tfrac{1}{2}\epsilon - \tfrac{1}{30}d' \text{ . cot. } \tfrac{1}{2}\epsilon \text{ . cot. } p.$$

$$\text{Let } M = \tfrac{1}{30}d' \text{ . tan. } l \text{ . cosec. } \tfrac{1}{2}\epsilon$$

$$\text{,, } N = \tfrac{1}{30}d' \text{ . cot. } \tfrac{1}{2}\epsilon \text{ . cot. } p;$$

then the equat. of equal alt. $x = M - N$.

To determine the algebraic signs of M *and* N.

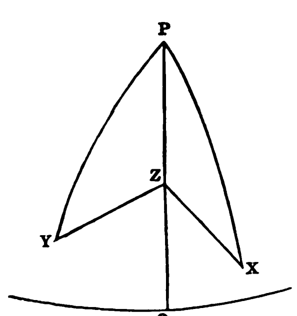
The algebraic signs of M and N will evidently depend on the magnitudes of the angles in the trigonometrical ratios which make up M and N. We will therefore now proceed to determine the algebraic signs of M and N for all conditions of the factors in M and N.

First. Let the declination be *increasing*, and of the *same name* as the latitude.

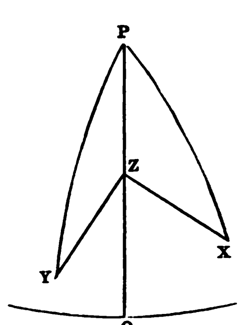
By making a figure (fig. 1) to suit this case, it will be seen that ZPX is less than half-elapsed time; $\therefore$ equation of equal alt. x must be subtracted, or

$$ZPX = \tfrac{1}{2}\epsilon - x = \tfrac{1}{2}\epsilon - M + N;$$

and since the latitude and pol. distance are both less than 90°, the proper signs of M and N are not on this account altered.



(fig. 1.)



(fig. 2.)

Second. Let the declination be *decreasing*, and of the same name as the latitude.

By making a figure (fig. 2) to suit this case, it will be seen that ZPX is greater than ZPY, and $\therefore$ the equation of equal alt. x must be added, or

$$\text{ZPX}=\tfrac{1}{2}\epsilon+x=\tfrac{1}{2}\epsilon+\text{M}-\text{N};$$

and since, as in the first case, the quantities in M and N are positive, the proper signs of M and N are not altered.

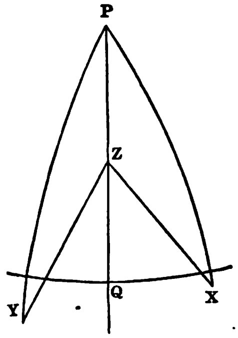
Third. Let the declination be *increasing*, and of a *different* name from the latitude.

By making a figure (fig. 1) to suit this case, it will be seen that ZPX is greater than ZPY, and ∴ the equation of equal alt. x must be added, or

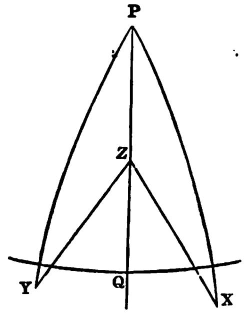
$$\text{ZPX}=\tfrac{1}{2}\epsilon+x=\tfrac{1}{2}\epsilon+\text{M}-\text{N}.$$

But the proper sign of N is negative, since one of its factors, namely cot. p, is negative, p being greater than 90° (for in this case $\text{N}=\frac{1}{30}d'\ \overset{+}{\text{cot.}}\ \tfrac{1}{2}\epsilon\ .\ \overset{-}{\text{cot.}}\ p$, or $-\text{N}=\frac{1}{30}d'$ cot. $\tfrac{1}{2}\epsilon$. cot. p),

$$\therefore\ \text{ZPX}=\tfrac{1}{2}\epsilon+\text{M}-(-\text{N})$$
$$=\tfrac{1}{2}\epsilon+\text{M}+\text{N}.$$



(fig. 1.)



(fig. 2.)

Lastly. Let the declination be *decreasing*, and of a *different* name from the latitude.

By making a figure (fig. 2) to suit this case, it will be seen that ZPX is less than ZPY, and ∴ the equation of equal alt. x must be subtracted, or

$$\text{ZPX}=\tfrac{1}{2}\epsilon-x=\tfrac{1}{2}\epsilon-\text{M}+\text{N}.$$

But, as in the last case, the proper sign of N is negative, since cot. p is negative,

$$\therefore\ \text{ZPX}=\tfrac{1}{2}\epsilon-\text{M}+(-\text{N})=\tfrac{1}{2}\epsilon-\text{M}-\text{N}.$$

By inspecting these four results, it appears that M is *positive* when decl. is decreasing and of same sign as latitude, or increasing and of a different sign.

M is *negative* when decl. is decreasing and of a different name from the latitude, or increasing, and of the same name.

N is positive when declination is increasing.

N is negative when declination is decreasing.

In the rule given in *Navigation*, Part I., the values of M and N are expressed in proportional logarithms, and a small table is used for facilitating the computation. We will now show how M and N are adapted to proportional logarithms, and also the construction of the table contained in the following page.

First. To adapt $M = \frac{1}{30} d' \tan. l . \operatorname{cosec.} \frac{1}{2}\epsilon$ to proportional logarithms:

Let δ = change of declination in 24 hours;

$$\text{then } d' : \delta :: \epsilon : 24, \therefore d' = \frac{\epsilon\delta}{24}$$

$$\therefore M = \frac{1}{30} \cdot \frac{\epsilon\delta}{24} \cdot \tan. l . \operatorname{cosec.} \tfrac{1}{2}\epsilon$$

$\therefore \log. M = -\log. 30 + \log. \epsilon - \log. 24^h + \log. \delta + \log. \tan. l + \log. \operatorname{cosec.} \frac{1}{2}\epsilon - 20$, subtracting both sides from log. 3^h,

$\log. 3^h - \log. M = \log. 30 + \log. 24^h - \log. \epsilon + \log. 3^h - \log. \delta - \log. \tan. l - \log. \operatorname{cosec.} \frac{1}{2}\epsilon + 20$

$= \log. 30 + (\log. 24^h - \log. \epsilon) + (\log. 3^h - \log. \delta) + \log. \cot. l + \log. \sin. \frac{1}{2}\epsilon - 20$

or prop. log. M = log. 30 + Gr. date log. sun for ϵ + prop. log. δ + log. cot. l + log. sin. $\frac{1}{2}\epsilon - 20$ (p. 138).

Second. To adapt N to proportional logarithms:

$$N = \tfrac{1}{30} d' \cot. \tfrac{1}{2}\epsilon . \cot. p.$$

Proceeding in a similar manner to the above, we have

prop. log. N = log. 30 + Gr. date log. sun for ϵ + prop. log. δ + log. cot. decl. + log. tan. $\frac{1}{2}\epsilon - 20$.

By these two formulæ the values of M and N may be found in proportional logarithms, and thence the equation of equal altitudes x.

The labour of finding M and N is considerably diminished, if we calculate the quantities

log. 30 + Gr. date log. of sun for ϵ + log. sin. $\frac{1}{2}\epsilon$ in M,

and log. 30 + Gr. date log. of sun for ϵ + log. tan. $\frac{1}{2}\epsilon$ in N,

for every ten minutes of elapsed time, and form the results into a table.

This Table is constructed as follows:

Calculate the values of A and B when the elapsed time $\epsilon = 5^h\ 30^m$.

A = log. 30 + Gr. date log. sun for ϵ + log. sin. $\frac{1}{2}\epsilon$

B = log. 30 + Gr. date log. sun for ϵ + log. tan. $\frac{1}{2}\epsilon$

A.		B.	
log. 30	1·47712	log. 30	1·47712
Gr. date log. sun for $5^h\ 30^m$	0·63985	. . . .	0·63985
log. sin. $2^h\ 45^m$	9·81911	log. tan. $2^h\ 45^m$	9·94299
	A = 1·93608		B = 2·05996

And in a similar manner may the values of A and B in the following table be computed for other values of the elapsed time.

VALUES OF A AND B FOR COMPUTING THE EQUATION OF EQUAL ALTITUDES.

Elapsed time.	A	B	Elapsed time.	A	B	Elapsed time.	A	B
1 30	1·97148	1·97991	4 30	1·94886	2·02901	7 30	1·90212	2·15738
1 40	1·97082	1·98123	4 40	1·94692	2·03356	7 40	1·89876	2·16854
1 50	1·97009	1·98272	4 50	1·94490	2·03833	7 50	1·89531	2·18033
2 0	1·96930	1·98435	5 0	1·94281	2·04334	8 0	1·89177	2·19280
2 10	1·96843	1·98614	5 10	1·94064	2·04861	8 10	1·88815	2·20602
2 20	1·96750	1·98808	5 20	1·93840	2·05414	8 20	1·88444	2·22003
2 30	1·96649	1·99017	5 30	1·93608	2·05996	8 30	1·88064	2·23493
2 40	1·96541	1·99243	5 40	1·93368	2·06605	8 40	1·87676	2·25081
2 50	1·96426	1·99484	5 50	1·93122	2·07246	8 50	1·87278	2·26775
3 0	1·96305	1·99743	6 0	1·92866	2·07918	9 0	1·86870	2·28587
3 10	1·96176	2·00019	6 10	1·92604	2·08624	9 10	1·86454	2·30531
3 20	1·96040	2·00312	6 20	1·92333	2·09365	9 20	1·86029	2·32623
3 30	1·95897	2·00623	6 30	1·92054	2·10143	9 30	1·85593	2·34882
3 40	1·95747	2·00954	6 40	1·91767	2·10961	9 40	1·85148	2·37334
3 50	1·95589	2·01303	6 50	1·91473	2·11821	9 50	1·84692	2·40003
4 0	1·95424	2·01671	7 0	1·91170	2·12725	10 0	1·84427	2·42928
4 10	1·95252	2·02060	7 10	1·90859	2·13678	10 10	1·83752	2·46152
4 20	1·95073	2·02470	7 20	1·90539	2·14680	10 20	1·83267	2·49733

The equation of equal altitudes x being thus found, is applied to $\frac{1}{2}\epsilon$, and thus the angle ZPX is obtained as measured by the chronometer. This being added to the time shown by the chronometer at the first observation, the result will be the time of apparent noon as shown by the chronometer.

The correct mean time of apparent noon is then to be found by applying the equation of time to 0^h or 24^h (see last problem); the result is the true time of apparent noon.

The difference between this and the time of apparent noon, as shown by the chronometer, will evidently be the error of the chronometer on mean time at the place.

The annexed blank form will facilitate the working out examples for finding the error and rate of a chronometer by equal altitudes.

FORM FOR FINDING ERROR AND RATE OF CHRONOMETER BY EQUAL ALTITUDES.

A.M. time. h m s

P.M. time. h m s

(1.)

	h m
App. time at noon (0^h or 24^h). . .	
Equation of time nearly	
Mean time at noon	
Longitude in time	
Gr. date on ( ) for eq. of time . .	
(4) Diminished by half elapsed time.	
Gr. date on ( ) for decl.	
☉'s decl. (5) . . .	
On . . .	
On . . .	
Cor.	=D
☉'s decl.	

M.

A (from table)	
Prop. log. D	
Log. cot. latitude.	
Prop. log. M	
" M*	
" N	
Equation of equal altitudes . . .	

* M is *pos.* when decl. is decreasing and of same name as lat., or increasing and of a different name; otherwise *neg.* N is *pos.* or *neg.* according as decl. is increasing or decreasing (*Rule*, p. 182).

(2.)

Equation of time.

On . .	
On . .	

N.

B (from table) .	
Prop. log. D .	
Cot. decl. . .	
Prop. log. N .	
" N .	

(3.)

A.M. time.

Sub. from P.M. + 12^h.	
Elapsed time	
$\frac{1}{2}$ elapsed time	
A.M. time + $\frac{1}{2}$ elapsed time . . .	
Approx. time of ap. noon . . .	
Equation of equal altitude . . .	
Time by chronometer at noon. .	
Ap. time at noon (0^h or 24^h) . .	
Equation of time.	
Mean time at noon	
Longitude in time	
Greenwich mean time	
Time by chronometer	
Error on Gr. mean time on ( ) .	
Error on Gr. mean time on ( ) .	
Diff. divided by number of days.	
Rate.	

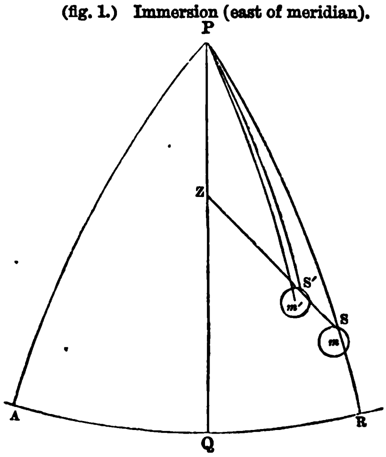
LONGITUDE BY OCCULTATION.

The longitude of a place is determined with great accuracy by noting the time of an occultation of a star or planet by the moon; the mean time at the place, at the instant of the disappearance or reappearance of the star, being supposed to be accurately known. The corresponding mean time at Greenwich can be computed by a rule deduced from the following problem.

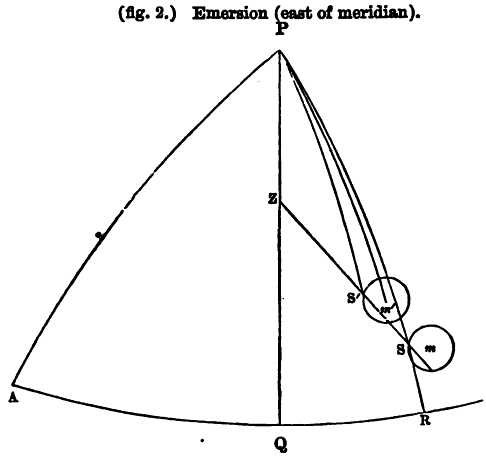
PROBLEM LI.

Given the mean time at the place at the instant of an occultation of a star by the moon; to determine the longitude of the place of observation.

(fig. 1.) Immersion (east of meridian).

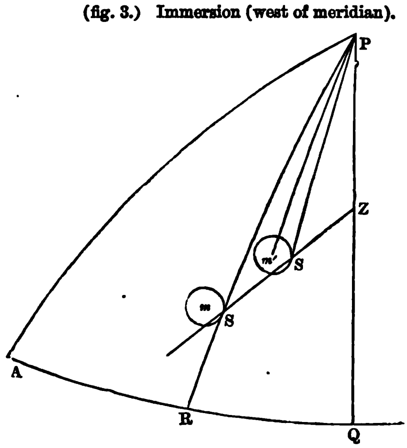


(fig. 2.) Emersion (east of meridian).

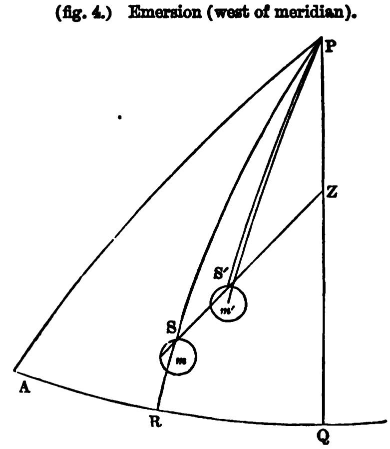


Let P be the pole, Z the reduced zenith of the spectator, and S the observed place of the point of contact of the star with the moon. Then,

(fig. 3.) Immersion (west of meridian).



(fig. 4.) Emersion (west of meridian).



since the moon is depressed by parallax, the true place of the point of contact is above S in a great circle passing through the reduced zenith Z.

Suppose S′ the true place of the point of contact, m the apparent place of the moon's center, and m' the true place.

Let AQ be the equator, and A the first point of Aries.

At the instant of the occultation, the apparent right ascension AR and decl. SR of the point of the moon's limb in contact with the star are known, since they must be the RA and decl. of the star itself; and these quantities are readily found in the *Nautical Almanac*. The object of the following investigation is to find the *right ascension of the true place of the moon's center* m', *namely the angle* A P m'; for by comparing this right ascension with the right ascensions found in the *Nautical Almanac* for two given times, the *Greenwich mean time* corresponding to the right ascension A P m' can be determined in the same manner as in the lunar.

In order to determine the angle A P m', we must investigate formulæ for computing the following quantities:

(1.) The angle SPS′ approximately.

(2.) The parallax in declination, namely PS—PS′.

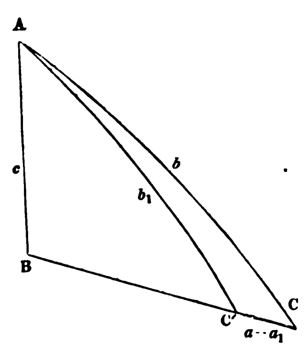
(3.) The parallax in right ascension, namely the angle SPS′, accurately.

(4.) The angle m'PS′, the semidiameter in right ascension.

(5.) The Greenwich mean time corresponding to the right ascension A P m'.

The angle APS, or arc AR, is known, since it is the right ascension of the star. It is therefore manifest that, if we can compute the two angles SPS′ and S′Pm', and apply them to the angle APS, we obtain the angle A P m', the right ascension of the moon's center at the time of the occultation.

In the figure, let A represent the pole, B the reduced zenith, C and C′ the apparent and true places of the point of contact, corresponding to S and S′ in the previous figures.



Let BAC the star's hour-angle=A,

BAC′ the angular dist. of C′ from the meridian=A′;

∴ the parallax in right ascension CAC′=A—A′

Let AB the reduced colatitude=c,

AC the polar distance of C, and therefore of the star=b,

AC′ the polar distance of C′=b',

∴ the parallax in declination=$b-b'$

Let BC=a, and BC′=a'. Then, in the two triangles BAC, BAC′, are given the colatitude c, the polar distance of star b, and the star's hour-angle A (found from the time at the place, which is supposed to be known accurately); to find the parallax in right ascension $A-A_1$, and parallax in declination $b-b_1$.

(1.) *To find* A—A′, *the* PARALLAX IN RIGHT ASCENSION, *approximately.*

Since C is the apparent place, and C the true place of the point of contact of the moon, the arc CC′, or $a-a_1$, is the diurnal parallax.

$$\therefore \sin.(a-a_1)=\sin. \text{H} . \sin. a \text{ (p. 125).}$$

In triangle ACC′, $\frac{\sin. \text{CAC}' \text{ or } \sin. (\text{A}-\text{A}_1)}{\sin. \text{C}}=\frac{\sin. \text{CC}'}{\sin. b_1}$

$$=\frac{\sin. (a-a_1)}{\sin. b_1}=\frac{\sin. \text{H} . \sin. a}{\sin. b_1}$$

$$\therefore \sin. (\text{A}-\text{A}_1)=\frac{\sin. \text{H} . \sin. a . \sin. \text{C}}{\sin. b_1} \quad . . . (1)$$

In triangle ABC, $\frac{\sin. \text{C}}{\sin. \text{A}}=\frac{\sin. c}{\sin. a} \therefore \sin. \text{C}=\frac{\sin. c . \sin. \text{A}}{\sin. a}$

Substituting this value of sin. C in (1), we have

$$\sin. (\text{A}-\text{A}_1)=\frac{\sin. \text{H} . \sin. c . \sin. \text{A}}{\sin. b_1} \quad . . . . (a)$$

From this expression the value of A—A′ cannot be found exactly, since we do not know b_1, the polar distance of the true point of contact; but if we use b, the star's polar distance, for b_1, we shall get an approximate value of A—A′, and therefore of A′, since A is already known. This value of A′ will be required, and will be sufficiently correct to enable us to calculate the parallax in declination $b-b_1$, as follows.

(2.) *To find the* PARALLAX IN DECLINATION $b-b_1$.

In triangle ABC′, $\cot. b_1 . \sin. c=\cot. \text{B} . \sin. \text{A}'+\cos. c . \cos. \text{A}'$

" ABC, $\cot. b . \sin. c=\cot. \text{B} . \sin. \text{A} +\cos. c . \cos. \text{A}.$

Multiply the first equation by sin. A, and the second by sin. A′, and subtracting the results, we have

$$\cot. b_1 . \sin. c . \sin. \text{A}=\cot. \text{B} . \sin. \text{A}' . \sin. \text{A}+\cos. c . \cos. \text{A}' . \sin. \text{A}$$
$$\cot. b . \sin. c . \sin. \text{A}'=\cot. \text{B} . \sin. \text{A} . \sin. \text{A}'+\cos. c . \cos. \text{A} . \sin. \text{A}'$$
$$(\cot. b_1 . \sin. \text{A}-\cot. \text{B} . \sin. \text{A}') . \sin. c=\cos. c . (\cos. \text{A}' . \sin. \text{A}-\cos. \text{A} . \sin. \text{A}')$$
$$=\cos. c . \sin. (\text{A}-\text{A}')$$
$$\therefore \cot. b_1 . \sin. \text{A}-\cot. b . \sin. \text{A}'=\frac{\cos. c . \sin. (\text{A}-\text{A}')}{\sin. c}$$
$$\text{or } \cot. b_1-\frac{\cot. b . \sin. \text{A}'}{\sin. \text{A}}=\frac{\cos. c . \sin. (\text{A}-\text{A}_1)}{\sin. c . \sin. \text{A}}$$

By (α) $\sin. (\text{A}-\text{A}')=\frac{\sin. \text{H} . \sin. c . \sin. \text{A}}{\sin. b_1}$

Substituting this value of sin. (A—A′), we have

$$\cot. b_1-\frac{\cot. b . \sin. \text{A}'}{\sin. \text{A}}=\frac{\sin. \text{H} . \cos. c}{\sin. b_1}$$

Subtracting cot. b from each side,

$$\text{cot. } b_1 - \text{cot. } b = \frac{\text{sin. H . cos. } c}{\text{sin. } b_1} - \text{cot. } b + \frac{\text{cot. } b \text{ . sin. A}'}{\text{sin. A}}$$

$$= \frac{\text{sin. H . cos. } c}{\text{sin. } b_1} - \frac{\text{sin. A} - \text{sin. A}'}{\text{sin. A}} \text{ . cot. } b.$$

But $\text{cot. } b_1 - \text{cot. } b = \frac{\text{sin. } (b - b_1)}{\text{sin. } b \text{ . sin. } b_1}$ (*Trig.* Example 180),

and $\text{sin. A} - \text{sin. A}' = 2 \text{ cos. } \frac{1}{2}(\text{A} + \text{A}') \text{ sin. } \frac{1}{2}(\text{A} - \text{A}')$ (*Trig.* p. 31).

Making these substitutions, we have

$$\frac{\text{sin. } (b - b_1)}{\text{sin. } b \text{ . sin. } b_1} = \frac{\text{sin. H . cos. } c}{\text{sin. } b_1} - \frac{2 \text{ cos. } \frac{1}{2}(\text{A} + \text{A}') \text{ . sin. } \frac{1}{2}(\text{A} - \text{A}')}{\text{sin. A}} \cdot \frac{\text{cos. } b}{\text{sin. } b}$$

$$\therefore \text{sin. } (b - b_1) = \text{sin. H . cos. } c \text{ . sin. } b - \frac{2 \text{ cos. } \frac{1}{2}(\text{A} + \text{A}') \text{ . sin. } \frac{1}{2}(\text{A} - \text{A}') \text{ . cos. } b \text{ . sin. } b_1}{\text{sin. A}}$$

and $2 \text{ sin. } \frac{1}{2}(\text{A} - \text{A}') = \text{A} - \text{A}'$ nearly (since $\text{A} - \text{A}'$ is small)

$$= \text{sin. } (\text{A} - \text{A}') = \frac{\text{sin. H . sin. } c \text{ . sin. A}}{\text{sin. } b_1} \text{ by } (\alpha)$$

$$\therefore \text{sin. } (b - b_1 = \text{sin. H . cos. } c \text{ . sin. } b - \text{sin. H . cos. } \tfrac{1}{2}(\text{A} + \text{A}') \text{ . cos. } b \text{ . sin. } c \ldots (\beta)$$

Although we do not know A′ exactly, we have an approximate value of A′ in (α) (p. 189) sufficiently near to give the value of $b - b_1$ without any sensible error.

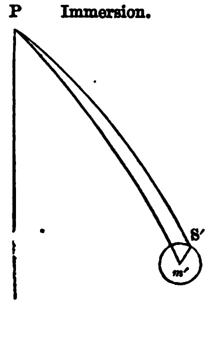
(3.) *To find the* PARALLAX IN RIGHT ASCENSION $\text{A} - \text{A}_1$.

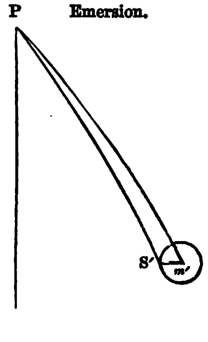
As neither of the formulæ (α) and (β) gives a direct value of the quantity sought, we must begin with the one most likely to obtain the nearest approximation to the truth. We shall find it better to calculate a near value of A′ by (α) *using b for b_1:* we shall then be able to find $\frac{1}{2}(\text{A} + \text{A}')$, a quantity that occurs in (β), sufficiently correct to compute $b - b_1$, and thence b_1.

We can then compute $\text{A} - \text{A}'$ by means of formula (α) with the correct value of b_1 thus obtained; the result will be the parallax in right ascension $\text{A} - \text{A}'$ required.

(4.) *To find the* SEMIDIAMETER IN RIGHT ASCENSION, *namely the angle m′*PS′.

Let S′ be the true point of contact, m' the true place of moon's center, and P the pole; then m'PS′ is the moon's semidiameter in right ascension,



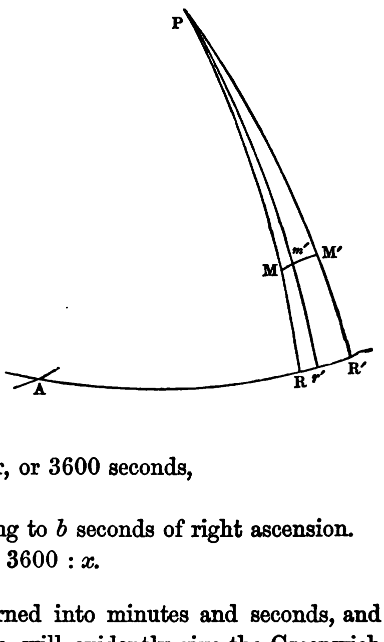


which may be computed by considering m'PS$'$ as a spherical triangle, whose three sides are given, namely,

PS$'$=polar distance b_1 of true point of contact, found by (β),
m$'$S$'$=moon's horizontal semi. for the Greenwich date,
Pm'=polar distance of moon, taken from the *Nautical Almanac* for a Greenwich date not differing much from Greenwich mean time.

The values of A—A$'$=SPS$'$; and m'PS$'$, found as above, being applied with the proper sign to the angle APS, the right ascension of the star, the result will be the angle APm', or the true right ascension of the moon's center at the instant of the observation.

(5.) *To find the instant at Greenwich corresponding to the right ascension of the moon's center* APm'.



Let M be the place of the moon's center at the hour of the Greenwich date, M$'$ the place one hour afterwards, and m' the place of the moon's center when the observation was taken, AR$'$ the celestial equator, and A the first point of Aries. Then RR$'$ is the change of moon's right ascension in 1 hour, or 3600 seconds, and Rr' the difference of right ascension between the calculated right ascension of Ar' of moon's center and AR the right ascension for the hour of Greenwich date, taken from the *Nautical Almanac.*

Let a=RR$'$=change of RA in 1 hour, or 3600 seconds,
b=Rr',
and x=seconds in time, corresponding to b seconds of right ascension.

$$\text{Then } a : b :: 3600 : x.$$

The value of x thus found, and turned into minutes and seconds, and added to the hour of the Greenwich date, will evidently give the Greenwich mean time when the moon's right ascension was Ar', that is, the Greenwich time at the instant of the observation. The difference between which and the mean time at the place, found in the usual manner (as in the lunar or chronometer), is the *longitude in time.*

From the formulæ investigated above may be deduced a direct and practical rule for finding mean time at Greenwich at the instant of the observation. To do this, the formulæ must be reduced to logarithms, and the several parts arranged in the manner pointed out in the following investigation.

PRACTICAL RULE FOR FINDING THE LONGITUDE BY AN OCCULTATION OF A FIXED STAR.

Reduction of Formulæ.

Since in $\sin.\ (A-A')=\dfrac{\sin.\ H\ .\ \sin.\ c\ .\ \sin.\ A}{\sin.\ b_1}$ . . . . (α)

and $\sin.\ (b-b_1)=\sin.\ H\ .\ \cos.\ c\ .\ \sin.\ b-\sin.\ H\ .\ \cos.\ \frac{1}{2}(A+A')\ .\ \cos.\ b\ .\ \sin.\ c$ (β) the angles $A-A'$, H, and $b-b_1$ are small; the circular measures of these angles may be substituted for their sines, that is, $\dfrac{\text{arc}\ (A-A')}{\text{rad.}}$ for $\sin.\ (A-A')$, $\dfrac{\text{arc}\ H}{\text{rad.}}$ for sin. H, and $\dfrac{\text{arc}\ (b-b_1)}{\text{rad.}}$ for $\sin.\ (b-b_1)$.

Making these substitutions, the formulæ become

$$b-b_1=H\ .\ \cos.\ c\ .\ \sin.\ b-H\ .\ \cos.\ b\ .\ \sin.\ c\ .\ \cos.\ \tfrac{1}{2}(A+A'),$$

and $A-A'=H.\ \sin.\ c\ .\ \sin.\ A\ .\ \text{cosec.}\ b_1$ . . . . (in arc)

or $A-A'=\frac{1}{15}\ H\ .\ \sin.\ c\ .\ \sin.\ A\ .\ \text{cosec.}\ b_1$ . . . . (in time)

Before we can compute $b-b_1$, we must know $\frac{1}{2}(A+A')$.

Now $\frac{1}{2}(A+A')=A-\frac{1}{2}(A-A')=A-\frac{1}{30}H\ .\ \sin.\ c\ .\ \sin.\ A\ .\ \text{cosec.}\ b_1$ . . (γ)

This expression would determine $\frac{1}{2}(A+A')$, supposing we knew cosec. b_1 (the cosec. of polar distance of the *true* place of point of contact); but since b_1 is not yet found, we can use cosec. b or sec. star's decl. instead (for $b=b_1$ very nearly, and therefore this may be done without any practical error).

Assuming at present that cosec. b_1=cosec. b=sec. star's decl., we have

$\frac{1}{2}(A-A')=\frac{1}{30}\ H\ .\ \sin.\ c\ .\ \sin.\ A\ .$ sec. star's decl.

$=\frac{1}{30}$. red. hor. par. × cos. red. lat. × sin. star's hour-angle × sec. star's decl.

To reduce this formula to logarithms, we shall find it convenient to add together the logarithms that remain unaltered (namely all the quantities excepting sec. star's decl.), as they will be required hereafter, when $A-A'$ is to be found accurately.

Let ∴ $C=\frac{1}{30}\ H$. cos. lat. sin. hour-angle,

∴ $\frac{1}{2}(A-A')=C$. sec. star's decl. : reducing to logarithms,

log. C=log. H+log. cos. lat.+log. sin. hour-angle—log. 30—20

=log. H+log. cos. lat.+log. sin. hour-angle+ar. co. log. 30—30

=log. H+log. cos. lat.+log. sin. hour-angle+8·522879—30.

log. $\frac{1}{2}(A-A')$=log. C+log. sec. star's decl.—10,

and $\frac{1}{2}(A+A')=A-\frac{1}{2}(A-A')$.

Hence this practical rule for finding $\frac{1}{2}(A+A')$:

Rule (*a*), under head (1). (See Example, p. 197.)

Add together log. red. hor. par. (in seconds),

log. cos. reduced lat.,

log. sin. star's hour-angle,

and constant log. 8·522879;

the result, rejecting the tens in the index, will be log. C.

To log. C add log. sec. star's decl.; the sum, rejecting the tens in index, will be log. $\frac{1}{2}(A-A')$ in seconds, the nat. number corresponding to which will be $\frac{1}{2}(A-A')$ in seconds. Turn this into minutes and seconds, and subtract it from the star's hour-angle A; the result will be $\frac{1}{2}(A+A')$ nearly.

This approximate value of $\frac{1}{2}(A+A')$ will be sufficiently correct to enable us to compute $b-b_1$, the parallax in declination.

Reduction of $b-b_1$ (p. 192), the parallax in declination, to logarithms.

$$b-b_1=H\ .\ \cos.\ c\ .\ \sin.\ b-H\ .\ \cos.\ b\ .\ \sin.\ c\ .\ \cos.\ \tfrac{1}{2}(A+A').$$

Let $M=H\ .\ \cos.\ c\ .\ \sin.\ b=H\ .$ sin. red. lat. cos. star's decl.

,, $N=H\ .\ \cos.\ b\ .\ \sin.\ c\ .\ \cos.\ \frac{1}{2}(A+A')$

$=H$. cos. red. lat. sin. star's decl. cos. $\frac{1}{2}(A+A')$.

In logarithms,

log. M=log. red. hor. par. + log. sin. red. lat. + log. cos. star's decl. − 20.

,, N=log. red. hor. par. + log. cos. red. lat. + log. sin. star's. decl.

+ log. cos. $\frac{1}{2}(A+A')-30$.

Hence this practical rule for finding the parallax in declination:

Rule (b). Under heads (2) and (3) (see Example, p. 198), put down the following quantities:

Under (2) and (3) } ......log. red. horizontal parallax (in seconds).

,, (2).........log. sin. reduced lat.

,, (3).........log. cos. reduced lat.

,, (2).........log. cos. star's decl.

,, (3).........log. sin. star's decl.

,, (3).........log. cos. $\frac{1}{2}(A+A')$.

Add together the logarithms under (2) and (3); the natural numbers corresponding to which, turned into minutes and seconds, will be the values of M and N, the two parts of the parallax in declination.

To determine the algebraic signs of M *and* N.

The algebraic sign of M is always positive, since the trigonometrical factors which make up M are all positive. In the value of N $\{=H\ .\ \cos.\ b\ .\ \sin.\ c\ .\ \cos.\ \frac{1}{2}(A+A')\}$ it may be seen that two of the factors, namely the polar dist. b and $\frac{1}{2}(A+A')$, may be greater or less than 90°, since b is reckoned from the elevated pole. If they are both greater, or both less than 90°, their signs will not affect the negative value of N. (This will be evident by putting the signs + and − over the quantities in N, according to the Rule in *Trig.* Part I. p. 31.) But if one be greater and the other less than 90°, the value of N will be rendered positive.

Hence this practical rule for determining the signs of the two parts of the parallax in declination.

Rule (c). If the polar distance and $\frac{1}{2}(A+A')$, or rather the hour-angle itself of the star, be one greater and one less than 90°, the second part N must be added to the first part M of the parallax in declination; but if the polar distance and hour-angle be both greater or both less than 90°, the second part N must be subtracted from the first part M. The result will be the parallax in declination $(b-b_1)$. Apply this to the star's declination, so as to diminish its polar distance (b), and we obtain the true declination (namely the complement of b_1) of the point of the moon's limb observed, subject to the small error arising from using in the calculation of $\frac{1}{2}(A+A')$, p. 193, the secant of star's declination instead of the secant of true declination of the point of contact. This error, however, will be always very small, and, if necessary, could be entirely removed by recomputing $\frac{1}{2}(A+A')$ with the approximate value of b_1 just found.

Reduction of $A-A'$, *the parallax in right ascension to logarithms.*

With the value of b_1, determined as above, we may now find the exact value of $A-A'$, the parallax in right ascension.

For $\frac{1}{2}(A-A')=\frac{1}{30}\,.\,H\,.\,\sin.\,c\,.\,\sin.\,A\,.\,\text{cosec.}\,b_1$ (p. 192),

$=C\,.\,\text{cosec.}\,b_1$ (see p. 192, where log. C is already found),

and b_1=true pol. dist. of point of moon's limb in contact with the star found above;

$\therefore\ A-A'=2C\,.\,\text{cosec.}\,b_1;$

$\therefore$ log. $(A-A')$=log. C+log. sec. decl.+·301030−10.

Hence this practical rule to determine the parallax in right ascension:

Rule (d). (See Example, p. 198.)

Under head (4) put down log. C already found, log. secant of the true decl. of point observed, and constant log. ·301030. The sum (rejecting 10 in index) will be the log. parallax in right ascension $(=A-A')$.

When the star is west of meridian, *add* the parallax in right ascension $(A-A')$ to the star's right ascension.

When east of meridian, *subtract.*

The result will be the true right ascension of the point of the moon's limb in observed contact with the star.

See figures p. 187, where it is evident that to find the angle APS′ we must subtract SPS′ $(=A-A')$ from APS when east of meridian, and add it when west, to obtain the angle APS′.

To calculate the moon's semidiameter in right ascension m'PS′.

Let moon's hor. semi. for. Gr. date=m'S′ (fig. p. 190),

$$PS'=90\mp\delta,\quad Pm'=90\mp D$$

(− or + according as the decl. and lat. are the same or different names) where δ=decl. of true point of contact,

D=decl. of moon's center, taken from *Nautical Almanac* for the Gr. date.

Then, considering $\text{P}m'\text{S}'$ a spherical triangle, we have (*Trig.* Part II. p. 62),

$$\sin.^2 \tfrac{1}{2}m'\text{PS}' = \text{cosec.PS}' . \text{cosec P}m' . \sin. \tfrac{1}{2}(m'\text{S}' + \overline{\text{P}m' \backsim \text{PS}}) \sin. \tfrac{1}{2}(m\text{S}' - \overline{\text{P}m' \backsim \text{PS}'})$$

But the quantities $m'\text{PS}'$, $m'\text{S}' + \overline{\text{P}m' \backsim \text{PS}'}$, and $m'\text{S}' - \overline{\text{P}m' \backsim \text{PS}'}$ are very small, and $\therefore \sin. m'\text{PS} = \dfrac{\text{arc } m'\text{PS}'}{\text{rad.}}$, &c.

Making these substitutions, and reducing, we have

$$\frac{(m'\text{PS})^2}{4} = \text{cosec. PS}' . \text{cosec. P}m' . \frac{m'\text{S} + \overline{\text{P}m' \backsim \text{PS}'}}{2} . \frac{m'\text{S}' - \overline{\text{P}m' \backsim \text{PS}'}}{2}$$

$$\text{or } (m'\text{PS}')^2 = \text{sec. } \delta . \text{sec. D} . (m'\text{S}' + \overline{\text{P}m' \backsim \text{PS}'}) . (m'\text{S}' - \overline{\text{P}m' \backsim \text{PS}'}) \text{ (in arc)}$$

$$\text{and } \therefore (m'\text{PS}')^2 = \frac{1}{15^2} . \text{sec. } \delta . \text{sec. D} . (m'\text{S}' + \overline{\text{P}m' \backsim \text{PS}'}) . (m'\text{S}' - \overline{\text{P}m' \backsim \text{PS}'}) \text{(in time)}$$

$$\therefore 2 \log. m'\text{PS}' = \log. \sec. \delta + \log. \sec. \text{D} + \log. (m'\text{S}' + \overline{\text{P}m' \backsim \text{PS}'})$$
$$+ \log. (m'\text{S}' - \overline{\text{P}m' \backsim \text{PS}'}) + \text{ar. co. log. } 15^2 \ (=7{\cdot}647818) - 30.$$

Hence this practical rule for finding semidiameter in right ascension $m'\text{PS}'$: Rule (*e*). (See Example, p. 199.)

Under the declination (δ) of true point of contact put the declination D of moon for Greenwich date; take the difference, which bring into seconds, and under it put moon's horizontal semidiameter in seconds. Take the sum and difference.

To the log. secants of the first two terms in this form add the logs. of the two last terms, and also the constant log. 7·647818. Half the sum will be the log. of the moon's semidiameter in right ascension $m'\text{PS}'$ in seconds.

When an immersion is observed, subtract the semi. in right ascension from the right ascension of true point of contact APS'; when an emersion is observed, add it thereto. The result will be the true right ascension of the moon's center APM' at the time of the observation.

See diagrams in p. 187, where it is evident that an emersion takes place to the east of the moon's center, and therefore at a greater distance from the first point of Aries; and the contrary at an emersion.

To calculate the time at Greenwich corresponding to the moon's right ascension $\text{AP}m'$ just found.

If a=change of moon's right ascension in 1^{h}, or 3600 seconds,

b=difference of right ascension between the calculated right ascension $\text{AP}m'$ and the one for hour of Greenwich date, taken out of the *Nautical Almanac*.

and x=seconds of time corresponding to b seconds in RA, it has been shown (p. 191) that $a : b :: 3600 : x$,

$$\therefore \log. x = \log. b + \log. 3600 - \log. a.$$

Hence this practical rule for finding x, the number of seconds that

must be added to the hour of the Greenwich date to get the Greenwich mean time at the instant of the observation:

Rule (*f*). From the true right ascension of moon's center, found by Rule (*e*), subtract the right ascension for the hour of the Greenwich date, and thus get *b*.

To the log. difference of right ascension in seconds (*b*) add constant log. 3·556302 (the log of 3600), and from the sum subtract the log. of the hourly change (in seconds) in right ascension. The result will be the log. of a number of seconds (*x*) in time, which take from the tables and, turned into minutes and seconds, add to the hour of the Greenwich date.

The result will be the GREENWICH MEAN TIME* at the instant of the occultation.

To find MEAN TIME *at the place.*

This is found in the usual manner from an altitude of a heavenly body, as in the chronometer or lunar observation.

Then the difference between Greenwich mean time and the time at the place will be the *longitude in time.*

EXAMPLE (AN IMMERSION, EAST OF MERIDIAN).

266. Aug. 25, 1839, at $8^h\ 14^m\ 34{\cdot}6^s$ mean time at the Royal Naval College, in lat. 50° 48′ N. and longitude in time $4^m\ 24{\cdot}2^s$ W., observed the immersion of φ Aquarii.

Before the rule can be applied, we should take out of the *Nautical Almanac* the following quantities and correct them for a Greenwich date: namely (1.) Moon's semi.; (2.) Moon's hor. parallax (corrected also for spheroidal fig. of earth, p. 129); (3.) Moon's declination; (4.) Right ascension of mean sun; and (5.) Star's right ascension and declination. The lat. must also be corrected for spheroidal fig. of earth, p. 123; and (6.) The moon's right ascension must be taken out for the hour of Greenwich date, and for the hour following, in order to get the horary motion in right ascension (see working form, p. 204).

R. N. College, Aug. 25	8^h	14^m	$34{\cdot}6^s$
Long. in time		4	24·2 W.
Greenwich, Aug. 25	8	18	58·8

* If the Greenwich time differs much from the Greenwich date, it may be necessary to recompute some part of the work, especially the declination of the moon's center, which is an element required to be found with great accuracy, using the Greenwich time last found as a new Greenwich date.

	(1.) Moon's semi.		(2.) Moon's hor. par.		Latitude.
Noon............	16 10·6″	....	59 21·9″		50 48 0″ N.
Midnight	16 14·2	....	59 35·2	Reduction ..	11 0
	3·6		13·3	Red. lat. ...	50 37 0 N.
Cor....	2·5	Cor...	9·2		
	16 13·1		59 31·1		
	60	Reduction...	7·0—(p. 129)		
∴ hor. semi.=	973·1		59 24·1		
			60		
		∴ red. hor. par.=	3564·1		

	(3.) Moon's decl.		(6.) Moon's RA.
8^h	6° 21′ 35·2″ S....		23^h 2^m $5{\cdot}47^s$
9^h	4 44·2 S....		23 4 13·13
	16 51·0		2 7·66
Cor. ...	5 19·7		60
	6 16 15·5 S.		127·66

(4.) RA mean sun.	
Aug. 25.........	10^h 12^m $15{\cdot}93^s$
Cor.	1 21·97
RA mean sun...	10 13 37·90

(5.) Star's RA and decl.	
Star's RA	23^h 6^m $2{\cdot}68^s$
Star's decl. ...	6 54 33·8 S.

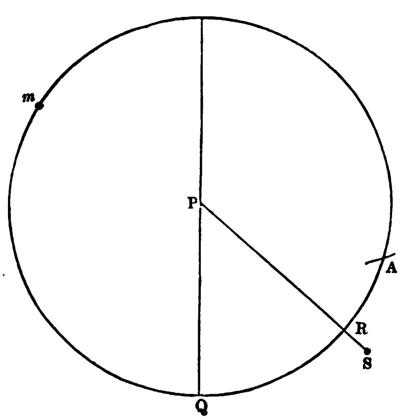
To find star's hour-angle A.

Mean time at place.........	8^h	14^m	$34{\cdot}60^s$	$=\text{Q}m$
RA mean sun	10	13	37·90	$=\text{A}m$
RA meridian (+24).........	18	28	12·50	$=\text{A}m\text{Q}$
Star's RA	23	6	2·68	$=\text{AQR}$
	19	22	9·82	$=\text{R}m\text{Q}$
	24			
∴ star's hour-angle=	4	37	50·18	$=\text{RPQ}$

Rule (*a*). To compute $\frac{1}{2}(\text{A}+\text{A}')$.

(1.)

Log. red. HP............	3·551950
„ cos. red. lat.......	9·802435
„ sin. hour-angle...	9·971469
„ const. log.	8·522879
log. c=	1·848733
„ sec. star's decl....	0·003165
„ $\frac{1}{2}(\text{A}-\text{A}')$	1·851898



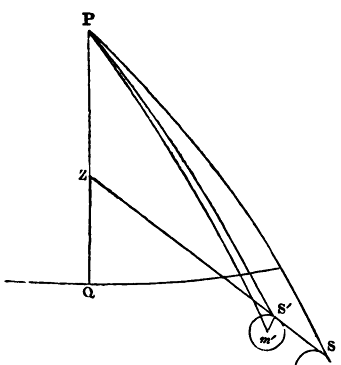
$\therefore \frac{1}{2}(A-A')=71{\cdot}10$

or $\frac{1}{2}A-\frac{1}{2}A'=$	1^m	$11{\cdot}10^s$
and $A=4$	37	$50{\cdot}18$
$\therefore \frac{1}{2}(A+A')=4$	36	$39{\cdot}08$ nearly.

Rule (*b*). To compute the parallax in declination $M-N$.

(2.)		(3.)	
Log. red. HP	3·551950	Log. red. HP	3·551950
„ sin. red. lat.	9·888133	„ cos. red. lat.	9·802435
„ cos. star's decl.	9·996835	„ sin. star's decl.	9·080198
„ M	3·436918	„ cos. $\frac{1}{2}(A+A')$	9·551107
$\therefore M=2734{\cdot}7$		„ N	1·985690
			$\therefore N=96{\cdot}8$

Diagram for this Example.



Rule (*c*). To determine the signs of M and N.

$$M=\overset{+}{H}\ .\ \overset{+}{\text{sin. red. lat.}}\ .\ \overset{+}{\text{sin. pol. dist.}}$$

$$-N=\overset{+}{-H}\ .\ \overset{+}{\text{cos. red. lat.}}\ .\ \overset{-}{\text{cos. pol. dist.}}\ .\ \overset{+}{\cos.\tfrac{1}{2}(A+A')}.$$

Putting the proper signs over the factors (*Trig.* Part. I. p. 31), we see that M is +, and $-$ N is + ;

$\therefore M=2734{\cdot}7+$

$-N=\ \ 96{\cdot}8+$

$\therefore M-N=2831{\cdot}5$

or par. in decl.=	47′	11·5″=PS−PS′	
star's decl.=6	54	33·8 S.=PS−90	
∴ true decl. of point of moon's limb observed =6	7	22·3 S.=PS′−90=δ	

Rule (*d*). To compute angle $SPS'=A-A'$, the parallax in right ascension, and thence RA of true point of contact S′.

(4.)

Log. C	1·848733
„ sec. (PS′−90°)	0·002484
	0·301030
	2·152247

$\therefore A-A'=$		$141{\cdot}98^s$
or $A-A'=$	2	21·98
but RA of star S=23	6	2·68
∴ RA of S′=23	3	40·70

Rule (*e*). To calculate the angle m'PS$'$, the semidiameter in right ascension, and thence RA of moon's center.

PS$'-90°$	6°	7′	22·3″ S.		
☾ decl.	6	16	15·5 S.		
		8	53·2		
		60			
		533·2		sec. PS$'-90°$	0·002484
☾ hor. semi.		973·1		sec. ☾ decl.	0·002606
		1506·3			3·177911
		439·9			2·643354
					7·647818
					2)3·474173
					1·737086

∴ semi. in RA= 54·58^s=m'PS$'$

RA of S$'$=23^h 3^m 40·70

∴ RA of moon's center m' at time of observation } =23 2 46·12

Rule (*f*). To calculate the corresponding time at Greenwich, when the moon's RA=23^h 2^m 46·12^s.

RA at observation	23^h	2^m	46·12^s
RA on Aug. 25, at 8^h	23	2	5·47
			40·65

Log. 40·65	1·609065
„ const. log.	3·556302
	5·165367
„ 127·66	2·106055
„ x	3·059312

∴ x= 1146·4=0 19^m 6·4^s

	8		
Greenwich mean time, Aug. 25	8	19	6·4
R. N. Coll. mean time, Aug. 25	8	14	34·6
Long. in time=		4	31·8 W.

EXAMPLE (AN EMERSION, EAST OF MERIDIAN).

267. Aug. 25, 1839, at 9^h 16^m 42·6^s mean time at the Royal Naval College, in latitude 50° 48′ N. and long. in time 4^m 24·2^s W., observed the emersion of φ Aquarii.

Before the rule can be applied, we must take out of the *Nautical*

Almanac the following quantities for a Greenwich date, namely (1.) Moon's semi.; (2.) Moon's hor. parallax (corrected for spheroidal figure of earth, p. 129); (3.) Moon's declination; (4.) Right ascension of mean sun; and (5.) Star's right ascension and decl. The latitude must also be corrected for spheroidal figure of earth (p. 123); and (6.) The moon's right ascension must be taken out for the hour of Greenwich date, and for the hour following (see working form, p. 204).

R. N. Coll. Aug. 25	9^h	16^m	$42{\cdot}6^s$
Long in time		4	24·2 W.
Greenwich, Aug. 25	9	21	6·8

	(1.) ☾ semi.		(2.) ☾ hor. par.		Latitude.
Noon............	16′ 10·6″	....	59′ 21·9″		50° 48′ N.
Midnight	16 14·2	....	59 35·2	Red............	11
	3·6		13·3	Red. lat.	50 37 N.
Cor....	2·8	Cor....	10·4		
	16 13·4		59 32·3		
	60	Red....	6·8		
Hor. semi....	973·4		59 25·5		
			60		
		Red. hor. par....	3565·5		

	(3.) ☾'s decl.	(6.) ☾'s RA.		(4.) RA mean sun.
9^h	6° 4′ 44·2″ S.	.........23^h 4^m $13{\cdot}13^s$	25	10^h 12^m $15{\cdot}93^s$
10^h	5 47 50·8 S.	.........23 6 20·74	Cor.	1 32·17
	16 53·4	2 7·61		10 13 48·10
Cor....	5 56·0	60		
	5 58 48·2 S.	127·61		

(5.) Star's RA and decl.
RA......23^h 6^m $2{\cdot}68^s$
Decl. ....6 54 33·8″ S.

To find star's hour-angle (see fig. p. 197).

Mean time at place.........	9^h	16^m	$42{\cdot}6^s$	$=\text{Q}m$
RA mean sun	10	13	48·1	$=\text{A}m$
RA meridian (24)	19	30	30·70	$=\text{A}m\text{Q}$
Star's RA.....................	23	6	2·68	$=\text{AQR}$
	20	24	28·02	
	24			
∴ star's hour-angle=	3	35	31·98	$=\text{RQ}$

Rule (*a*). To compute $\frac{1}{2}(A+A')$.

(1.)

Log. red. hor. par.....................3·552120
„ cos. red. lat. ....9·802435
„ sin. star's hour-angle ....9·907314
„ const. log.........................8·522879

$\therefore$ log. c=1·784748

„ sec. star's decl..................0·003165

„ $\frac{1}{2}(A-A')$ ....1·787913

$\therefore \frac{1}{2}(A-A')=$ 61·37

or=		1	1·37
A=	3	35	31·98
$\therefore \frac{1}{2}(A+A')$=	3	34	30·61

Rule (*b*). To compute the par. in decl. $M-N$.

(2.)

Log. red. hor. par. ......3·552120
„ sin. red. lat. ......9·888133
„ cos. star's decl. ...9·996835
„ M...................3·437088

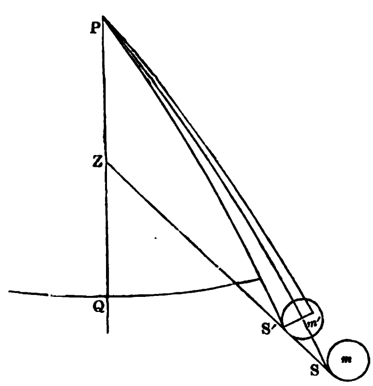
$\therefore M=2736·1$

(3.)

Log. red. hor. par. ......3·552120
„ cos. red. lat. ......9·802435
„ sin. star's decl. ...9·080198
„ cos. $\frac{1}{2}(A+A')$......9·773080
„ N....................2·207833

$\therefore N=161·4$

Diagram for this Example.



Rule (*c*). To determine the signs of M and N (see *Trig.* Part I. art. 33).

$$M-N = \overset{+}{\text{hor. par.}} \cdot \overset{+}{\text{sin. lat.}} \cdot \overset{+}{\text{sin. pol. dist.}}$$
$$- \overset{+}{\text{hor. par.}} \cdot \overset{+}{\text{cos. lat.}} \cdot \overset{-}{\text{cos. pol. dist.}} \cdot \overset{+}{\text{cos.}\ \tfrac{1}{2}(A+A')}.$$

Putting the proper signs over the factors, we see that M is $+$, and $-$ N is $+$.

$$\therefore \text{M} = 2736{\cdot}1 +$$
$$-\text{N} = \underline{161{\cdot}4} +$$
$$\therefore \text{M}-\text{N} = 2897{\cdot}5$$

or par. in decl. $= \quad 48' \quad 17{\cdot}5'' = \text{PS}-\text{PS}'$ (fig. p. 201)

star's decl. $= 6 \quad 54 \quad 33{\cdot}8 = \text{PS}-90^\circ$

$\therefore$ true decl. of point of moon's limb observed $\Big\} = 6 \quad 6 \quad 16{\cdot}3$ S. $= \text{PS}'-90^\circ$.

Rule (*d*). To compute SPS′ or A—A′, the parallax in right ascension.

(4.)

Log. C ....1·784748
„ sec. (PS′—90°)...0·002469
0·301030

log. (A—A′)..............2·088247

∴ A—A′= 122·53ˢ

or= 2ᵐ 2·53=parallax in RA.

star's RA=23 6 2·68=RA of app. point of contact.

RA of S=23 4 0·15=RA of true point of contact.

Rule (*e*). To calculate the angle S′Pm', the moon's semi. in right ascension.

6° 6′ 16·3″ S.	0·002469
5 58 48·2 S.	0·002366
7 28·1	3·152646
60	2·720407
448·1	7·647818
973·4	2)3·525706
1421·5	1·762853
525·3	

semi. in RA= 57·92ˢ=m'PS′

RA of point S′=23ʰ 4ᵐ 0·15

∴ RA of moon's center m' at time of observation=23 4 58·07

Rule (*f*). To calculate the corresponding time at Greenwich when the moon's right ascension=23ʰ 4ᵐ 58·07ˢ.

RA at observation ...23ʰ 4ᵐ 58·07ˢ		Log. 44·94	1·652633
RA Aug. 25, at 9ʰ ...23 4 13·13		Const. log.	3·556302
44·94			5·208935
		Log. 127·61	2·105885
		„ x..........................	3·103050

$\therefore x = 1267{\cdot}8^s$, or=	0^h	21^m	$7{\cdot}8^s$
	9		
Greenwich, Aug. 25...	9	21	7·8
R. N. Coll. Aug. 25...	9	16	42·6
Long. in time		4	25·2 W.

If the Greenwich time had come out very differently from the assumed Greenwich date, it might have been necessary to recalculate the RA of mean sun, the moon's decl., hor. semi., and hor. par., using the computed date as a new Greenwich date: but this is seldom required; the element that is most affected by an error in the Greenwich date is the moon's declination (see note, p. 196).

The blank form in the next page will considerably reduce the labour of working out an occultation, as the student will see that several of the logarithmic quantities, placed in a horizontal line, can be taken out at the same opening of the tables.

The following is the order in which the form should be filled up:

1. Get a Greenwich date.
2. Find moon's semidiameter in seconds for Greenwich date.
3. „ „ horizontal parallax for Greenwich date (corrected by Inman's tables (*h*), or by formula, p. 129).
4. Latitude, corrected by table (*h*), or by formula, p. 122.
5. Right ascension of mean sun for Greenwich date.
6. Moon's horary change in right ascension.
7. Moon's declination for Greenwich date.
8. Take out star's RA and decl. for the day of the month.
9. Find the star's hour-angle, either by a diagram or by the following formula deduced from Prob. IX. p. 34: viz. star's hour-angle= mean time + RA mean sun—star's RA.

Then proceed to compute the several quantities by means of the rules (*a*), (*b*), (*c*), (*d*), (*e*), and (*f*): namely, by rule (*a*) compute $\frac{1}{2}(A+A')$; by (*b*) compute parallax in decl. M—N; by (*c*) determine the algebraic signs of M and N by the rule in the author's *Trigonometry*, Part I.; by (*d*) compute parallax in RA; by (*e*) compute semi. in RA; by (*f*) compute Greenwich mean time at the moment of observation—the difference between which and the mean time at the place at the time of the observation, obtained from an altitude of a heavenly body, as in the rule for chronometer, or otherwise, is the longitude of the observer.

This method of determining the longitude by an occultation will be found by the student, after a little practice, much easier and more certain than by a lunar observation.

FORM FOR FINDING LONGITUDE BY OCCULTATION OF STAR.

1. Greenwich date.

Mean time at place . .	
Long. in time . . .	
Gr. date	

2. Moon's semi.

At	
At	
In seconds . .	

3. Hor. par.

At	
At	
Red.	
In seconds. .	

4. Latitude.

Red.	
Red. Lat. . .	

5. RA mean sun.

At	
Cor.	

6. Moon's RA.

At ( ) hour. . . .	
At ( ) hour. . . .	
Hourly change . . .	
In seconds	

7. Moon's decl.

At ( ) hour . .	
At ( ) hour . .	
	$=\delta_1$

8.

Star's RA . . .	
Star's decl. . . .	

9. To find star's hour-angle.

Mean time at place .	
+	
RA mean sun . . .	
RA meridian . . .	
−	
Star's RA	
∴ star's hour-angle .	$=$ A

1.

Log. hor. par. (in seconds)	
„ cos. red. lat. . . .	
„ sin. A	
Const. log.	8·522879
Log. C	
+	
„ sec. star's decl. . .	
„ $\frac{1}{2}$(A—A′) nearly . .	
∴ $\frac{1}{2}$(A—A′)	
sub. from A	
∴ $\frac{1}{2}$(A+A′) nearly . . .	

(*a*) M has the same sign (N or S) as star's decl., if lat. and star's decl. have the same name; otherwise a different name.
(*b*) N has a different name from star's decl.; except when star's hour-angle is between 6^h and 18^h.
(*c*) Add if like, subtract if unlike, names.
(*d*) + if west, — if east, of meridian.
(*e*) + if emersion, — if immersion.

Star's RA	
(*d*) ± par. in RA. . . .	
(*e*) ± semi. in RA . . .	
∴ RA ☾'s center . . .	
—	
☾'s RA at hour of Gr. date	
Diff.	
In seconds=D′	

2.

Log. hor. par. . . .	
„ sin. red. lat. .	
„ cos. star's decl. .	
„ sum	
Nat. No.=M (*a*) . .	
=N (*b*) . .	
Par. in decl. . . .	
Or in min. & sec. (*c*) ±	
Star's decl.	
∴ δ	
☾'s decl. δ_1 . . .	
Diff.	
In seconds	
☾'s semi. in seconds .	
Sum=S.	
Diff.=D.	
Log. D′	
+Const. log. . . .	3·556302
Sum.	
—Log. hourly change	
Log. diff.	
Nat. No.	
In min. and sec. . .	
+hour of Gr. date .	
∴ Green. mean time.	
Ship mean time . .	
∴ long. in time . .	

3.

Log. hor. par. . .	
„ cos. red. lat. .	
„ sin. star's decl.	
„ cos. $\frac{1}{2}$(A+A′) .	
Log. N.	
∴ N	
Log. sec. δ . . .	
„ sec. δ_1 . . .	
„ S.	
„ D.	
Const. log. . . .	7·647818
	2)
Log.	
Nat. No.	
In min. and sec. .	
=semi. in RA . .	

4.

Log. C. . . .	
„ 2 . . .	0·301030
„ sec. δ . .	
Sum	
Nat. No. . . .	
In min. and sec.	
=par. in RA. .	

EXAMPLES FOR PRACTICE.

268. January 7, 1836, at $10^h\ 45^m\ 53{\cdot}2^s$ mean time, in latitude 52° 8′ 28″ N. and estimated longitude 1^m W., observed the immersion of ν Leonis (east of meridian); to determine the longitude.

Elements from *Nautical Almanac*, corrected for a Greenwich date, Jan. 7, at $10^h\ 47^m$:

(1.) Moon's hor. semi. 15′ 16·1″; (2.) Moon's red. hor. par. 55′ 54·9″; (3.) Moon's decl. 15° 49′ 40″ N.; (4.) RA mean sun, $19^h\ 6^m\ 8{\cdot}5^s$; (5.) Star's RA, $10^h\ 23^m\ 26{\cdot}4^s$, star's decl. 14° 58′ 38·8″ N.; (6.) Moon's RA at 10^h, $10^h\ 18^m\ 55{\cdot}5^s$; at 11^h, $10^h\ 20^m\ 58{\cdot}5^s$. *Ans.* Long. $0^h\ 2^m\ 7^s$ W.

269. Feb. 12, 1835, at $2^h\ 29^m\ 40{\cdot}5^s$ A.M. mean time, in lat. 50° 49′ N. and estimated longitude $4^m\ 17^s$ W., observed the immersion of γ Cancri (west of meridian); to determine the longitude.

Elements from *Nautical Almanac*, corrected for a Greenwich date, Feb. 11, at $14^h\ 34^m$:

(1.) Moon's hor. semi. 15′ 46·6″; (2.) Moon's red. hor. par. 57′ 47·6″; (3.) Moon's decl. 22° 42′ 59·1″ N.; (4.) RA mean sun, $21^h\ 25^m\ 42{\cdot}61^s$, (5.) Star's RA, $8^h\ 33^m\ 44{\cdot}21^s$, star's decl. 22° 3′ 25·8″ N.; (6.) Moon's RA at 14^h, $8^h\ 33^m\ 24{\cdot}37^s$; at 15^h, $8^h\ 35^m\ 49{\cdot}31^s$. *Ans.* Long. $0^h\ 3^m\ 48^s$ W.

270. Feb. 12, 1835, at $2^h\ 10^m\ 20^s$ A.M. mean time, in lat. 55° 57′ 20″ N. and estimated longitude $12^m\ 43{\cdot}6^s$ W., observed the immersion of γ Cancri (west of meridian); to determine the longitude.

Elements from *Nautical Almanac*, corrected for a Greenwich date, Feb. 11, at $14^h\ 23^m$:

(1.) Moon's hor. semi. 15′ 47·6″; (2.) Moon's red. hor. par. 57′ 50·6′; (3.) Moon's decl. 22° 44′ 12″ N.; (4.) RA mean sun, $21^h\ 25^m\ 40{\cdot}8^s$; (5.) and (6.) see last example. *Ans.* Long. $0^h\ 12^m\ 40^s$ W.

By means of the blank form for finding the longitude from an occultation of a star, the labour of working out the observation will be considerably diminished. The rule and form may be used, with some slight alterations, for determining the longitude from the observed occultation of a *planet;* the necessary changes to be observed are the following:

1. Instead of the moon's red. hor. par., use the difference between the moon's red. hor. par. and the planet's horizontal parallax.

2. Instead of the moon's equatorial semi., use the *sum* of the moon's equatorial semi. and the planet's semi. if the interior contact is observed, and their *difference* if the exterior contact is observed. This rule may be neglected if the mean of the two contacts, or the time of contact of the planet's center with the moon's limb, is used, as in the first example following.

EXAMPLES OF AN IMMERSION AND AN EMERSION OF A PLANET.

271. June 20, 1853, observed at Auckland, New Zealand, in lat. 36° 50′ 41″ S. and estimated longitude 11^h 39^m 16^s E. (with telescope (G. H. Jones), smallest inverting power, focal length 4 feet, aperture 2·7 inches), the immersion of the eastern and western limbs of Jupiter (west of meridian) at the following times by chronometer:

Contact of 1st limb...............11^h 45^m $42{\cdot}2^s$ P.M
,, 2d ,, ...11 47 40·7

The error of chronometer on mean time at the place was 23^m $52{\cdot}37^s$ fast; to determine the longitude.

Elements from the *Nautical Almanac*, corrected for a Greenwich date, June 19, 23^h 43^m 33^s, corresponding to the mean of the times of contact, namely, June 20, at 11^h 22^m $48{\cdot}88^s$ P.M. at the place:

(1.) Moon's hor. semi. 16′ 40·18″; (2.) moon's red. hor. par.—planet's hor. par., 60′ 57·29″; (3.) moon's declination, 22° 48′ 1·6″ S.; (4.) RA mean sun, 5^h 54^m $23{\cdot}62^s$; (5.) planet's RA, 17^h 9^m $32{\cdot}02^s$, planet's decl. 22° 26′ 9·05″ S.; (6.) moon's RA at 23^h, 17^h 6^m $36{\cdot}54^s$; at 24^h, 17^h 9^m $17{\cdot}83^s$.

Ans. Long. 11^h 39^m 23^s E.

272. June 21, 1853, at 0^h 25^m $40{\cdot}83^s$ A.M., observed the emersion of the planet's western limb (west of meridian) at the same place; to determine the longitude.

Elements from *Nautical Almanac*, corrected for a Greenwich date, June 20, 0^h 46^m $24{\cdot}83^s$.

(1.) Moon's hor. semi. + planet's semi. (16′ 41·2″ + 21·6″) 17′ 2·8″; (2.) moon's red. hor. par.—planet's hor. par., 61′ 1·1″; (3.) moon's declination, 22° 55′ 6·1″ S.; (4.) RA mean sun, 5^h 54^m $33{\cdot}86^s$; (5.) planet's RA, 17^h 9^m $30{\cdot}61^s$, planet's decl. 22° 26′ 7·7″; (6.) moon's RA at 0^h, 17^h 9^m $17{\cdot}83^s$; change to next hour, $161{\cdot}5^s$.

Ans. Long. = 11^h 39^m 25^s E.

The above occultation of the planet Jupiter was observed by Captain Byron Drury, while employed in his survey of New Zealand. The longitude of his station determined by other methods was 11^h 39^m 16^s E.

THE END.

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